Exercises 201 Math

HOME WORK PROBLEMS

Title of the book used for this course is: **Calculus**, The classic edition, by Swokowski.

In this course we cover chapters 11, 16 and 17, but we start with chapter 11 ,then finally chapter 16 and chapter 17 , also section 14.6 should be read by the students.

Chapter11: Infinite series

11.1 Sequences

11. 2 Convergent or Divergent Series

11. 3 Positive -Term Series

11. 4The Ratio and Root Tests

11. 5 Alternating Series and Absolute Convergence

11. 6 Power Series

11. 7 Power Series Representations of Functions

11. 8 Maclaurin and Taylor Series

Exercises:-

11. 1: 3,5,7,11,12,13,16,17,18,23,24,28,29,30,31,32,33,34,36,37,39,41,42.

11. 2: 2,4,5,6,8,10,14,15,18,20,25,28,30,34,37,39,42,43,45,46.

11.3: 2,3,5,7,9,11,14,15,16,18,20,22,24,25,30,31,33,34,35,39,40,42,43,45,46,51,52,57,58.

11. 4: 2,4,6,8,10,11,14,15,18,20,21,23,25,27,28,29,**31,33,35,38.**

11. 5: 2,3,5,7,9,10,12,13,16,18,20,21,22,27,29,32,33,35,38,41,43,45,46.

11. 6: 5,6,7,14,15,19,23,25,27,30,35,36,41,42.

11. 7**:** 2,4,6,7,10,13,14,16,19,22,25,29,30,32,33,34,37.

11. 8: 2,4,8,10,13,15,18,19,21,26,29,32,34,36,38,39,42.

Chapter 16:

Section 16.1: Functions of Several Variables

Do the following problems from the book

1, 3, 5, 8, 9, 14, 15, 17, 23, 37, 40, 41, 47.

Section 16.2: Limits and Continuity

1. Do the following problems from the book

   3, 5, 6, 9, 12, 14, 16, 19, 20, 25, 28, 29, 32, 36, 38, 42.

2. Find the following limits, if they exist:

1) $lim_{(x,y,z)\to(0,0,0)} \dfrac{zy^2}{x^2+y^2+z^2}$, 2) $lim_{(x,y)\to(2,1)} \dfrac{(y-1)(x-2)^2}{(y-1)^3+(x-2)^3}$.

3) $lim_{(x,y)\to(0,0)} \dfrac{xy^3}{x^3+y^6}$, 4) $lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^4+y^2}$, 5) $lim_{(x,y)\to(0,0)} \dfrac{10xy}{5x^3+2y^3}$

6) $lim_{(x,y,z)\to(0,0,0)} \dfrac{y^3+x^3 sinz^3}{x^2+y^2+z^2}$, 7) $lim_{(x,y)\to(0,0)} \dfrac{x^3-x^2y+xy^2-y^3}{x^2+y^2}$

8) $lim_{(x,y)\to(0,0)} [\dfrac{4x^2y}{x^4+y^2}+\dfrac{y^4}{x^2+y^2}]$, 9) $lim_{(x,y)\to(1,-1)} \dfrac{2x-y}{x^2+y^2}$

3. Discuss the continuity of the following functions on their domain:

   1. $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

   2. $f(x,y,z) = \begin{cases} \frac{x^3+y^3+z^3}{x^2+y^2+z^2}, & (x,y,z)\neq(0,0,0) \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

   3. $f(x,y,z) = \begin{cases} \frac{xz-y^2}{x^2+y^2+z^2}, & (x,y,z)\neq(0,0,0) \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

   4. $f(x,y) = e^{x^2+5xy+y^3}$.

   5. $h(x,y) = sin(\sqrt{y-4x^2})$.

   6. $k(x,y,z) = ln(36-4x^2-y^2-9z^2)$.

Section16.3: Partial Derivatives

1. Do the following problems from the book

   4, 6, 8, 12, 13, 16, 17, 21, 23, 27, 29, 32, 34, 36, 39, 42, 44, 47.

2. Do the following problems;

1. Using the definition, find f_x, f_y of the function

$$f(x,y) = 3x^2 - 2xy + y^2.$$

2. Discuss the continuity of the function f at (0,0), where

$$f(x,y) = \begin{cases} \frac{\sin xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Does f_x and f_y exist at $(0,0)$.

3. Let $f(x,y) = \begin{cases} \frac{x^2 y}{x^3+y^3}, & x^3+y^3 \neq 0 \\ 0, & x^3+y^3 = 0 \end{cases}$

Find f_x, f_y at (0, 0), if they exist

4. Let $f(x,y) = \begin{cases} \frac{3x^2}{2x-y} + \frac{y^3}{y-2x}, & y \neq 2x \\ 12, & y = 2x \end{cases}$

Find f_x, f_y at (1, 2), if they exist

5. Let $f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$

Find f_x, f_y at (1, 1) and at (0, 0), if they exist

6. Let $f(x,y) = e^{x-y}\sin(x+y)$. Show that

$$(f_x)^2 + (f_y)^2 = \frac{2(f(x,y))^2}{\sin^2(x+y)}.$$

Section 16.4: Increments and Differentials

1. Do the following problems from the book

   2, 9, 11, 12, 16, 20, 24(b), 41, 42.

2. Use the differential to approximate the change in the function

$$w = f(x, y, z) = x^2 ln\ (z^2 + y^2)$$

   as (x, y, z) changes from $(1, 2, 3)$ to $(0.9, 1.9, 3.1)$.

3. Use the differential to approximate the change in the function

$$w = f(x, y) = yx^{\frac{2}{5}} + x\sqrt{y}$$

   as (x, y) changes from (52, 16) to (35, 18).

4. Discuss the continuity and the differentiability of the functions in problem 3 and 4 in section 16.3 (part II) as the indicated points.

5. Discuss the differentiability of the function f, where

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

   At (0, 0) and at (-1, 1)

6. Let $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

   Show that f is continuous at (0, 0) but not differentiable at (0, 0).

7. R Let $f(x,y,z) = \begin{cases} \frac{xy^2z}{x^4+y^4+z^4}, & (x,y,z) \neq (0,0,0) \\ 0, & (x,y,z) = (0,0,0) \end{cases}$

   - Show that f_x (0,0,0), f_y(0,0,0) and f_z(0,0,0) exist.
   - Discuss the differentiability of f at (0, 0, 0).

8. Let $f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

   - Show that f_x and f_y exist for all $(x, y) \in R^2$.
   - Show that f_x and f_y are not continuous at point (0, 0).
   - Show that f is not differentiable at (0, 0).

Section 16.5: Chain rule

1. Do the following problems from the book

   2,4,6,10,12,14,18,19,22,26,38,40,42.

2. If $w = f(x, y)$ such that $x = r\cos\theta$, and $y = r\sin\theta$. Find $g(r, \theta)$ such that the equation below holds

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + g(r, \theta)\left(\frac{\partial w}{\partial \theta}\right)^2.$$

3. If $u = f(x, y)$ where $y = e^t$ and $x = e^k$. Find

$$\frac{\partial^2 u}{\partial k^2} + \frac{\partial^2 u}{\partial t^2}.$$

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4. If $w = x^2 + y^2 + z^2$, where $x = rcos\theta$, $y = rsin\theta$ and $z = r$. Use the differential to show that$d\ w = 4rdr$.

5. Let $z = f(x, y)$ be determined implicitly by$yx^2 + z^2 + \cos(xyz) - 4 = 0$. Find

   $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. Then show that

$$2y\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial x} = \frac{xyz\sin(xyz)}{2z - xy\sin(xyz)}.$$

Section 16.8: Extrema of functions of several variables

1. Do the following problems from the book;
   5,11, 15, 20, 23, 24, 26, 30, 31, 32.

2. Find the extrema of the function$f(x, y) = (x - 4)^2 + y^2$, on the region R bounded by $y = 4\sqrt{x}$ and$y = 4x$.
3. Let$f(x, y) = xy + \frac{1}{x} + 1/y$, where$x \neq 0, y \neq 0$. Find the local extrema and saddle points of f if they exist.
4. Find the maximum and the minimum of the function $f(x, y) = x^3 - 3x + y^2$ on the region bounded by $x^2 - 2x + y^2 = 0$

Section 16.9: Lagrange multipliers;

1. **Do the following problems from thebook;**

   1, 2, 3, 11.

Chapter 17: Multiple integral

Section 13.1: Double integral;

1. Do the following problems from the book;

   1 - 10, 13, 16, 18, 19, 20, 21, 23, 25, 26, 27, 29, 31, 32, 33, 37, 38, 39, 43, 44, 50.

2. Sketch the region bounded by the graphs of the given equations, and then evaluate the given integral

   a) $y = x, y = \sqrt{x}, x = 0; \quad \int\int_R \sin y^2 dA.$

   b) $y = x^{3/2}, y = 0, x = 1; \quad \int\int_R y e^{x^2} dA.$

3. Evaluate the double integral

$$\int_0^2 \int_{y/2}^1 e^{x^2} dx\, dy.$$

Section 13.2: Area and Volume;

1. Do the following problems from the book;

   2, 4, 6, 7, 11, 14, 18, 22, 24, 27, 28, 30, 31, 32.

   i. Sketch the region bounded by the graphs of the equation

   $y = \sin x, y = \cos x, x = 0, x = \frac{\pi}{2}$

   Then use the double integral to find its area.

   ii. Sketch the region bounded by the graphs of the equation

   $x = -\sqrt{9 - y^2}, y = -2x + 9, y = -3, y = 3$

   Then use the double integral to find its area.

Section 13.3: Double Integral by Polar Coordinate;

1. Do the following problems from the book;

   1 - 12, 13, 15, 17, 18, 19, 21, 23, 24.

2. Use polar coordinate to evaluate the double integral

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} (x^2 + y^2)^{3/2}\, dy\, dx.$$

Section 13.5: Triple Integral;

1. Do the following problems from the book;

   2, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 23, 26, 28.

2. Sketch the region bounded by the graphs of the equations

   a) $z = x^2 + y^2$, $y + z = 2$.

   b) $z + y^2 = 4$, $x + z = 4$, $x = 0$, $z = 0$.

   c) $z = 9 - y^2$, $z = 0, x = -1$, $x = 2$.

3. Set up a triple integral for the volume of the region in the first octant, bounded above by the cylinder $z = 1 - y^2$ and lying between the vertical planes $x + y = 1$ and $x + y = 3$.