King Saud University
Department of Mathematics

Exam1 in Math151
S.1, 1448 H.

Q1. (a) Construct the truth table of $A = [(p \wedge \neg q) \rightarrow r] \vee \neg p$. (3)

(b) Without using truth tables, prove that

$$(p \rightarrow q) \rightarrow r \equiv (\neg r \rightarrow p) \wedge (q \rightarrow r). \quad (3)$$

Q2. (a) Let n be an integer. Using a direct proof, show that if $n+1$ is divisible by 3, then $2n^2 + n - 1$ is divisible by 9. (2)

(b) Let a be an irrational number. Use a proof by contradiction to show that $\frac{3a-2}{5}$ is an irrational number. (2)

Q3. (a) Show that $5 + 13 + 21 + \cdots + (8n-3) = 4n^2 + n$ for all integers $n \geq 1$. (4)

(b) Let $\{u_n\}_{n=2}^{\infty}$ be a sequence defined by:

$$u_2 = 7, u_3 = 17 \text{ and } u_{n+1} = 2u_n - u_{n-1} + 4, \text{ for } n \geq 3.$$

Prove that $u_n = 2n^2 - 1$ for all $n \geq 2$. (4)

Q4. (a) Let R be the relation from $A = \{-1, 0, 1, 2, 3\}$ to $B = \{1, 2, 3, 4\}$ defined as follows:

$$aRb \Longleftrightarrow 3 \mid (a^2 + b).$$

(i) List all ordered pairs of R. (2)
(ii) Represent R with a matrix. (1)
(ii) Find the domain and image (range) of R. (1)

(b) Let $S = \{(1,3), (2,2), (2,4), (3,3), (4,2)\}$ be a relation on the set $E = \{1, 2, 3, 4\}$.
(i) Represent S with a digraph. (1)
(ii) Find S^2. (2)

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