PHYSICS 507 – SPRING 2020
1st HOMEWORK- Solutions
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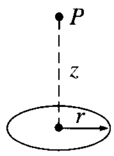
Hand in: Sunday 9th of February at 23:59

1. The electric field of a distance z above the center of a circular loop as shown in the figure, which carries a uniform line charge λ, is given by: $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(r^2+z^2\right)^{3/2}}\hat{\mathbf{k}}$



What is the value of the field at a distance z such that $z << r$.? Give a qualitative explanation of the result. (2 marks)

Solution:

When $z << r$ we have

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(r^2+z^2\right)^{3/2}}\hat{\mathbf{k}} \Rightarrow \mathbf{E} \approx \frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(r^2\right)^{3/2}}\hat{\mathbf{k}} \Rightarrow \mathbf{E} \approx \frac{1}{4\pi\varepsilon_0}\frac{qz}{r^3}\hat{\mathbf{k}} \to 0$$

This is natural because in this case z is very close to the center of the ring. In this case due to symetry (anti-diametric elementary charges dq on the ring produce opposite electric fields at the center) the total electric is zero

2. (a) Find the value of the electric flux through the surface of a sphere containing 12 protons and 10 electrons. $|e| = 1.6\times10^{-19}\ C, \varepsilon_0 = 8.85\times10^{-12}\ F/m$.

(2 marks)

(b) Does the size of the sphere matter in the answer of question (a)?

(1 mark)

Solution:

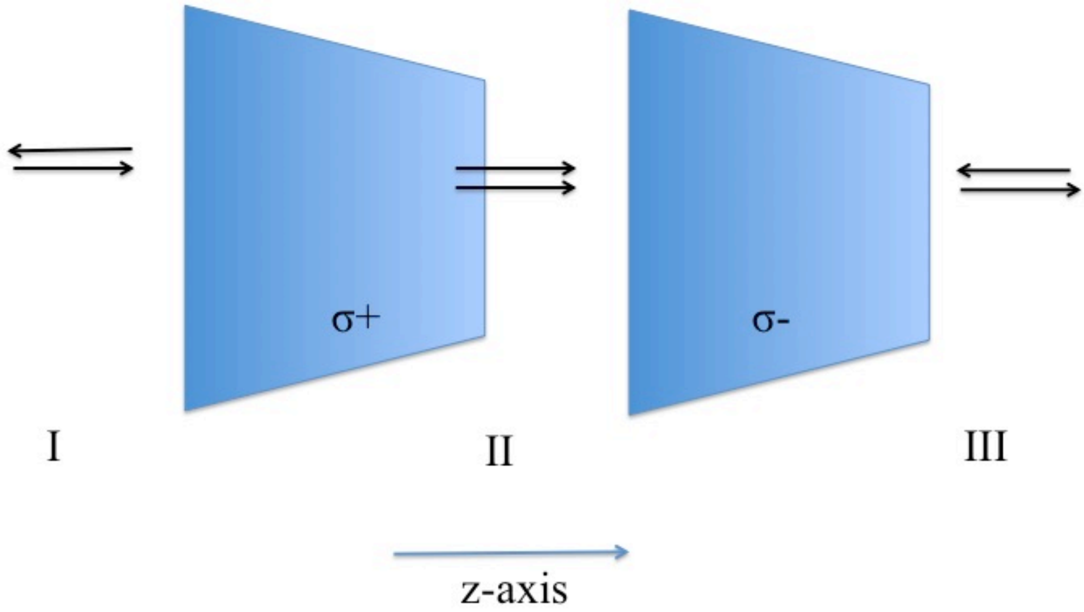
(a) The electric flux through the spherical surface is given by $\Phi = q_{\text{net}}/\varepsilon_0$. Thus

$$\Phi = q_{\text{net}}/\varepsilon_0 = 2|e|/\varepsilon_0 = 3.61\times10^{-8}\ \text{Nm}^2/\text{C}$$

(b) No, the size plays no role.

3. Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$. Find the field in each of three regions: (i) to the left of both, (ii) between them, (iii) to the right of both. (5 marks)

Solution:



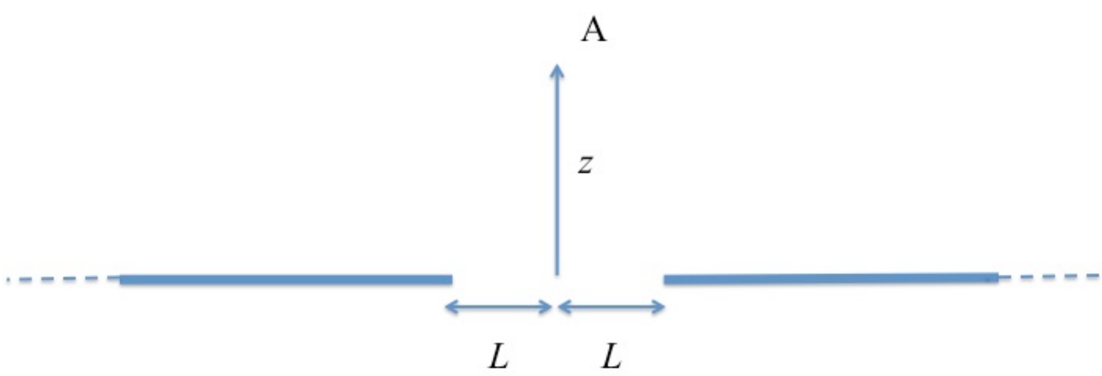
We know that an infinite uniformly charged plane creates an electric field of magnitude $E = \sigma/2\varepsilon_0$. The direction of the fields created by the two planes are shown in the figure. So the total electric field is:

Region (I): $$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} = 0\,.$$

Region (II): $$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{z}}\,.$$

Region (III): $$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} = 0$$

4. An infinitely long wire carries positive charge with uniform linear charge density λ. As the figure shows there is an interruption of the wire of total length $2L$. Find the total electric field at point A at a distance z from the center of the interrupted region (5 marks).



Solution:

The problem can be considered as a superposition of two problems:

One infinite wire of positive linear charge density λ and one segment of length $2L$ and negative charge density $-\lambda$.

From Q2.17 we have shown in the class that the field of the infinite wire is:

$$\mathbf{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 z}\hat{\mathbf{k}}$$

From Q2.3 we have shown in the class that

$$\mathbf{E}_2 = -\frac{\lambda L}{2\pi\varepsilon_0 z\sqrt{z^2+L^2}}\hat{\mathbf{k}}$$

Thus the total field at A is:

$$\mathbf{E}_A = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 z}\hat{\mathbf{k}} - \frac{\lambda L}{2\pi\varepsilon_0 z\sqrt{z^2+L^2}}\hat{\mathbf{k}} = \frac{\lambda}{2\pi\varepsilon_0 z}\left(1-\frac{L}{\sqrt{z^2+L^2}}\right)\hat{\mathbf{k}}$$

5. Find the charge density ρ if the electric field in the region is given by the relation

$$\mathbf{E} = \frac{az}{r}\hat{r} + br\hat{\phi} + cr^2z^2\hat{k}$$

where *a, b, c* are known positive constants and the vectors shown are the unit vectors in spherical coordinates. (5 marks)

Solution:

From the Gauss law (differential form) we have:

$$\vec{\nabla}\mathbf{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0\vec{\nabla}\mathbf{E}$$

In spherical coordinates we know that:

$$\vec{\nabla}\cdot\mathbf{E} = \frac{1}{r}\frac{\partial}{\partial r}\left(rE_r\right) + \frac{1}{r}\frac{\partial E_\varphi}{\partial\varphi} + \frac{\partial E_z}{\partial z} \Rightarrow$$

$$\vec{\nabla}\cdot\mathbf{E} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{az}{r}\right) + \frac{1}{r}\frac{\partial\left(br\right)}{\partial\varphi} + \frac{\partial\left(cr^2z^2\right)}{\partial z} \Rightarrow$$

$$\vec{\nabla}\cdot\mathbf{E} = \frac{a}{r}\frac{\partial z}{\partial r}\left(z\right) + \frac{b}{r}\frac{\partial r}{\partial\varphi} + cr^2\frac{\partial z^2}{\partial z} \Rightarrow$$

$$\vec{\nabla}\cdot\mathbf{E} = 2czr^2$$

So $\rho = 2c\varepsilon_0 z r^2$