Dynamic Response of a MDOF System subjected to Harmonic and Impulsive Loadings and Free Vibration: An Analytical Approach

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Abstract

The influence of different kinds of dynamic forces on multistory structures is notable. This paper studies the effect of harmonic and impulsive loading and free-vibration on a three-story shear frame structure. A persistent pulse loading was considered at the upper level of the structure, and the resulted mathematical model of the time-dependent multi-level displacements were derived. For harmonic force, normalized response amplitudes under the applied harmonic loading is plotted against the frequency ratio ω/ω_1. These frequency-response curves show three resonance conditions at $\omega=\omega_1$, $\omega=\omega_2$ and $\omega=\omega_3$; at these exciting frequencies, the steady-state response is unbounded. At other exciting frequencies, the vibration is finite and could be calculated from the derived equations. When the structure is excited with harmonic loading over a range of frequencies, the structure experiences resonance at some frequency. Resonance occurs when the frequency of the excitation is equal to the natural frequency of the structure. At the resonant frequency, the structure experiences its largest response as compared to any other frequency of loading. For rectangular pulse force, the time-history of the multi-level displacements were presented. The results indicate that the extreme displacement occurs at the top-level. Additionally, when the persistent pulse loading had been expired, the maximum top-level displacement response was obtained. As the impact of the preliminary conditions is essential, the final response of the structure to the pulse loading is not being "steady-state. For free vibration, floor displacements versus t/T_1 was plotted and observed the relative contributions of the three vibration modes to the response that was produced by the given initial displacement. Although all three modes contribute to the response, the dominant response is due to the first mode since the shape of the given initial displacement is similar to the configuration of the first mode.

Keywords: Harmonic loading, impulsive loading, free vibration, shear frame, MDOF system

INTRODUCTION

Shear frames are used in all the major reinforced concrete buildings and structures in all over the world. These frames are subjected to various static and dynamic loads. The most common static load is the gravity load identified as dead and live loads. Among the dynamic loads, we have high probable wind and wave loads and less likely earthquake, impulsive and harmonic loads. The harmonic loading is an external subjection with a sinusoidal behavior and a specified regularity of occurrence. Due to the probability of resonance, the concern of structural system response to harmonic loading is of substantial importance. The resonance Phenomena is quite likely to develop when the natural frequency of the structural system is approaching that for the external loading conditions. This phenomena is accompanied by infinite movements of the structure, which cause plastic deformations and also may lead catastrophic devastation.

Substantial research exists on the response of shear frames and buildings subjected to wind, wave and earthquake loads. Tuken [7] proposed an analytical procedure to estimate the lateral displacement of a mixed (frame + shear wall) structure subject to earthquake forces. The analytical procedure was then applied to a 3-D building with different heights. The analytical lateral displacements matched reasonably well with SAP2000 results. Tuken and Siddiqui [8] proposed a simple-to-apply analytical method based on "dual system" concept to determine the amount of shear walls which can satisfy the strength, stiffness and ductility requirements imposed by the seismic codes on RC moment resisting frame buildings. The proposed methodology was then applied to a 10-story RC building containing shear walls. It was shown that the amount of shear walls which is enough to satisfy the strength requirements also fulfills the stiffness criteria (i.e. story drift limitation) required by the seismic codes. Dahish *et al.* [9] studied the influence of shear walls in controlling the lateral response of the RC frame building by varying the shear wall thicknesses, height, configuration and opening locations. The seismic load was considered from one direction only while studying the effect of the first two parameters (i.e., thickness and height) as the building and shear wall arrangements were symmetric along the two orthogonal directions. On the other hand; since the opening location and the shear wall configuration were not symmetric in the two orthogonal directions, the seismic load was considered from the two directions separately in studying these two parameters. As a result, authors obtained the optimum amount and the most appropriate arrangement of shear walls for a given RC frame building against a specified earthquake loading. Tuken and Siddiqui [10] proposed a simple-to-apply analytical method to determine the number of shear walls

necessary to make reinforced concrete buildings seismic-resistant against moderate to severe earthquakes. The method is based on the following design strategy: (i) The total design base shear must be resisted by shear walls; (ii) equal amounts of shear walls must be placed in both orthogonal directions of the structure; and (iii) the moment resisting frame elements, which are beams and columns, must independently be able to resist 25% of the total design base shear. In this design methodology, the optimum amount of shear walls in such a system was obtained by equating the total design base shear to the total shear resistance of all shear walls in one direction. Since the seismic load may come from any direction, an equal amount of shear walls should be provided in both orthogonal directions. A particular method is also presented to check the stiffness requirement of any seismic code for the prescribed amount of shear walls. The complete analytical procedure was implemented on a ten-story RC building. Tuken and Siddiqui [11] proposed an analytical method based on the dual-system concept and the Saudi Building Code (SBC 301, SBC 304) provisions to determine the optimum quantity of shear walls. For displacement and curvature ductility of RC shear wall-moment resisting frame buildings, they also outlined a detailed plastic analysis based on the assumption that plastic hinges form at the base of the shear walls. The proposed methodology was then applied to a 10-story RC building. Authors showed that the optimum amount of shear walls, which is enough to satisfy the strength requirements, also fulfills the stiffness requirement of SBC and the ductility requirements imposed by SBC can easily be satisfied by using the same amount of shear walls. Tuken *et al.* [12] presented a detailed procedure for reliability assessment of RC shear wall-frame building subjected to earthquake loading against serviceability limit state. Monte

Carlo simulation was used for the reliability assessment. The procedure was then implemented on a 10-story RC building to demonstrate that the shear walls improve the reliability substantially. The annual and lifetime failure probabilities of the studied building were estimated by employing the information of the annual probability of earthquake occurrence and the design life of the building. A risk-based cost assessment methodology that relates the total cost of the building with both the target reliability and the structural lifetime failure probability was then presented. The structural failure probability by considering human errors was also studied. It was shown that concrete strength and human error in the estimation of total load changes the reliability keenly. Elhelloty [2] carried out the modal and transient analysis to study the effect of lateral loads resisting systems on the response of buildings subjected to dynamic loads. In his study, two multi-story (with 3- and 5-levels) steel structures were studied for the probability of resonance, mode shapes and internal stresses. Two (with or without) bracing systems (shear walls of steel and laminated composite plates) were considered. A comparative study is conducted to evaluate the effect of lateral loads resisting systems on the performance of buildings subjected to dynamic loads using the finite element system ANSYS16. Jayatilake *et al.* [3] examined three-dimensional nonlinear dynamic responses of typical tall RC buildings (with 20-stories) with and without setbacks under blast loading. This structure had been designed for dead, live and wind loads only. The focus of the investigation was on the effects of lateral impulsive loading, considering the maximum displacements, inertia forces, and other internal loadings. Additionally, the prospect of progressive collapse following the formation of plastic centers were reported. A commercial 3-D finite element-based software was utilized to simulate the time-history of the tall building. The results indicates that structures with setbacks (to guard the setbacks level and the below) exhibits an improved structural performance (with regard to maximum drifts) compared to that without setbacks. Moreover, the report established that rate of change of angular velocities are directly correlated to the phases of the angular modes. Abrupt changes in moments and shears are experienced near the levels of the setbacks. Typical twenty story tall buildings with shear walls and frames that are designed for only normal loads perform reasonably well, without catastrophic collapse, when subjected to a blast that is equivalent to 500 kg TNT at a standoff distance of 10 m. Patel *et al.* [4] reviewed the work carried out in the past few years on blast effects on structures. A blast explosion inside or surrounding the structure can cause severe damage to the structural and non-structural members. The structure can be made blast resistant but not a blast-proof in reality and also it is not an economical option. The objective of this study is to shed light on blast-resistant building design theories, the enhancement of building security against the effects of explosives in the structural design process and the design techniques that should be carried out. The paper includes introduction and detail explanation on blast wave phenomenon as well as a review of various research on blast load and their effect on the structure studied in the past.

The above review shows that there is substantial research available on the response of shear frames and buildings subjected to wind, wave and earthquake loads, but the studies on shear frames and structures subjected to harmonic and impulsive loads and free vibration are limited. Tuken [1] analytically formulated the response of a 3-story shear frame subjected to impulsive loading. He applied a rectangular pulse force at the top floor

and obtained the equations for the floor displacements as functions of time. Then the floor displacements under the applied rectangular pulse force is plotted against time. He observed that the displacement values at the top floor are maximum where the impulsive load is applied, and the peak value of the response at the top floor is obtained at the end of the rectangular pulse force duration. Baig *et al.* [5] adopted the harmonic response technique on ANSYS platform for a 15-story structure bare frame and evaluated the displacements of the structure at various floor levels by employing mode superposition method. Peak displacement is then visualized from the frequency v/s displacement graph obtained from mode superposition of reduced modes at forcing frequencies. Saatcioglu *et al.* [13] investigated 10-story moment resisting frames with or without shear walls subjected to blast loads consisting of different charge-weight and standoff distance combinations. The results are presented with regard to ductility and drift demands. They show better performance in seismic-resistant buildings subjected to blast loads, pertaining the progressive collapse potential, overall structural response and local column performance. Khan *et al.* [14] carried out a reliability assessment of Tension Leg Platform (TLP) tethers against maximum tension (i.e., Tension exceeding yield) under combined action of extreme wave and impulsive forces. With this object in mind, a non-linear dynamic analysis of TLP was done in the time domain. By employing the Von-Mises failure theory, a limit state function for maximum tension was derived. Reliability assessment of the TLP tethers was then performed utilizing the established function of limit state together with the time-history responses to various dynamic excitations (namely: half-triangular, triangular and sinusoidal). The tethers were designed by finding the most optimal condition (by solution of the bounded optimization issue). In addition to that, a sensitivity analysis was conducted in order to investigate the impact of different arbitrary parameters on the safety of tethers. A parametric study was done to observe the effects of variable submergence, material yield strength and angle of impact on the tether reliability. In order to show the importance of quality control in the various design parameters, the effect of uncertainty on the overall tether reliability was also discussed. Andac Lulec [15] predicted the response of shear-critical structures accurately under quasi-static conditions by utilizing the VecTor family of nonlinear finite element programs and using a macro-element smeared rotating crack approach. For this purpose, VecTor3 and VecTor6 were adapted for the blast and impact analyses of reinforced and prestressed concrete structures in 3-D, respectively. The experimentally observed results were close to those obtained from the simulations. Moreover, a semi-analytical expression was derived for the prediction of perforation velocity from missile impact. This expression, which is based on the Modified Compression Field Theory, considers the influence of longitudinal and shear reinforcement differently from the other commonly used empirical formulae. The derived expression was then validated with many missile impact data available in the literature, and satisfactory accuracy was observed.

The above computational researches on harmonic and impulsive loads are primarily based on advanced numerical (e.g., finite element or boundary element) softwares. Simple analytical methods were used in very limited studies for obtaining the response of shear frames or buildings subjected to harmonic and impulsive loads. In the present study, the effect of harmonic and impulsive load and also free vibration on a 3-story shear frame was studied using a simple analytical approach.

The shear frame consists of beams and columns rigidly connected at the ends. The columns which are fixed at the base primarily provide the lateral stiffness. For dynamic analysis, the entire shear frame was modeled as Multi Degree of Freedom (MDOF) system having rigid beams and lumped masses of each story placed at the middle of the beams.

ANALYTICAL FORMULATION
In this section, the analytical formulation of a 3-story shear frame is presented for obtaining its response against harmonic loading. The dimensions, floor masses and story stiffnesses of this frame are shown in Fig. 1. This shear frame is subjected to a harmonic force $p(t)=p_0.sin\omega t$ at the top floor. The equations for the floor displacements as functions of time is derived and the normalized response amplitudes are plotted against the frequency ratio ω/ω_1. The same structure is also subjected to a rectangular pulse force at the top floor as shown in Fig. 2. The pulse has an amplitude $p_0 = 900$ kN and duration $t_d = T_1/2$ where T_1 is the fundamental vibration period of the system. The equations for the floor displacements as functions of time is derived and the floor displacements under rectangular impulsive loading is plotted against time. The free vibration response of the same undamped system was also determined if the structure is displaced as shown in Fig. 3 and then released. Floor displacements versus t/T_1 was plotted and the relative contributions of the three vibration modes to the response that was produced by the given initial displacement was observed.
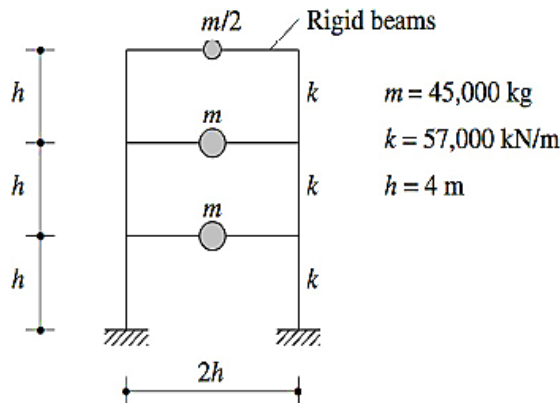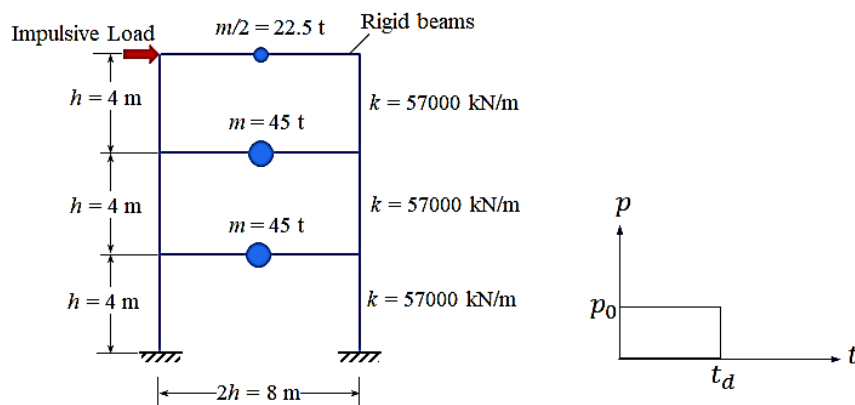

Fig. 1: *Schematic view of the 3-story shear frame*

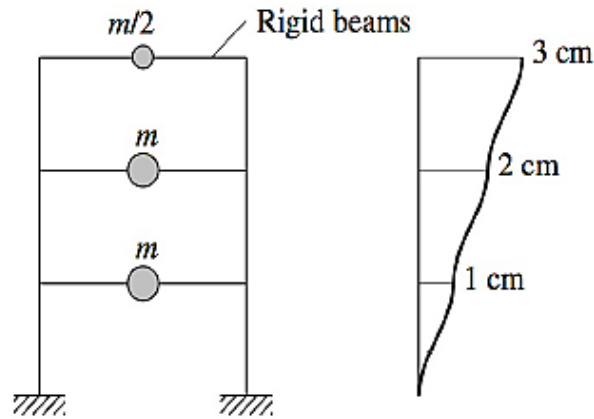**Fig. 2**: *The impulsive load applied to the top floor*

Fig. 3: *Given initial displacement for free vibration*

Mass Matrix

Since the beams are rigid in flexure and axial deformation is neglected in columns, three DOFs associated with each story represent the properties of this three-story shear frame. The corresponding story masses are:

$$m_1 = m; \quad m_2 = m; \quad m_3 = \frac{m}{2}$$

$$\mathbf{m} = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

Here, $m = 45$ t $= 45,000$ kg (as shown in Fig. 1)

Stiffness Matrix

The stiffness coefficients k_{i1}, k_{i2} and k_{i3} were obtained by applying unit displacement at each degree of freedom respectively as shown in Fig. 4. Keeping all the stiffness coefficients in the form of a matrix, we obtain the stiffness matrix as given below:

$$\mathbf{k} = k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

where,

$k = 2\left(\frac{12EI}{h^3}\right) = \frac{24EI}{h^3} = 57,000$ kN/m; E = modulus of elasticity of concrete; I = moment of inertia of each column and h = story height.

The Equation of Motion for Harmonic and Impulsive Loading and Free Vibration

Having derived the mass matrix and stiffness matrix, one can write the governing equation of motion for the studied shear frame under harmonic loading applied at the top floor as follows:

$$m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} \sin \omega t$$

The governing equation of motion for the same structure under rectangular pulse force applied at the top floor can also be obtained as follows:

$$m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} \quad \text{where} \quad p_0 = 900 \text{ kN}$$

The governing equation of motion for the same structure can also be easily written for free vibration under the given initial displacement as below:

$$m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$
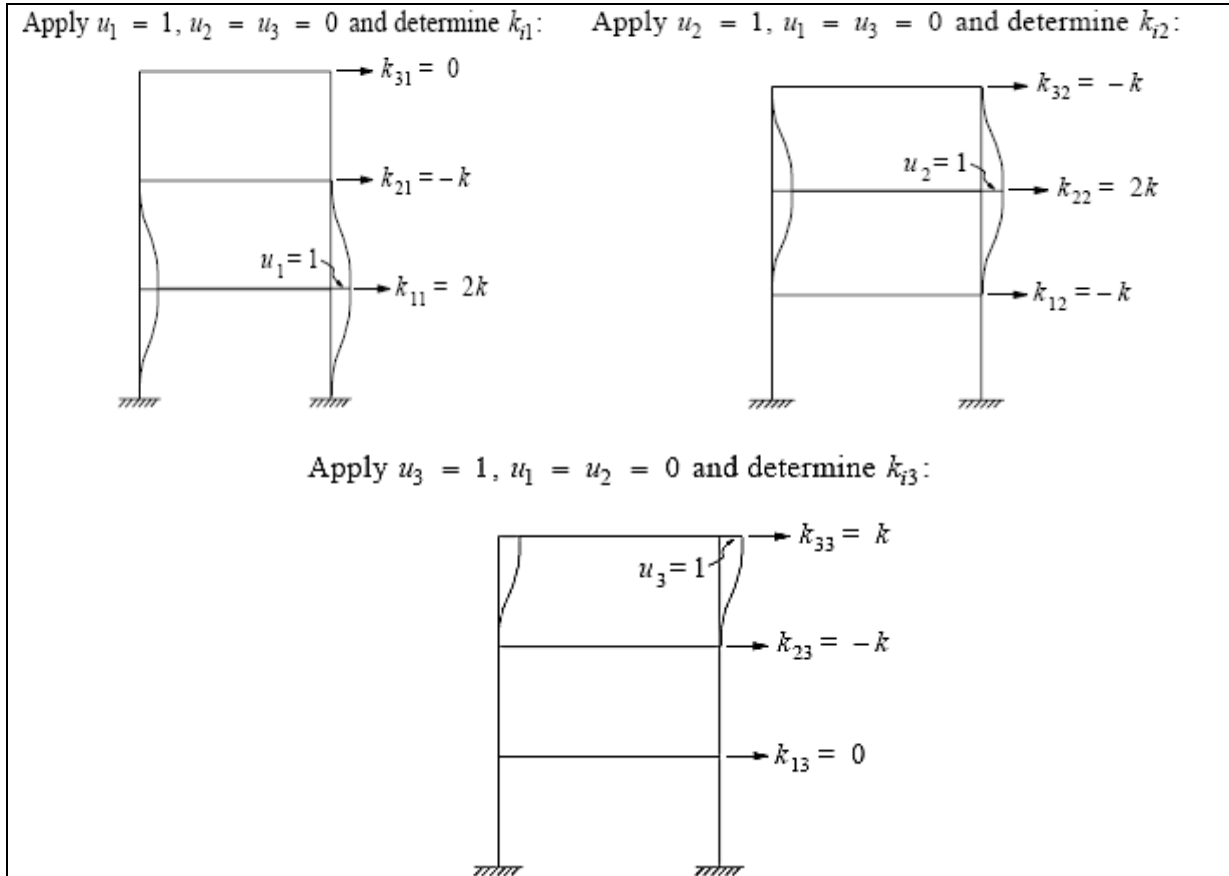

Fig. 4: Determination of stiffness coefficients

RESPONSE ANALYSIS

Natural Frequencies and Mode Shapes
The natural frequencies and mode shapes can be determined from:

$$\mathbf{k} - \omega^2\mathbf{m} = \frac{24EI}{h^3}\begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-0.5\lambda \end{bmatrix} \quad \text{where } \lambda = \frac{mh^3}{24EI}\omega^2$$

$\det[\mathbf{k} - \omega^2\mathbf{m}] = 0$ gives the frequency equation as follows

$$\lambda^3 - 6\lambda^2 + 9\lambda - 2 = 0$$

The solution of this 3rd-degree frequency equation gives
$$\lambda_1 = 2 - \sqrt{3} = 0.2679, \quad \lambda_2 = 2 \quad and \quad \lambda_3 = 2 + \sqrt{3} = 3.7321$$

The corresponding natural frequencies are

$$\omega_1 = 2.5359\sqrt{\frac{EI}{mh^3}}; \; \omega_2 = 6.9282\sqrt{\frac{EI}{mh^3}}; \; \omega_3 = 9.4641\sqrt{\frac{EI}{mh^3}}$$

And the mode shapes are as follows:

$$\emptyset_1 = \begin{Bmatrix} 0.5 \\ 0.866 \\ 1 \end{Bmatrix} \quad \emptyset_2 = \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix} \quad \emptyset_3 = \begin{Bmatrix} 0.5 \\ -0.866 \\ 1 \end{Bmatrix}$$

Determination of Response for Harmonic and Impulsive Loading and Free Vibration

i. The response of harmonic loading

The response of the shear frame structure subjected to harmonic load is obtained following the procedure presented in Chopra [6].

The steady-state response is assumed as

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{Bmatrix} = \begin{Bmatrix} u_{10} \\ u_{20} \\ u_{30} \end{Bmatrix} \sin \omega t$$

Where

$$\begin{Bmatrix} u_{10} \\ u_{20} \\ u_{30} \end{Bmatrix} = [\mathbf{k} - \omega^2\mathbf{m}]^{-1} \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} = \frac{1}{\det[\mathbf{k}-\omega^2\mathbf{m}]} \operatorname{adj} [\mathbf{k} - \omega^2\mathbf{m}] \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix}$$

In which,
$$\det[\boldsymbol{k} - \omega^2\boldsymbol{m}] = m_1 m_2 m_3 (\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2)(\omega_3^2 - \omega^2)$$

$$= \frac{m^3}{2}\left(1 - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right)\left(1 - \frac{\omega^2}{\omega_3^2}\right)\omega_1^2\omega_2^2\omega_3^2$$

$$= k^3\left(1 - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right)\left(1 - \frac{\omega^2}{\omega_3^2}\right)$$

And

$$\operatorname{adj} [\mathbf{k} - \omega^2\mathbf{m}] \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} = k^2 p_0 \begin{Bmatrix} 1 \\ 2\left(1 - \frac{\omega^2}{\omega_2^2}\right) \\ 4\left(1 - \frac{\omega^2}{\omega_2^2}\right) - 1 \end{Bmatrix}$$

Therefore,

$$\begin{Bmatrix} u_{10} \\ u_{20} \\ u_{30} \end{Bmatrix} = \frac{p_0}{k} \frac{1}{\left(1 - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right)\left(1 - \frac{\omega^2}{\omega_3^2}\right)} \begin{Bmatrix} 1 \\ 2\left(1 - \frac{\omega^2}{\omega_2^2}\right) \\ 4\left(1 - \frac{\omega^2}{\omega_2^2}\right) - 1 \end{Bmatrix}$$

Finally, the floor displacements can be obtained by the following equations

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{Bmatrix} = \frac{p_0}{k} \frac{1}{(1 - \frac{\omega^2}{\omega_1^2})(1 - \frac{\omega^2}{\omega_2^2})(1 - \frac{\omega^2}{\omega_3^2})} \begin{Bmatrix} 1 \\ 2(1 - \frac{\omega^2}{\omega_2^2}) \\ 4\left(1 - \frac{\omega^2}{\omega_2^2}\right) - 1 \end{Bmatrix} \sin \omega t$$

On the other hand, defining

$$C_n = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)}$$

Normalized response amplitudes can be expressed as follows

$$\frac{u_{10}}{p_0/k} = C_1 C_2 C_3$$

$$\frac{u_{20}}{p_0/k} = 2C_1 C_3$$

$$\frac{u_{30}}{p_0/k} = C_1 C_2 C_3 (\frac{4}{C_2^2} - 1)$$

ii. The response of impulsive loading

The response of the same structure subjected to an impulsive load is also obtained following the procedure presented in Chopra [6].

Generalized modal masses and stiffnesses

$M_{n1} = \emptyset_1^T . m . \emptyset_1 = 67500$ kg

$M_{n2} = \emptyset_2^T . m . \emptyset_2 = 67500$ kg

$M_{n3} = \emptyset_3^T . m . \emptyset_3 = 67500$ kg

$K_{n1} = \emptyset_1^T . k . \emptyset_1 = 22909.66$ kN/m

$K_{n2} = \emptyset_2^T . k . \emptyset_2 = 171000$ kN/m

$K_{n3} = \emptyset_3^T . k . \emptyset_3 = 319090.34$ kN/m

Generalized modal forces (constants)

$$P_{10} = \emptyset_1^T \begin{Bmatrix} 0 \\ 0 \\ P_0 \end{Bmatrix} = 900 \text{ kN}$$

$$P_{20} = \emptyset_2^T \begin{Bmatrix} 0 \\ 0 \\ P_0 \end{Bmatrix} = 900 \text{ kN}$$

$$P_{30} = \emptyset_3^T \begin{Bmatrix} 0 \\ 0 \\ P_0 \end{Bmatrix} = 900 \text{ kN}$$

Duration of rectangular pulse

$$T_1 = \frac{2\pi}{w_1} = 0.3411 \text{ s}$$

and the rectangular pulse force duration at the third floor is, $t_d = T_1/2 = 0.1705$ s

$$T_2 = \frac{2\pi}{w_2} = 0.1248 \text{ s} \quad and \quad T_3 = \frac{2\pi}{w_3} = 0.0914 \text{ s}$$

Determination of lateral displacements

$$Q_1 = \frac{P_{10}}{K_{n1}} = 3.9285 \text{ cm} \quad and \quad R_1 = \frac{P_{10}}{K_{n1}} 2\sin\left(\frac{\pi t_d}{T_1}\right) = 7.8569 \text{ cm}$$

$$Q_2 = \frac{P_{20}}{K_{n2}} = 0.5263 \text{ cm} \quad and \quad R_2 = \frac{P_{20}}{K_{n2}} 2\sin\left(\frac{\pi t_d}{T_2}\right) = -0.9608 \text{ cm}$$

$$Q_3 = \frac{P_{30}}{K_{n3}} = 0.2821 \text{ cm} \quad and \quad R_3 = \frac{P_{30}}{K_{n3}} 2\sin\left(\frac{\pi t_d}{T_3}\right) = -0.2305 \text{ cm}$$

$$q_1(t) = \begin{cases} Q_1\left(1 - \cos\left(\frac{2\pi t}{T_1}\right)\right) & \text{if} \quad 0 \le t \le t_d \\ R_1 \sin\left[2\pi\left(\frac{t}{T_1} - \frac{1}{2}\frac{t_d}{T_1}\right)\right] & \text{if} \quad t > t_d \end{cases}$$

$$q_2(t) = \begin{cases} Q_2\left(1 - \cos\left(\frac{2\pi t}{T_2}\right)\right) & \text{if} \quad 0 \le t \le t_d \\ R_2 \sin\left[2\pi\left(\frac{t}{T_2} - \frac{1}{2}\frac{t_d}{T_2}\right)\right] & \text{if} \quad t > t_d \end{cases}$$

$$q_3(t) = \begin{cases} Q_3\left(1 - \cos\left(\frac{2\pi t}{T_3}\right)\right) & \text{if} \quad 0 \le t \le t_d \\ R_3 \sin\left[2\pi\left(\frac{t}{T_3} - \frac{1}{2}\frac{t_d}{T_3}\right)\right] & \text{if} \quad t > t_d \end{cases}$$

Substituting ϕ_n and $q_n(t)$ in the following equation gives the lateral displacements $u(t)$.

$$u(t) = \sum_{n=1}^{N} u_n(t) = \sum_{n=1}^{N} \emptyset_n q_n(t) = \emptyset_1 q_1(t) + \emptyset_2 q_2(t) + \emptyset_3 q_3(t)$$

$$\Rightarrow u(t) = \begin{Bmatrix} 0.5 \\ 0.866 \\ 1 \end{Bmatrix} \begin{Bmatrix} Q_1\left(1 - \cos\left(\frac{2\pi t}{T_1}\right)\right) & \text{if } 0 \le t \le t_d \\ R_1 \sin\left[2\pi\left(\frac{t}{T_1} - \frac{1}{2}\frac{t_d}{T_1}\right)\right] & \text{if } t > t_d \end{Bmatrix}$$

$$+ \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix} \begin{Bmatrix} Q_2\left(1 - \cos\left(\frac{2\pi t}{T_2}\right)\right) & \text{if } 0 \le t \le t_d \\ R_2 \sin\left[2\pi\left(\frac{t}{T_2} - \frac{1}{2}\frac{t_d}{T_2}\right)\right] & \text{if } t > t_d \end{Bmatrix}$$

$$+ \begin{Bmatrix} 0.5 \\ -0.866 \\ 1 \end{Bmatrix} \begin{Bmatrix} Q_3\left(1 - \cos\left(\frac{2\pi t}{T_3}\right)\right) & \text{if } 0 \le t \le t_d \\ R_3 \sin\left[2\pi\left(\frac{t}{T_3} - \frac{1}{2}\frac{t_d}{T_3}\right)\right] & \text{if } t > t_d \end{Bmatrix}$$

iii. The response of free vibration

The free vibration response of the same system to the given initial displacement is obtained from

$$u(t) = \sum_{n=1}^{3} \emptyset_n \left[q_n(0) \cos \omega_n t + \frac{\dot{q}_n(0)}{\omega_n} \sin \omega_n t\right]$$

where

$$q_n(0) = \frac{\emptyset_n^T.m.u(0)}{\emptyset_n^T.m.\emptyset_n} \quad and \quad \dot{q}_n(0) = \frac{\emptyset_n^T.m.\dot{u}(0)}{\emptyset_n^T.m.\emptyset_n}$$

Since the initial conditions are as follows:

$$u(0) = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad and \quad \dot{u}(0) = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Then;

$$q_1(0) = 2.4800 \quad and \quad \dot{q}_1(0) = 0$$

$$q_2(0) = 0.3333 \quad and \quad \dot{q}_2(0) = 0$$

$$q_3(0) = 0.1786 \quad and \quad \dot{q}_3(0) = 0$$

Substituting these values in the above response equation gives u(t) as follows:

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{Bmatrix} = \begin{Bmatrix} 1.2440 \\ 2.1547 \\ 2.4880 \end{Bmatrix} \cos \omega_1 t + \begin{Bmatrix} -0.3333 \\ 0 \\ 0.3333 \end{Bmatrix} \cos \omega_2 t + \begin{Bmatrix} 0.0893 \\ -0.1547 \\ 0.1786 \end{Bmatrix} \cos \omega_3 t$$

DISCUSSION OF RESULTS

The normalized response amplitudes were plotted against the frequency ratio ω/ω_1 and shown in Fig. 5. These frequency-response curves show three resonance conditions at $\omega=\omega_1$, $\omega=\omega_2$ and $\omega=\omega_3$; at these exciting frequencies the steady-state response is unbounded. At other exciting frequencies, the vibration is finite and could be calculated from the above three equations.
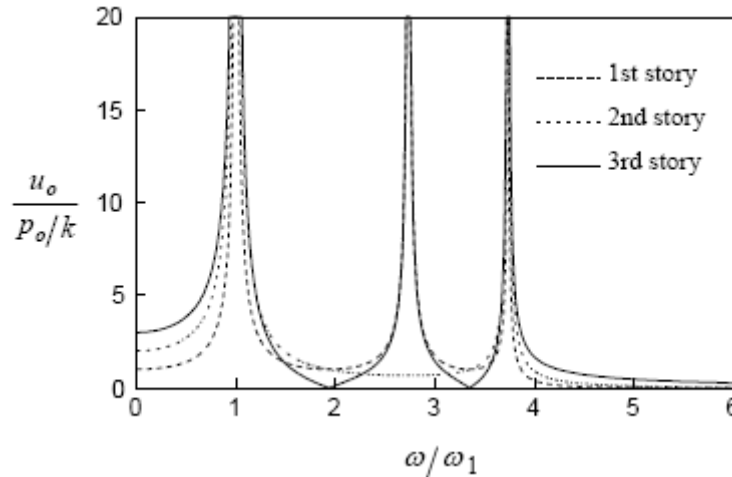

Fig. 5: Normalized response amplitudes against the frequency ratio ω/ω_1

The response for impulsive load was obtained by changing the values of time t in the above-derived equation. The values of times versus floor displacements were plotted and shown in Fig. 6. One can easily observe that the displacement values at the top floor is maximum where the impulsive load is applied. On the other hand, the peak value of the response at the top floor is obtained at the end of the rectangular pulse force duration.

The response of the system to pulse excitation does not reach a steady-state condition; the effects of the initial conditions must be considered. One of several analytical methods can determine the response to such pulse excitations: (1) the classical method for solving differential equations, (2) evaluating Duhamel`s integral and (3) expressing the pulse as the superposition of two or more simpler functions for which response solutions are already available or more straightforward to determine. In the present study, the last of these approaches was used through modal analysis concept.
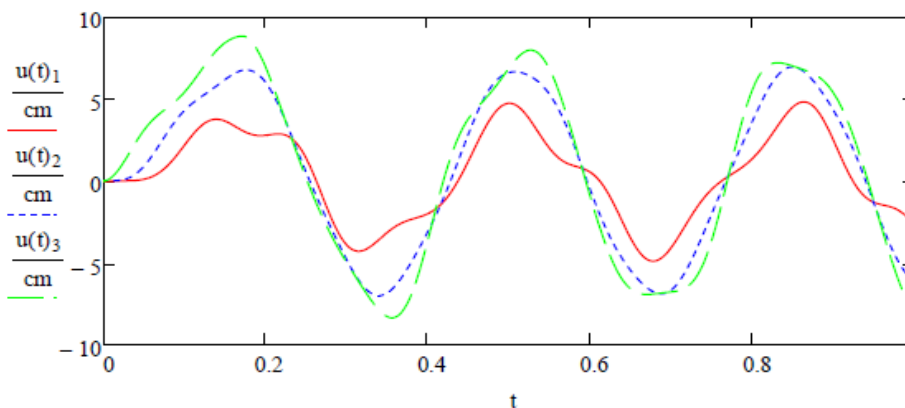

Fig. 6: Floor displacements under rectangular impulsive loading.

As for the free vibration response, floor displacements versus t / T_1 was plotted as shown in Fig. 7 and the relative contributions of the three vibration modes to the response that was produced by the given initial displacement was observed.

Although all three modes contribute to the response, the dominant response is due to the first mode since the shape of the given initial displacement is similar to the shape of the first mode.
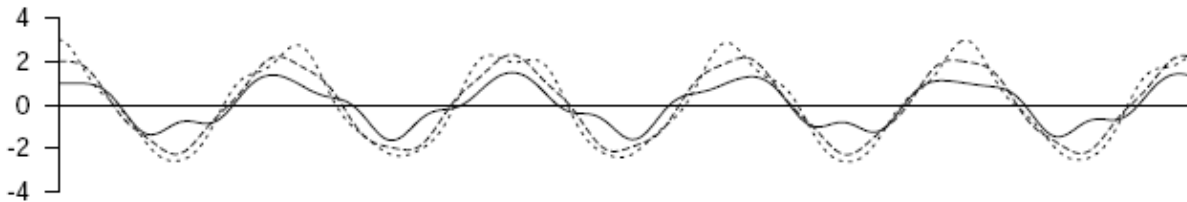

Fig. 7: *Floor displacements (cm) versus t / T_1 under the given initial displacement*

CONCLUSIONS

Followings are the main outcomes of the present study:

- When the structure is excited with harmonic loading over a range of frequencies, the structure experiences resonance at some frequency. Resonance occurs when the frequency of the excitation is equal to the natural frequency of the structure. At the resonant frequency, the structure experiences its most substantial response as compared to any other frequency of loading.
- Frequency-response curves show three resonance conditions at $\omega=\omega_1$, $\omega=\omega_2$, and $\omega=\omega_3$; at these exciting frequencies the steady-state response is unbounded. At other exciting frequencies, the vibration is finite and could be calculated from the derived equations.
- Also, it was observed that the amplitude of the motion changes with the excitation frequency. As the excitation frequency is brought closer to the natural frequency, the amplitude of the vibrations will become very large.
- Analysis from the damage caused to structural elements by impulsive loading can estimate the overpressure produced on structural elements. Although the methods cannot yet be precisely validated, different analyses do compare reasonably well and the results obtained can provide a statistical and rational base for the safe design of structures in the future.
- Impulsive and/or blast loading are applied very quickly and maintained for a very short period of time. If the duration of such loading is less than one-tenth of the fundamental natural period of the structure, then the specific impulse is the dominant characteristic of the load and the loaded area of the structure takes a velocity in a very small displacement. Then, effective mass and the impulse determine the work done into the structure and an energy balance can be used to analyze the produced distortion. On the other hand, the peak pressure is dominant if the duration of such loading is greater than the fundamental natural period of the structure.
- For blast analyses, the blast loads can be modeled as impulse loads assuming a uniform pressure applied to whole surface of the structure.
- Although all three modes contribute to the free vibration response, the dominant response is due to the first mode since the shape of given initial displacement is similar to the shape of the first mode.

REFERENCES

1. Tuken A., Analytical Formulation and Response Analysis of a 3 Story Shear Frame subjected to Impulsive Loading, MAT Journals, Journal of Civil and Construction Engineering (e-ISSN: 2457-001X) 2018; 4(2).
2. Elhelloty A., Effect of Lateral Loads Resisting Systems on Response of Buildings Subjected to Dynamic Loads, International Journal of Engineering Inventions 2017; 6(10):62-76.
3. Jayatilake I.N., Dias W.P.S., Jayasinghe M.T.R. and Thambiratnam, D.P., Response of tall buildings with symmetric setbacks under blast loading, Journal of the National Science Foundation of Sri Lanka 2010; 38(2):115-123.
4. Patel P.D., Dhameliya H.K. and Patel K.S., A Review on Dynamic Analysis of Building under Blast and Seismic Loading, International Journal of Advance Engineering and Research Development 2017; 4(11):429-434.
5. Baig S.S., Mogali M. and Hampali M.P., Harmonic Response Analysis of Multi-Storey Building, International Journal of Current Engineering and Technology 2014; 4(4):2387-2391.
6. Chopra A.K., Dynamics of Structures, 4th Global Edition, Pearson, Prentice Hall; 2014.
7. Tuken A., Analysis and Assessment of Seismic Drift of Reinforced Concrete Mixed (Shear Wall-Frame) Structures, Engineering Science and Technology, an International Journal (JESTECH) (formerly Technology) 2004; 7(4):523-532.
8. Tuken, A. and Siddiqui, N.A., A simplified analytical procedure to determine the amount of shear walls in reinforced concrete buildings, 7th International Conference on Analytical Models and New Concepts in Concrete and Masonry Structures, AMCM 2011, Krakow, Poland, 13-15 June 2011
9. Dahesh, M.A. Tuken, A. and Siddiqui, N.A., Controlling the earthquake induced lateral displacement of RC buildings using shear walls: Parametric study, Arabian Journal of Geosciences, Springer 2015; 8(11):9913–9927.
10. Tuken, A. and Siddiqui, N.A., Assessment of shear wall quantity in seismic-resistant design of reinforced concrete buildings, Arabian Journal for Science and Engineering (AJSE), Springer 2013; 38(10):2639-2648.
11. Tuken A. and Siddiqui, N.A., SBC-based assessment of shear wall quantity in moment resisting frame buildings, KSCE Journal of Civil Engineering, Springer 2015; 19(1):183-199.
12. Tuken, A., Dahesh, M.A. and Siddiqui, N.A., Reliability assessment of RC shear wall-frame buildings subjected to seismic loading, Computers and Concrete, An International Journal 2017; 20(6):719-729.
13. Saatcioglu, M., Ozbakkaloglu, T., Naumoski, N. and Lloyd, A., Response of earthquake-resistant reinforced-concrete buildings to blast loading, Canadian Journal of Civil Engineering 2009; 36(8):1378-1390.
14. Khan, R.A., Siddiqui, N. A., Naqvi, S.Q.A. and Ahmad, S., Reliability analysis of TLP tethers under impulsive loading, Reliability Engineering and System Safety 2006; 91(1):73-83.
15. Lulec, A., Simplified analytical tools for impact and impulsive loading analysis of reinforced concrete structures, *Ph.D. Thesis*, Department of Civil Engineering, University of Toronto, Canada; 2017.