KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1430/1431)

Solution First Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	c	a	c	b	b	a	c	b	a	d

Q. No: 1 If $\sum_{k=1}^{n}(k+\alpha)=\frac{n^2}{2}\ (n\geq 1)$, then the value of α is equal to:

(a) $-\frac{n}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

Q. No: 2 The value of the integral $\int_{-1}^{1}\sinh(x)\,dx$ is equal to:

(a) 0 (b) $2e$ (c) $2e^{-1}$ (d) $\frac{1}{2}e$

Q. No: 3 The number z that satisfies the conclusion of the Mean value Theorem for $f(x)=x^2+1$ on $[-2,1]$ is:

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) -1 (d) 0

Q. No: 4 If $\log_2\left(\frac{x}{x-1}\right)=1$, then x is equal to:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) -1

Q. No: 5 If $\int_{0}^{x^2} f\left(\sqrt{t}\right)dt=x$ for $x>0$, then $f(x)$ is equal to:

(a) 1 (b) $\frac{1}{2x}$ (c) $\frac{1}{x^2}$ (d) $\frac{1}{x}$

Q. No: 6 The derivative of the function $f(x)=\tan^{-1}(\sinh x)$ is equal to:

(a) $sech\,(x)$ (b) $csch\,(x)$ (c) $\tanh(x)$ (d) $-sech\,(x)$

Q. No: 7 The value of the integral $\int_{0}^{1}5^x dx$ is equal to:

(a) $\frac{4\ln 5}{5}$ (b) $\frac{\ln 5}{4}$ (c) $\frac{4}{\ln 5}$ (d) $\frac{5\ln 5}{4}$

Q. No: 8 For the integral $\int(2x-1)^{10}dx$ the substitution $u=2x-1$ simplifies the integral to:

(a) $\int u^{10}du$ (b) $\frac{1}{2}\int u^{10}du$ (c) $2\int u^{10}du$ (d) $\int(2u-1)^{10}du$

Q. No: 9 The value of the integral $\int \frac{\sin x}{\sqrt{2+\cos x}} dx$ is equal to:

(a) $-2\sqrt{2+\cos x}+c$ (b) $\sqrt{2+\cos x}+c$ (c) $-\sqrt{2+\cos x}+c$ (d) $2\sqrt{2+\cos x}+c$

Q. No: 10 If $f(1)=3$, $f(4)=7$, $f(2)=4$ and $f(14)=23$, then the value of the integral $\int_1^2 (x^2+1) f'(x^3+3x)\, dx$ is equal to:

(a) $\frac{1}{3}$ (b) 16 (c) 1 (d) $\frac{16}{3}$

Full Questions

Question No: 11 Approximate the integral $\int_0^1 \frac{4}{1+x^2} dx$ using the **Simpson's rule** with $n=4$. [**3**]

Solution:

Let $f(x) = \frac{4}{1+x^2}$.

$\Delta x = \frac{1}{4} = 0.25$

$x_0 = 0,\ x_1 = 0.25,\ x_2 = 0.5,\ x_3 = 0.75$ and $x_4 = 1$.

$$\begin{aligned}\int_0^1 f(x)dx &\approx \tfrac{1-0}{3\times 4}\{f(0)+4f(0.25)+2f(0.5)+4f(0.75)+f(1)\}\\ &= \tfrac{1}{12}\{4+4(3.7647)+2(3.2)+4(2.56)+2\}\\ &= \tfrac{1}{12}\{4+15.0588+6.4+10.24+2\}\\ &= \tfrac{1}{12}\{37.6988\} \approx 3.1416\end{aligned}$$

Question No: 12 If $y = x^{(e^x)}$, then find y'. [**2**]

Solution:

$\ln y = e^x \ln x$

$\frac{y'}{y} = e^x\left(\ln x + \frac{1}{x}\right)$.

So $y' = e^x\left(\ln x + \frac{1}{x}\right) x^{(e^x)}$.

Question No: 13 Evaluate the integral $\int \frac{\sqrt{x^3}}{\sqrt{1+x^5}}dx.$ **[3]**

Solution:

Let $u = x^{\frac{5}{2}} \implies du = \frac{5}{2}x^{\frac{3}{2}}dx$

$$\begin{aligned}
\int \frac{\sqrt{x^3}}{\sqrt{1+x^5}}dx &= \int \frac{x^{\frac{3}{2}}}{\sqrt{1+\left(x^{\frac{5}{2}}\right)^2}}dx \\
&= \tfrac{2}{5}\int \frac{1}{\sqrt{1+u^2}}du \\
&= \tfrac{2}{5}\sinh^{-1}(u) + c \\
&= \tfrac{2}{5}\sinh^{-1}\left(x^{\frac{5}{2}}\right) + c
\end{aligned}$$

Question No: 14 Evaluate the integral $\int \frac{1}{\sqrt{e^{2x}-1}}dx.$ **[2]**

Solution:

Let $u = e^x \implies du = e^x dx$

$$\begin{aligned}
\int \frac{1}{\sqrt{e^{2x}-1}}dx &= \int \frac{e^x}{e^x\sqrt{(e^x)^2-1}}dx. \\
&= \int \frac{1}{u\sqrt{u^2-1}}du \\
&= \sec^{-1}(u) + c \\
&= \sec^{-1}(e^x) + c
\end{aligned}$$