College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1446 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Discrete Mathematics		اسم المقرر
Course Code	151 Math		رمز المقرر
Exam Date	2024-12-23	1446-06-22	تاريخ الامتحان
Exam Time	08: 00 AM		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 9 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculator does not allowed.

تعليمات عامة:

- عدد صفحات الامتحان 9 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- لا يسمح باستخدام الالة الحاسبة.

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1.1(1+(1+2)=4 marks)	QII(a(iii)), QII(b(ii-iii)		
2	C.L.O 1.2(1+4=5 marks)	QII(a(i)) QV(a)		
3	C.L.O 2.1(9+2=11 marks)	QI QIV(b)		
4	C.L.O 2.2(3+3=6 marks)	QII(a(ii)), QII(b(i))		
5	C.L.O 2.3(7+4=11 marks)	QIII QIV(a-c)		
6	C.L.O 2.4(3 marks)	QV(b)		
7				
8				

EXAM COVER PAGE

<table>
<tr><th colspan="2">Question</th><th>Marks</th></tr>
<tr><td colspan="2">QI</td><td></td></tr>
<tr><td rowspan="3">QII(a)</td><td>(i)</td><td rowspan="3"></td></tr>
<tr><td>(ii)</td></tr>
<tr><td>(iii)</td></tr>
<tr><td rowspan="3">QII(b)</td><td>(i)</td><td rowspan="3"></td></tr>
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<tr><td colspan="2">QIII</td><td></td></tr>
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<tr><td colspan="2">QIV(c)</td><td></td></tr>
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<u>Question I</u>: (3+4+2=9 points)

(a) Without using truth tables, show that $[(p \rightarrow \neg q) \wedge p] \rightarrow \neg q$ is a tautology.

(b) Use induction to prove that $3|(16^n + 2)$ for all $n \geq 0$.

(c) Let a and b be integers such that $4|(a^2 + 5b^2)$. Use a proof by contradiction to show that a is even or b is even.

Question II: ((1+3+1)+(3+1+2)=11 points)

(a) Let
$E = \{(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5)\}$
be a relation on $A = \{1, 2, 3, 4, 5\}$.

(i) Represent E with a digraph.

(ii) Show that E is an equivalence relation.

(iii) Find all distinct equivalence classes of E.

(b) Let P be the relation on the set $\mathbb{Z}^+$ of positive integers defined by aPb if and only if $there\ exists\ an\ integer\ n \geq 0\ such\ that\ a = 2^n b.$

(i) Show that P is a partial ordering.

(ii) Is P a total ordering? (Justify your answer.)

(iii) Draw the Hasse diagram of P on the subset $\{1, 2, 3, 4, 5, 6, 7, 8\}$ of $\mathbb{Z}^+$.

Question III: (1+2+(1+1+2)=7 points)

(a) Does a bipartite graph have to always be connected? (Justify your answer.)

(b) Let G be a simple graph with 15 edges such that its complement $\bar{G}$ has 13 edges. Find the number of vertices of G. (Justify your answer.)

(c) Let J be the (undirected) graph represented with the following adjacency matrix.

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(i) Determine whether J is bipartite. (Justify your answer.)

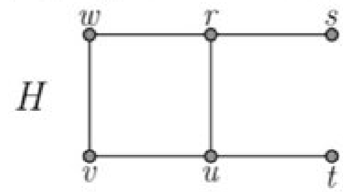
(ii) Draw the complement $\bar{J}$ of J.

(iii) Determine if J is isomorphic to $\bar{J}$. (Justify your answer.)

Question IV: (2+(1+1)+2=6 points)

(a) Let T be a tree with degree-sequence $3, 2, 2, 2, 2, x, y, z$. Find all possible solutions for the triple (x, y, z). (Justify your solutions.)

(b) For the graph H below, find a spanning tree with root r,



(i) using *depth-first* search;

(ii) using *breadth-first* search.

(c) Using alphabetical order, form a binary search tree for the words:
$Car, Train, Boat, Aeroplane, Bus, Helicopter, Bicycle.$

Question V: ((2+2)+(1+2)= 7 points)

(a) For the Boolean function $f(x, y, z) = x + \bar{y}z$, find

(i) the complete sum-of-products expansion (CSP);

(ii) the complete product-of-sums expansion (CPS).

(b) Let $g(x, y, z) = xyz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z$ be a Boolean function.

(i) Build the Karnaugh map of g.

(ii) Simplify g (i.e., write it in MSP form).

Good Luck