King Saud University
Department Of Mathematics.
M-203 [Final Examination]
(Differential and Integral Calculus)
(II-Semester 1435/1436)

Max. Marks: 40 Time: 3 hrs

Marking Scheme: Q.No:1[4+4+4], Q.No:2[4+4+4], Q.No:3[4+4+4+4]

Q. No: 1 (a) Find the sum of the series $\sum_{n=0}^{\infty}\left[\frac{1}{(2n+1)(2n+3)}\right]$.

(b) Find the **interval of convergence** and **radius of convergence** of the power series $\sum_{n=1}^{\infty}\frac{(2x-5)^n}{\ln(n+1)}$.

(c) Find the power series representation for the function $f(x)=\tan^{-1}(x)$ and use its first three non-zero terms to calculate $\tan^{-1}(0.1)$.

Q. No: 2 (a) Evaluate the integral $\iint_R (x+2y)\,dA$, where R is the plane region bounded by the graphs of $y=2x^2$ and $y=1+x^2$.

(b) Find the volume of the solid in the first octant bounded by the graphs of $x+y=6$ and $z=9-y^2$.

(c) Evaluate $\iiint_Q \sqrt{x^2+y^2+z^2}\,dxdydz$, where Q is the solid bounded by the sphere $x^2+y^2+z^2=2z$.

Q. No: 3 (a) Use Green's theorem to evaluate $\oint_C (y+e^{\sqrt{x}})dx+(2x+\cos y^2)dy$, where C is the boundary of the region enclosed by the parabolas $y=x^2$ and $x=y^2$.

(b) Show that the line integral $\int_{(0,\pi/2)}^{(1,\pi)}(e^x\cos y)dx-(e^x\sin y)dy$ is independent of path, and find its value.

(c) Let S be the surface of the solid region bounded by the graphs of the equations $z=6-x^2-y^2$ and $z=x^2+y^2$. Use divergence theorem to find the flux of $\vec{F}(x,y,z)=2x\,\vec{i}-y\,\vec{j}+z\,\vec{k}$ through the surface S.

(d) Verify Stokes' theorem for the force $\vec{F}(x,y,z)=-x\,\vec{i}+y\,\vec{j}+z\,\vec{k}$ and the surface S where S is the portion of the sphere $x^2+y^2+z^2=4$ cut off by the plane $z=\sqrt{3}$.

Q: 1 (a) $\dfrac{1}{(2n+1)(2n+3)} = \dfrac{A}{2n+1} + \dfrac{B}{2n+3}$ [Marks: 4] (1)

$$= \frac{A(2n+3) + B(2n+1)}{(2n+1)(2n+3)}$$

$1 = A(2n+3) + B(2n+1) \Rightarrow A = \frac{1}{2},\ B = -\frac{1}{2}$ (1)

$$S_n = \left[\frac{1/2}{1} - \frac{1/2}{3}\right] + \left[\frac{1/2}{3} - \frac{1/2}{5}\right] + \cdots + \left[\frac{1/2}{2n+1} - \frac{1/2}{2n+3}\right]$$ (1)

$$= \frac{1}{2} - \frac{1}{2(2n+3)} \Rightarrow \lim_{n\to\infty} S_n = \frac{1}{2}$$ (1)

(b) $\left|\dfrac{u_{n+1}}{u_n}\right| = \left|\dfrac{(2x-5)^{n+1}}{\ln(n+2)} \times \dfrac{\ln(n+1)}{(2x-5)^n}\right| = \dfrac{\ln(n+1)}{\ln(n+2)}\left|2x-5\right|$ [Marks: 4]

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = |2x-5|$$

for c'gence $|2x-5| < 1 \Rightarrow 2 < x < 3$ (1)

c'gence at $x = 2$

$\sum_{n=1}^{\infty} (-1)^n \dfrac{1}{\ln(n+1)}$

c'gt by AST (1)

c'gence at $x = 3$

$\sum_{n=1}^{\infty} \dfrac{1}{\ln(n+1)}$ d'gt by

B.C.T $\sum_{n=1}^{\infty} \dfrac{1}{n+1}$ (1)

Interval of c'gence $2 \le x < 3$

Radius of c'gence $\rho = \dfrac{3-2}{2} = \dfrac{1}{2}$ (1)

(c) $\tan^{-1} x = \int_0^x \frac{1}{1+t^2}\,dt = \int_0^x [1 - t^2 + t^4 - \cdots]\,dt$ [Marks: 4]

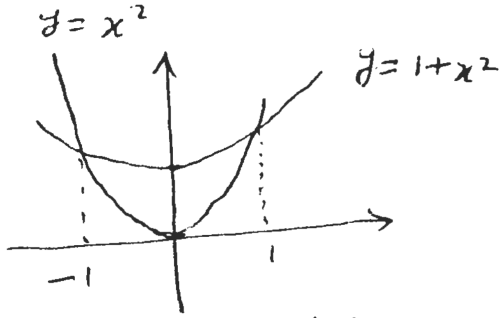
$$= \left[t - \frac{t^3}{3} + \frac{t^5}{5} \cdots\right]_0^x = x - \frac{x^3}{3} + \frac{x^5}{5} \cdots \quad (2)$$

$$\tan^{-1}(0.1) \approx (0.1) - \frac{(0.1)^3}{3} + \frac{(0.1)^5}{\cancel{3}\,5}$$

$$= 0.1 - 0.000333 + 0.000002 \quad (2)$$

$$\approx 0.099\cancel{9}669$$

Q: 2 (a) $\iint_R (x+2y)\,dA$ [Marks: 4]



$-1 \le x \le 1$

$x^2 \le y \le 1+x^2$

$$= \int_{-1}^{1}\int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx \quad (2)$$

$$= \int_{-1}^{1} \left[xy + y^2\right]_{x^2}^{1+x^2} dx$$

$$= \int_{-1}^{1} \left[(x^3 + x + 1 + 2x^2 + x^4) - (x^3 + x^4)\right] dx$$

$$= \int_{-1}^{1} [x + 1 + 2x^2]\,dx = \left.\frac{x^2}{2} + x + \frac{2}{3}x^3\right]_{-1}^{1} \quad (1)$$

$$= \left(\frac{1}{2} + 1 + \frac{2}{3}\right) - \left(\frac{1}{2} - 1 - \frac{2}{3}\right)$$

$$\cancel{= 2 + \frac{4}{3} = \frac{10}{3}} \quad (1)$$

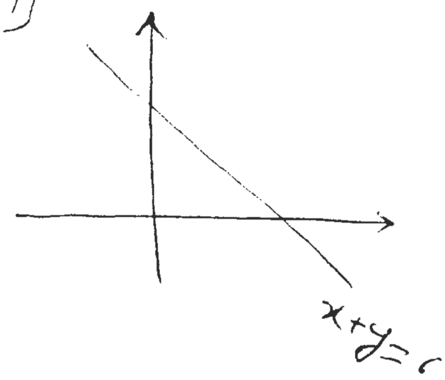
$$= \frac{32}{15} \approx 2.13$$

(b) $\int_0^6 \int_0^{6-x} \int_0^{9-y^2} dz\, dy\, dx$ [Marks: 4]

$dx\, dz\, dy$ (2)

$\int_0^3 \int_0^{9-y^2} \int_0^{6-y} dx\, dz\, dy$

$= \int_0^{6} \int_0^{6-x} (9-y^2)\, dy\, dx$ (3 written above the upper limit 6)



$= \int_0^6 \left[9y - \frac{y^3}{3}\right]_0^{6-x} dx$

$= \int_0^6 \left[9(6-x) - \frac{1}{3}(6-x)^3\right] dy$

$= \int_0^6 \left[54 - 9x - \frac{1}{3}(216 - 108x + 18x^2 - x^3)\right] dx$

$= \int_0^6 \left[54 - 9x - 72 + 36x - 6x^2 + \frac{1}{3}x^3\right] dx$ (1)

$= \int_0^6 \left[-18 + 27x - 6x^2 + \frac{1}{3}x^3\right] dx$

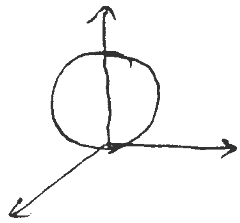
$= -18x + \frac{27}{2}x^2 - \frac{6x^3}{3} + \frac{1}{12}x^4 \Big]_0^6$

$= -18(6) + \frac{27}{2}(36) - 2(216) + 108 = -108 + 486 - 432 + 108$

$= 54$ (1) $\frac{351}{4} \approx 87.75$

Q: 2 (c) [Marks: 4]

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2\cos\varphi} \rho\,\rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta \quad (3)$$



$2\rho\cos\varphi = \rho^2$

$\rho = 2\cos\varphi$

$$= \int_0^{2\pi}\int_0^{\pi/2} \left.\frac{\rho^4}{4}\right]_0^{2\cos\varphi} \sin\varphi \, d\varphi\, d\theta$$

$$= \frac{1}{4}\int_0^{2\pi}\int_0^{\pi/2} 16\cos^4\varphi \sin\varphi \, d\varphi\, d\theta$$

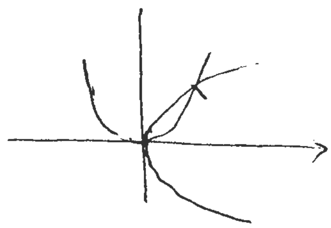
$$= -4\int_0^{2\pi} \left.\frac{\cos^5\varphi}{5}\right]_0^{\pi/2} d\theta = +\frac{4}{5}(2\pi) \quad (1)$$

$$= \frac{8}{5}\pi \approx 5.024$$

Q: 3 (a) [Marks: 4]

$$\oint_C (y + e^{\sqrt{x}})\,dx + (2x + \cos y^2)\,dy$$

$$= \int_0^1\int_{x^2}^{\sqrt{x}} (2-1)\,dy\,dx \quad (3)$$



$0 \le x \le 1$

$x^2 \le y \le \sqrt{x}$

$$= \int_0^1 (\sqrt{x} - x^2)\,dx = \left.\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3} = 1/3 \quad (1)$$

(b) $\frac{\partial M}{\partial y} = -e^{-x}\cos y = \frac{\partial N}{\partial x}$ [Marks: 4]

$f_x = e^x\cos y \Rightarrow f(x,y) = e^x\cos y + c(y)$ —— (1)

$\Rightarrow f_y(x,y) = -e^x\sin y + c'(y)$

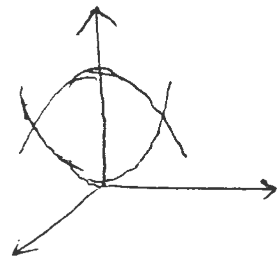
$\Rightarrow -e^x\sin x = -e^x\sin y + c'(y)$

$\Rightarrow c(y) = \text{Constant}$

Putting in (1) $\quad f(x,y) = e^x\cos y + c \quad (3)$

$$\Rightarrow \left[e^x\cos y + c\right]_{(0,\pi/2)}^{(1,\pi)} = e(-1) - 0 = -e \quad (1)$$

Q3(c) $\iiint_Q \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}\right) dV$ [Marks: 4]



$$= \iiint_Q (2-1+1)\, dV$$

$$= 2\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{6-r^2} r\, dz\, dr\, d\theta \quad (3)$$

$$= 2\int_0^{2\pi}\int_0^{\sqrt{3}} r(6-r^2-r^2)\, dr\, d\theta = 2\int_0^{2\pi}\int_0^{\sqrt{3}} (6r-2r^3)\, dr\, d\theta$$

$$= 2\int_0^{2\pi} \left[3r^2 - \frac{1}{2}r^4\right]_0^{\sqrt{3}} d\theta = 2\left(9-\frac{9}{2}\right)2\pi = 18\pi \quad (1)$$

Q:3(d) $\oint_C \bar{F}\cdot d\bar{r} = \oint_C -x\,dx + y\,dy + z\,dz$ [Marks: 4]

$x = \cos t,\ y = \sin t,\ z = \sqrt{3}$, $0 \le t \le 2\pi$
$dx = -\sin t\, dt$, $dy = \cos t\, dt$, $dz = 0$

$$= \int_0^{2\pi} \cos t \sin t\, dt + \sin t \cos t\, dt \quad (2)$$

$$= 2\int_0^{2\pi} \sin t \cos t\, dt = 2\left[\frac{\sin^2 t}{2}\right]_0^{2\pi} = 0$$

$\iint_S (\text{curl}\,\bar{F})\cdot\bar{n}\, ds$

$$\text{curl}\,\bar{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -x & +y & z \end{vmatrix}$$

$$= i(0-0) - j(0-0) + k(0-0)$$

$$= \langle 0,0,0\rangle$$

$\Rightarrow \iint_S (\text{curl}\,\bar{F})\cdot\bar{n}\, ds = 0$

(2)

Verified