

King Saud University
Department of Mathematics

Exam2 in Math151
S.1, 1447 H.

Q1. (a) Let R be the relation on $\mathbb{R}$ defined by: $xRy \Leftrightarrow x - y < 8$.

Determine whether the relation R is reflexive, symmetric, antisymmetric or transitive.

(4)

(b) Let E be the relation on $\mathbb{Z}$ defined by: $mEn \Leftrightarrow 3 \mid (7m^2 - n^2)$.

(i) Prove that E is an equivalence relation. (3)

(ii) Show that $(2m)E(4m)$ for all integers m . (1)

(c) Let $P = \{(a, a), (b, b), (b, c), (c, c), (d, a), (d, c), (d, d), (e, a), (e, c)\}$ be a relation on $A = \{a, b, c, d, e\}$.

(i) Represent P with a digraph. (1)

(ii) Show that P is a partial order. (3)

(iii) Is P a total order? (Justify your answer.) (1)

(iv) Represent P with a Hasse diagram. (2)

Q2. (a) Let G be a (simple) 1-regular graph with m vertices.

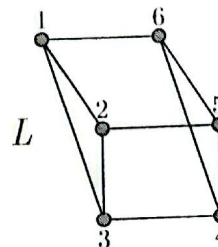
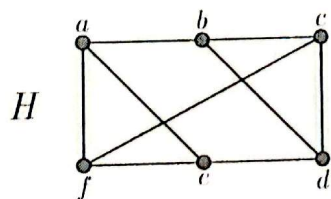
(i) Prove that m must be even. (1)

(ii) Knowing that $m = 14$, determine whether G is a connected graph. (Justify your answer.) (2)

(iii) For $m = 14$, find the number of edges of the complement $\bar{G}$ of G . (1)

(b) Find the number of vertices of a complete bipartite graph $K_{n,2n-4}$ that has 48 edges. (3)

(c) Determine whether the graphs H and L below are isomorphic. (2)



(d) Determine whether the graph H in **Q2(c)** is bipartite, and if this is the case, find a bipartite representation. (1)