King Saud University
Department of Mathematics

Exam1 in Math151
S.1, 1447 H.

Q1. (a) Construct the truth table of $A = (p \to \neg q) \wedge (q \vee \neg r)$. (3)

(b) Without using truth tables, prove that $B = (\neg p \wedge q) \to (p \to q)$ is a tautology. (2)

Q2. (a) Use a direct proof to show that if n is an odd integer, then $2n^2 - 18$ is divisible by 8. (2)

(b) Let m be an even integer. Use a proof by contradiction to show that $2m^2 - 18$ is not divisible by 8. (2)

(c) Let $a, b, c \in \mathbb{R}$. Use contraposition to prove that if $a = (b+1)c$, then

$a \geq 0$ or $b \geq -1$ or $c \geq 0$. (2)

Q3. (a) Prove that 3 divides $2n^3 + n$ for all integers $n \geq 1$. (4)

(b) Let $\{u_n\}$ be a sequence defined by:

$$u_1 = 3, u_2 = 5 \text{ and } u_{n+1} = \frac{nu_n - (n-1)u_{n-1} + 7}{2},$$

for $n \geq 2$. Prove that $u_n = 2n + 1$ for all $n \geq 1$. (4)

Q4. (a) Let R be the relation on $A = \{-2, -1, 0, 1, 2\}$ defined as follows:

$$aRb \iff ab \geq 2.$$

(i) List all ordered pairs of R. (1)
(ii) Find the domain and image (range) of R. (1)

(b) Let $S = \{(w,x), (w,y), (x,y), (x,z), (z,w), (z,x)\}$ be a relation on the set $E = \{w, x, y, z\}$.
(i) Find $\overline{S} - S^{-1}$. (2)
(ii) Find S^2. (2)

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Q1) 5
a) 3

p	q	r	¬q	p→¬q	¬r	q∨¬r	A
T	T	T	F	F	F	T	F
T	T	F	F	F	T	T	F
T	F	T	T	T	F	F	F
T	F	F	T	T	T	T	T
F	T	T	F	T	F	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	F	F	F
F	F	F	T	T	T	T	T

1 1 1

b) 2

$B \equiv (\neg p \wedge q) \longrightarrow (p \longrightarrow q) \equiv \neg(\neg p \wedge q) \vee (\neg p \vee q)$

$\equiv (p \vee \neg q) \vee (\neg p \vee q)$

$\equiv (p \vee \neg p) \vee (q \vee \neg q) \equiv T \vee T \equiv T.$

Q2) 6
a) we assume that n is odd $\Rightarrow n = 2k+1$, $k \in \mathbb{Z}$.

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$2n^2 - 18 = 2(2k+1)^2 - 18 = 2(4k^2+4k+1) - 18$

$= 8k^2 + 8k + 2 - 18 = 8(\underbrace{k^2 + k - 2}_{\in \mathbb{Z}})$

$\Rightarrow 8 \mid (2n^2 - 18)$

b) By contradiction, suppose that $8 \mid (2m^2 - 18)$;

$\Rightarrow 2m^2 - 18 = 8q, q \in \mathbb{Z} \Rightarrow 2m^2 = 8q + 18$

$\Rightarrow m^2 = 4q + 9 = 2(\underbrace{2q+4}_{\in \mathbb{Z}}) + 1$

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$\Rightarrow m^2$ is odd $\Rightarrow m$ is odd.

a contradiction $\Rightarrow 8 \nmid (2m^2 - 18)$

c) By contraposition, we show that, if $a < 0$ and $b < -1$ and $c < 0$.
then $(b+1).c \neq a$; we assume that $a < 0$ and $b < -1$ and $c < 0$

$\Rightarrow \left.\begin{matrix} b+1 < 0 \\ c < 0 \end{matrix}\right\} \Rightarrow (b+1).c > 0$

and we have $a < 0$

$\Rightarrow a \neq (b+1).c$

P_3) 8 a) $P(n)$: $3 \mid (2n^3+n)$. $\forall n \geq 1$.

B.S. $P(1)$: $2\times1^3+1=3$; $3|3 \Rightarrow P(1)$ is true.

I.S: let $k\geq1$, we prove that $P(k) \longrightarrow P(k+1)$

we assume that $P(k)$ is true. $3|(2k^3+k) \Rightarrow 2k^3+k=3a; a\in\mathbb{Z}$.

we prove that $P(k+1)$ is true. $3|(2(k+1)^3+k+1)$??

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$$2(k+1)^3+k+1 = 2(k^3+12k^2+6k+1)+k+1.$$
$$= 2k^3+24k^2+12k+2+k+1.$$
$$= (2k^3+k)+24k^2+12k+3.$$
$$= 3a+24k^2+12k+3 = 3(\underbrace{a+8k^2+4k+1}_{\in\mathbb{Z}})$$
$$\Rightarrow 3|(2(k+1)^3+k+1) \Rightarrow P(k+1) \text{ is true}$$
$$\Rightarrow \forall n\geq1,\ P(n) \text{ is true}.$$

b) $P(n)$: $u_n=2n+1$; $\forall n\geq 1$.

B.S: $P(1)$: $u_1=2+1=3$ true.

$P(2)$: $u_2=2\times2+1=5$ true.

I.S: let $k\geq2$, we assume that $P(1), P(2), \ldots, P(k)$ are true and we show that $P(k+1)$ is true. $u_{k+1}=2(k+1)+1=2k+3$??

$$u_{k+1}=\frac{k u_k - (k-1)u_{k-1}+7}{2}.$$

$P(k)$ true $\Rightarrow u_k=2k+1$.

$P(k-1)$ true $\Rightarrow u_{k-1}=2(k-1)+1=2k-1$.

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$$u_{k+1}=\frac{k(2k+1)-(k-1)(2k-1)+7}{2}=\frac{\cancel{2k^2}+k-\cancel{2k^2}+k+2k-1+7}{2}$$
$$=\frac{4k+6}{2}=2k+3.$$

$\Rightarrow P(k+1)$ is true $\Rightarrow \forall n\geq1$; $P(n)$ is true.

2)

Q4) 6

a)

i) $R = \{(-2,-2), (-2,-1), (-1,-2), (1,2), (2,1), (2,2)\}$. 1

ii) $\text{domain}(R) = \{-2,-1,1,2\}$. 1

$\text{image}(R) = \{-2,-1,1,2\}$.

b).

i) $S = \{(w,w), (w,z), (x,w), (x,x), (y,w), (y,x), (y,y), (y,z), (z,y), (z,z)\}$.

$S^{-1} = \{(x,w), (y,w), (y,x), (z,x), (w,z), (x,z)\}$.

2 $= \{(w,z), (x,w), (x,z), (y,w), (y,x), (z,x)\}$.

$S - S^{-1} = \{(w,w), (x,x), (y,y), (y,z), (z,y), (z,z)\}$.

ii) $S^2 = \{(w,y), (w,z), (x,w), (x,x), (z,x), (z,y), (z,z)\}$. 2