College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

First Midterm Exam
Academic Year 1446 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Discrete Mathematics		اسم المقرر
Course Code	151 Math		رمز المقرر
Exam Date	2024-10-02	1446-03-29	تاريخ الامتحان
Exam Time	12: 00 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculator does not allowed.

تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- لا يسمح باستخدام الالة الحاسبة.

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1-1	QI-QII		
2	C.L.O 2-1	QIII-QIV-QV		
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Questions	Q(I)	Q(II)	Q(III)	Q(IV)	Q(V)	Total
Marks						

Question I: (7 points)

Question Number	1	2	3	4	5	6	7
Answer							

Choose the correct answer, then fill in the table above:

1- The truth value of "$\forall x \in \mathbb{R}$, if $x^2 < 0$, then $x^2 + 1 = 3$" is:

(a) True.
(b) False.

2- The proposition $\neg q \leftrightarrow q$ for any proposition q is a:

(a) Tautology.
(b) Contradiction.
(c) Contingency.

3- The proposition $(q \wedge \mathbf{F}) \vee p$ is logically equivalent to:

(a) **T** (b) **F** (c) p (d) q

4- The proposition $\neg(p \rightarrow \neg q) \rightarrow (p \wedge q)$ is logically equivalent to:

(a) **T** (b) **F** (c) p (d) q

5- The negation of the statement $[\exists x\ (x^2 < x)]\ and\ [\forall x\ (x^2 \neq 3)]$ is
   (a) $\forall x(x^2 \geq x)\ or\ \exists x\ (x^2 = 3)$,
   (b) $\exists x\ (x^2 < x)\ or\ \forall x\ (x^2 = 3)$,
   (c) $\forall x(x^2 > x)\ or\ \exists x\ (x^2 = 3)$,
   (d) $\exists x(x^2 \leq x) or \forall x(x^2 \neq 3)$.

6- The statement "$x + 2 < 2$", is true when the domain is the set of:
   (a) for all positive real number,
   (b) for all negative real number,
   (c) for all real number,
   (d) for all real number except x=0.

7- Let $P(x)$ be the statement "$x^2 > 11$", and the universe of discourse consists of the positive integers not exceeding 4, then the following is equivalent to the statement $\exists x P(x)$:
   (a) $\left(P(1) \vee P(2)\right) \wedge \left(P(3) \wedge P(4)\right)$,
   (b) $P(1) \vee P(2) \vee P(3) \vee P(4)$,
   (c) $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$,
   (d) $\left(P(1) \vee P(2)\right) \vee \left(P(3) \wedge P(4)\right)$.

<u>Question II</u>: (3+2.5 points)

(A) Consider the proposition: **"If $\mathbf{1 + 3 > -2}$ then 2 is even number ".** Find the following**:**

(1) The truth value of the proposition.

(2) The converse and its truth value.

(3) The inverse and its truth value.

(4) The contraposition and its truth value.

(B) **<u>Without using the truth tables:</u>** prove that the following proposition is **Tautology**:

$$[\neg \boldsymbol{p} \land (\boldsymbol{p} \lor \boldsymbol{q})] \to \boldsymbol{q}$$

Question III: (2.5+2.5 points)

(A) **By Contradiction:** prove that if x is an irrational then $\frac{1}{x}$ is irrational.

(B) Prove that for all integer $\boldsymbol{n}$, if $\boldsymbol{n^2 + 3}$ is even, then $\boldsymbol{1 - n}$ is also even.

Question IV: (2.5+1 points)

(A) Prove that for all integers $\boldsymbol{a}$ and $\boldsymbol{b}$, if $\boldsymbol{a}$ is even and $\boldsymbol{b}$ is odd, then $\boldsymbol{a^2 + b^2}$ has the form **4j+1** for some integer **j**.

(B) Show that the statement: " For every positive integer $\boldsymbol{n}$, $\boldsymbol{n^2 \geq 2n}$ " is **false.**

Question V:(4 points)

The sequence $\{\boldsymbol{a_n}\}$ is defined recursively by

$$\boldsymbol{a_1 = 1, a_2 = 4, \quad a_n = 2a_{n-1} - a_{n-2}, \qquad \forall\ n \geq 3}$$

Use the Principle of Strong Mathematical Induction to prove that $\boldsymbol{a_n = 3n - 2,\ \forall\ n \geq 1.}$

Good luck