

M - 107 SOLUTION (SEMESTER I, 1432-1433) FIRST MID-TERM

Question: 1 (a) Let

$$A = \begin{bmatrix} x+3y+z & 2x+3y+4z \\ 4x+3y+5z & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} -y-z & 8-x-y-2z \\ -x-y-z & 3x+4y+6z \end{bmatrix}$$

Use Gauss Jordan method to find x , y and z such that A and B are equal. [8]

Solution: As $A = B$,

we can write the system of equations by using equality of matrices

$$x+3y+z = -y-z$$

$$2x+3y+4z = 8-x-y-2z$$

$$4x+3y+5z = -x-y-z$$

$$8 = 3x+4y+6z$$

rearranging both the sides

$$x+4y+2z = 0$$

$$3x+4y+6z = 8$$

$$5x+4y+6z = 0$$

$$3x+4y+6z = 8$$

Equation 2 and equation 4 are same so the system is reduced to three equations

$$x+4y+2z = 0$$

$$3x+4y+6z = 8$$

$$5x+4y+6z = 0$$

Solving system by Gauss Jordan method, rewriting the system in Augmented form and reducing the augmented matrix to reduced row echlon form

$$\begin{bmatrix} 1 & 4 & 2 & 0 \\ 3 & 4 & 6 & 8 \\ 5 & 4 & 6 & 0 \end{bmatrix} \xrightarrow{\text{performing row operations}} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution of the system is $x = -4$, $y = -1$ and $z = 4$.

1 (b) Find the values of λ for which the system of linear equations

$$2x + y = 5$$

$$x - 3y = -1$$

$$3x + 4y = \lambda$$

will have solutions and find the solutions.

[6]

Solution:

$$2x + y = 5$$

$$x - 3y = -1$$

$$3x + 4y = \lambda$$

Augmented form of the system is

$$\begin{aligned} \begin{bmatrix} 2 & 1 & 5 \\ 1 & -3 & -1 \\ 3 & 4 & \lambda \end{bmatrix} &\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & -1 \\ 2 & 1 & 5 \\ 3 & 4 & \lambda \end{bmatrix} \xrightarrow{R_2 - 2R_1, R_3 - 3R_1} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 7 & 7 \\ 0 & 13 & \lambda + 3 \end{bmatrix} \\ &\xrightarrow{\frac{1}{7}R_2, R_3 - 13R_2, R_1 + 3R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & \lambda - 10 \end{bmatrix} \end{aligned}$$

The system will have solution if and only if $\lambda - 10 = 0$ or $\lambda = 10$

System is reduced to the form

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \text{ hence the solution is } x = 2 \text{ and } y = 1$$

Question: 2 (a)

If $A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 2 & 0 \\ 11 & 3 & 2 \end{bmatrix}$, find the inverse of A by using Elementary matrix method. [8]

Solution:

$$[A|I] \approx$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 & 1 & 0 \\ 11 & 3 & 2 & 0 & 0 & 1 \end{bmatrix} \approx \dots \approx \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{6} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{12} & -\frac{1}{3} & \frac{1}{12} \end{bmatrix} \approx [I : A^{-1}]$$

$$A^{-1} = \frac{1}{12} \begin{bmatrix} -2 & -2 & 2 \\ 4 & 10 & -4 \\ 5 & -4 & 1 \end{bmatrix}$$

2(b) Let $A = \begin{bmatrix} a^2 & 1 \\ 4a & 16 \end{bmatrix}$, find all values of a for which A is not invertible.

[4]

Solution:

If matrix A is not invertible then $\det A = 0$

$$\det A = 16a^2 - 4a = 0$$

$$4a(4a - 1) = 0 \Rightarrow a = 0 \text{ or } a = \frac{1}{4}$$

Question: 3 (a) Evaluate the determinant by using elementary row operations

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \end{vmatrix}$$

[6]

Solution:

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \end{vmatrix}$$

using row operations, it is reduced to upper triangular form

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{vmatrix} = 5$$

3(b) Solve the system of linear equations by Cramer's Rule

$$4x + 5y = 2$$

$$11x + y + 2z = 3$$

$$x + 5y + 2z = 1$$

[8]**Solution:**

$$\begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow AX = B$$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \det(A) = -132,$$

$$A_1 = \begin{bmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \det(A_1) = -36$$

$$A_2 = \begin{bmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}, \det(A_2) = -24,$$

$$A_3 = \begin{bmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{bmatrix}, \det(A_3) = 12$$

By using Cramer's Rule

$$x = \frac{\det(A_1)}{\det(A)} = \frac{3}{11}, \quad y = \frac{\det(A_2)}{\det(A)} = \frac{2}{11}, \quad z = \frac{\det(A_3)}{\det(A)} = -\frac{1}{11}$$