King Saud University	College of Sciences		Department of Mathematics
First Examination	Math 151	Semester 2 (1447)	Time:1H30

Calculators are not allowed

Question 1: (9 marks)

1. Construct the truth table for the following proposition.

$$[(p \leftrightarrow \neg q) \wedge (q \leftrightarrow \neg r)] \wedge (r \leftrightarrow \neg p). \quad \text{(3 marks)}$$

2. Without using truth tables, prove that the following statement is a Tautology:

$$q \rightarrow [p \vee (\neg p \wedge q)]. \quad \text{(3 pts)}$$

3. Let n be an integer. Using a proof by contraposition, show that: if 4 divides n^2, then n is even. (2 marks)

4. Let n be an integer. Use a direct proof to show that: if n is odd, then $n^3 - 2n + 1$ is even. (2 marks)

Question 2: (8 marks)

1. Use mathematical induction to show that 3 divides $4^n - 1$, for every positive integer n $(n \geq 1)$. (4 marks)

2. Consider the sequence $\{u_n\}_{n=0}^{\infty}$ defined as follows: $\begin{cases} u_0 = 1 \\ u_1 = 2 \\ u_{n+1} = 5u_n - 6u_{n-1}; \quad n \geq 1 \end{cases}$

   Use mathematical induction to prove the following statement:

$$u_n = 2^n, \quad \text{for each integer } n, \text{ with } n \geq 0. \quad \text{(4 marks)}$$

Question 3: (8 marks)

Let R be the relation from the set $A := \{0, 1, 2, 3\}$ to the set $B := \{1, 2, 3\}$ defined as follows:

$$\text{for } a \in A \text{ and } b \in B, [(aRb) \Leftrightarrow (b^2 - a \geq 1)].$$

1. List all the ordered pairs in the relation R. (2 marks)

2. Represent the relation R with a matrix. (1 mark)

3. Find $\overline{R}$. (1 mark)

4. Let S be the relation from B to A defined by $S := \{(1,0), (2,1), (2,2), (3,3)\}$.

   (a) Find $S \circ R$ and . (2 marks)

   (b) Find $S^{-1} \setminus R$. (1 mark)

   (c) Draw the digraph of the relation $S \circ R$. (1 mark)