King Saud University
Department of Mathematics

Exam2 in Math151
S.1, 1448 H.

Q1. (a) Let R be the relation on $\mathbb{N}$ defined by: $mRn \Leftrightarrow m^2 - n^2 \geq 5$. Determine whether the relation R is reflexive, symmetric, antisymmetric or transitive. (Justify.) (4)

(b) Let $A = \mathbb{Z} - \{0\}$, and let E be the relation on $A \times A$ defined by:

$$(a, b)E(c, d) \text{ iff } ad = bc.$$

(i) Show that E is an equivalence relation. (3)

(ii) Prove that the classes $[(x, y)]$ and $[(-x, -y)]$ are equal for every $(x, y) \in A \times A$. (1)

(iii) Find the classes $[(1, 1)]$ and $[(-1, 1)]$. (2)

(c) Let $P = \{(a, a), (b, b), (c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, d)\}$ be a relation on $B = \{a, b, c, d\}$.

(i) Represent P with a digraph. (1)

(ii) Show that P is a partial order. (3)

(iii) Is P a total order? (Justify.) (1)

(iv) Represent P with a Hasse diagram. (1)

Q2. (a) Let a be a positive integer. Is there a simple graph with degree sequence $a, a + 1, a + 2, a + 3$? (Justify.) (2)

(b) Is there a complete bipartite graph $K_{m,m+2}$ with equal numbers of vertices and edges? (Justify.) (2)

(c) Determine whether the graph G, represented with the adjacency matrix M below, is bipartite or not. (Justify.) (2)

$$M = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(d) Let H be a graph with degree sequence $2, 2, 2, 3, 3$. Find an example of such a graph H. Also, find the number of edges of the complement $\overline{H}$, and then determine whether H is isomorphic $\overline{H}$. (Justify your answers.) (1+1+1)