King Saud University
Department of Mathematics

First Midterm Exam in Math 151, S2-1445H.

Calculators are not allowed

Q1. (a) Construct the truth table of $A = [p \vee (\neg q \to r)] \wedge \neg r$. [3]
(b) Without using truth tables, show that

$$p \to q \equiv (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q). \ [3]$$

(c) Let a and b be integers. Use a direct proof to show that if a and b are odd, then the number $(a-b)(a^2-b^2)$ is divisible by 8. [3]

Q2. (a) Use induction to show that $12+14+16+\ldots+(2n+10) = n(n+11)$ for all $n \geq 1$. [4]

(b) Let $\{v_n\}$ be a sequence defined by:

$$v_1 = 5, v_2 = 11, \text{ and } v_{n+1} = 2v_n - v_{n-1} + 4 \text{ for } n \geq 2.$$

Show that $v_n = 2n^2 + 3$ for all $n \geq 1$. [4]

Q3. (a) Let R be the relation from $A = \{-2, -1, 0, 1, 2\}$ to $B = \{2, 3, 4, 5\}$ defined by:

$$aRb \Leftrightarrow 3 \mid (a+b).$$

(i) List all ordered pairs of R. [1]
(ii) Represent R with a matrix. [1]
(iii) Find the domain and image (range) of R. [1]

(b) Let $S = \{(w,x), (x,y), (x,z), (z,w), (z,x), (z,z)\}$ be a relation on $E = \{w, x, y, z\}$.

(i) Represent S with a digraph. [1]
(ii) Find $\overline{S} - S^{-1}$. [2]
(iii) Find S^2. [2]