King Saud University **244 Math**

College of Science **First Midterm Exam**

Department of Mathematics **1 1/2 Hours**

Name:	**Student No.:**
Section / Group No.:	**Sequence No.:**

Question No.	**I**	**II(a)**	**II(b)**	**II(c)**	**II(d)**	**III**	**IV**	**V**	**Total**
Mark									

I. Determine whether the following statements are always true or sometimes false, and justify your answer with a logical argument or a counter example:

(a) Let A and B be two square matrices such that $AB = \mathbf{0}$. If A is invertible then $B = \mathbf{0}$.

☐ True ☐ False

Justification:

(b) If $tr(A) = 0$, then A is not invertible.

☐ True ☐ False

Justification:

(c) If A and B are two $n \times n$ matrices, then

$$(A+B)(A-B) = A^2 - B^2.$$

☐ True ☐ False

Justification:

(f) The equation $\sqrt{2}x_1 - 6x_2 + 3x_3 = \pi$ is a linear equation.

☐ True ☐ False

Justification:

II. Choose the correct answer:

(a) If $A^3 = \begin{bmatrix} 8 & 0 & 0 \\ 0 & -27 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then A^2 is equal to:

i. $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

ii. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

iii. $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

iv. $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

(b) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix}$ then A^{-2} is equal to:

i. $\begin{bmatrix} -3 & 2 \\ 1 & -1/2 \end{bmatrix}$.

ii. $\begin{bmatrix} 11 & -7 \\ -7/2 & 9/4 \end{bmatrix}$.

iii. $\begin{bmatrix} 9 & 28 \\ 14 & 44 \end{bmatrix}$.

iv. $\begin{bmatrix} 6 & -4 \\ -2 & 1 \end{bmatrix}$.

(c) The homogeneous system of linear equations:

$$\begin{aligned} 5x_1 - x_2 + x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 0 \end{aligned}$$

has:

i. No solution.

ii. One solution.

iii. Three solutions.

iv. Infinitely many solutions.

(d) If $A = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$, then A is invertible if:

i. $x = y$ and $x \neq 0$.

ii. $x \neq y$ or $x \neq -y$.

iii. $x \neq 0$ and $y \neq 0$.

iv. $x \neq y$ and $x \neq -y$.

III. Solve the following system by Gaussian elimination:

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & - & 4x_4 & = & 1 \\ 2x_1 & + & 6x_2 & + & 14x_3 & + & 4x_4 & = & 4 \\ x_1 & - & 12x_2 & - & 11x_3 & - & 16x_4 & = & -3 \end{array}$$

IV. What conditions must b_1, b_2, and b_3 satisfy in order for the following system of linear equations to be consistent:

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & x_3 & = & b_1 \\ -4x_1 & + & 5x_2 & + & 2x_3 & = & b_2 \\ -4x_1 & + & 7x_2 & + & 4x_3 & = & b_3 \end{array}$$

V. Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

Determine whether A is invertible, and if so, find its inverse?

Good Luck