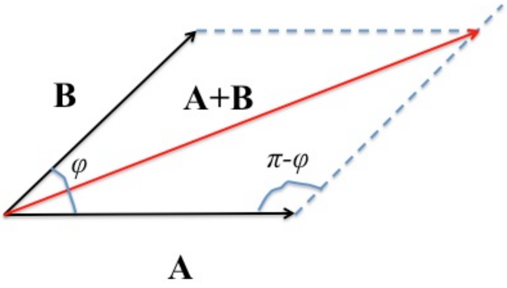
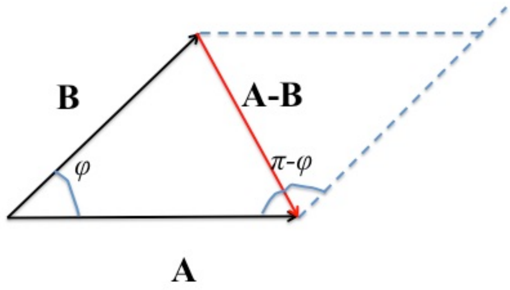
PHYSICS 501
Solutions of 1st Homework –FALL 2019
Prof. V. Lempesis

1. Two vectors **A**, **B** have precisely the same magnitudes. For the magnitude of **A+B** to be three times larger than the magnitude of **A-B** what must be the angle between them?

(5 marks)

Solution:





$$\left|\mathbf{A}+\mathbf{B}\right| = 3\left|\mathbf{A}-\mathbf{B}\right| \Rightarrow \left|\mathbf{A}+\mathbf{B}\right|^2 = 9\left|\mathbf{A}-\mathbf{B}\right|^2 \Rightarrow$$
$$\left(\mathbf{A}+\mathbf{B}\right)\cdot\left(\mathbf{A}+\mathbf{B}\right) = 9\left(\mathbf{A}-\mathbf{B}\right)\cdot\left(\mathbf{A}-\mathbf{B}\right) \Rightarrow$$
$$\mathbf{A}\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{B}+\mathbf{B}\cdot\mathbf{A}+\mathbf{B}\cdot\mathbf{B} = 9\left(\mathbf{A}\cdot\mathbf{A}-\mathbf{A}\cdot\mathbf{B}-\mathbf{B}\cdot\mathbf{A}+\mathbf{B}\cdot\mathbf{B}\right) \Rightarrow$$
$$A^2 + AB\cos\varphi + AB\cos\varphi + B^2 = 9A^2 - 9AB\cos\varphi - 9AB\cos\varphi + 9B^2 \underset{A=B}{\Rightarrow}$$
$$2+2\cos\varphi = 18-18\cos\varphi \Rightarrow 20\cos\varphi = 16 \Rightarrow \cos\varphi = 8/10 \Rightarrow \varphi = \arccos(8/10) \Rightarrow$$
$$\varphi = 36.8^0$$

2. Find the vector $\left(\mathbf{A}-\mathbf{B}\right)\times\left(\mathbf{A}+\mathbf{B}\right)$.

(5 marks)

Solution:

$$\left(\mathbf{A}-\mathbf{B}\right)\times\left(\mathbf{A}+\mathbf{B}\right)=\mathbf{A}\times\mathbf{A}+\mathbf{A}\times\mathbf{B}-\mathbf{B}\times\mathbf{A}-\mathbf{B}\times\mathbf{B}=\mathbf{A}\times\mathbf{B}-\left(-\mathbf{A}\times\mathbf{B}\right)=2\mathbf{A}\times\mathbf{B}$$

3. The points A(2, 4), B(5, 8), C(13, 8), D(10,4) define a parallelogram. Find the area of the parallelogram.

(5 marks)

Solution:

The parallelogram is defined by the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$. These vectors are given by:

$$\overrightarrow{\mathrm{AB}}=\left(5,\ 8\right)-(2,\ 4)=\left(5-2,\ 8-4\right)=\left(3,\ 4\right),$$

$$\overrightarrow{\mathrm{AD}}=\left(10,\ 4\right)-(2,\ 4)=\left(10-2,\ 4-4\right)=\left(8,\ 0\right)$$

The area of the parallelogram is given by

$$Area=\left|\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AD}}\right|=\left\|\begin{matrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 4 & 0\\ 8 & 0 & 0\end{matrix}\right\|=\left|\left(3\times0-8\times4\right)\mathbf{k}\right|=32$$

Answer must be given in this way. Other way, even correct, is not acceptable

4. We have two vectors **A** = (2, 4) and **B** = (-2, 1). The components are given with respect to a coordinate system *x-y*. We chose now another system of axis *x´- y´* which is rotated at an angle φ = -30^0 with respect to *x-y*. Find out: a) The components of the two vectors in the **new** system b) The scalar product of the two vectors in **both** systems

(5 marks)

Solution:

The components of the two vectors in the rotated system are given by:

$$\begin{matrix}A_x^{'}=A_x\cos\phi+A_y\sin\phi\\ A_y^{'}=-A_x\sin\phi+A_y\cos\phi\end{matrix}\Rightarrow\begin{matrix}A_x^{'}=2\cos\left(-30^0\right)+4\sin\left(-30^0\right)\\ A_y^{'}=-2\sin\left(-30^0\right)+4\cos\left(-30^0\right)\end{matrix}\Rightarrow\begin{matrix}A_x^{'}=2\cos\left(-30^0\right)+4\sin\left(-30^0\right)\\ A_y^{'}=-2\sin\left(-30^0\right)+4\cos\left(-30^0\right)\end{matrix}$$

$$\Rightarrow\begin{matrix}A_x^{'}=2\sqrt{3}/2-4/2\\ A_y^{'}=-2\left(-1/2\right)+4\sqrt{3}/2\end{matrix}\Rightarrow\begin{matrix}A_x^{'}=\sqrt{3}-2=-0.268\\ A_y^{'}=1+2\sqrt{3}=4.464\end{matrix}$$

$$\begin{aligned} B_x^{'} &= B_x \cos\phi + B_y \sin\phi \\ B_y^{'} &= -B_x \sin\phi + B_y \cos\phi \end{aligned} \Rightarrow \begin{aligned} B_x^{'} &= (-2)\cos(-30^0) + 1\sin(-30^0) \\ B_y^{'} &= -(-2)\sin(-30^0) + 1\cos(-30^0) \end{aligned} \Rightarrow \begin{aligned} B_x^{'} &= (-2)\sqrt{3}/2 + 1(-1/2) \\ B_y^{'} &= -(-2)(-1/2) + 1\sqrt{3}/2 \end{aligned}$$

$$\Rightarrow \begin{aligned} B_x^{'} &= -\sqrt{3} - 1/2 = -2.232 \\ B_y^{'} &= -1 + \sqrt{3}/2 = -0.134 \end{aligned}$$

The scalar product is given:

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = 2 \cdot (-2) + 1 \cdot 4 = 0$$

$$\mathbf{A}^{'} \cdot \mathbf{B}^{'} = A_x^{'} B_x^{'} + A_y^{'} B_y^{'} + A_z^{'} B_z^{'} = (\sqrt{3} - 2) \cdot (-\sqrt{3} - 1/2) + (1 + 2\sqrt{3}) \cdot (-1 + \sqrt{3}/2) = 0$$

The scalar product must be zero in any rotated frame of reference!