| King Saud University | College of Sciences | Department of Mathematics | |
|---|---|---|---|
| Second Examination | Math 151 | Semester 2 (1447) | Time:1H30 |

**Calculators are not allowed**

**Question 1: (15 marks)**

(a) Let $R$ be the relation on $\mathbb{R}$ defined by: $xRy \iff x - y \leq 9$.
Determine whether the relation $R$ is reflexive, symmetric, antisymmetric, or transitive. **(4 marks)**

(b) Let $E$ be the relation on $\mathbb{R}$ defined by: $xEy \iff (x^2 - y^2) \in \mathbb{Z}$

(i) Prove that $E$ is an equivalence relation. **(3 marks)**

(ii) Find $[1]$ (the equivalence classe of 1). **(1 mark)**

(c) Let $P = \{(1,1),(2,2),(2,3),(3,3),(4,1),(4,2),(4,3),(4,4),(5,2),(5,3),(5,5),(6,6)\}$ be a relation on $A = \{1,2,3,4,5,6\}$.

(i) Represent $P$ with a digraph. **(3 marks)**

(ii) Show that $P$ is a partial order. **(1 marks)**

(iii) Is $P$ a total order? Justify your answer. **(1 marks)**

(iv) Represent $P$ with a Hasse diagram. **(2 marks)**

**Question 2: (10 marks)**

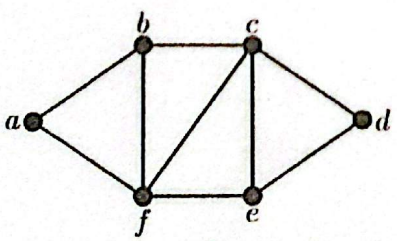
(a) Let $G$ be a graph with degree-sequence: $a-3, a-2, a-1, a, a+2$. Find the value of $a$ if $G$ has 8 edges. **(2 marks)**

(b) Find the number of edges of a graph $G$ with 9 vertices and knowing that the number of edges of its complement $\overline{G}$ are 14. **(2 marks)**

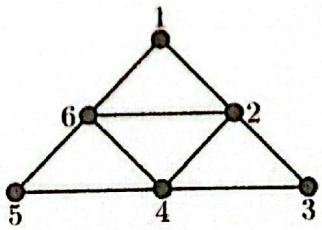
(c) Find the number of vertices of a complete bipartite graph $K_{m,2m}$ that has 50 edges. **(2 marks)**

(d) Determine whether the graphs $G_1$ and $G_2$ below are isomorphic. **(2 marks)**

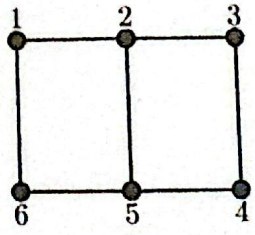
(e) Determine whether the graph $H$ is bipartite. If yes, give a bipartite representation. **(2 marks)**



$G_1$



$G_2$



$H$