King Saud University
Department of Mathematics

Final Exam in Math 151
1^{st} Semester, 1441 H.
(Duration: 3 Hours)

Calculators are not allowed

Q1. (a) Without using truth tables, show that $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$. (2 pts)

(b) Show that $3 + \frac{3}{4} + \frac{3}{4^2} + \cdots + \frac{3}{4^n} = \frac{4^{n+1}-1}{4^n}$ for all integers $n \geq 0$. (4 pts)

(c) Let $R = \{(x,x), (y,x), (y,y), (y,z), (z,y)\}$ be a relation on $A = \{x, y, z\}$. Determine whether R is reflexive, symmetric, antisymmetric, transitive. (4 pts)

Q2. (a) Find the **CSP** and **CPS** forms of $f(x,y,z) = \overline{x + \bar{x}\bar{y}z}$. (2+2 pts)

(b) Let g be the Boolean function represented by the K-map below.

	zw	$z\bar{w}$	$\bar{z}\bar{w}$	$\bar{z}w$
xy	1	1	1	
$x\bar{y}$	1			1
$\bar{x}\bar{y}$		1	1	
$\bar{x}y$		1	1	

(i) Write g in **MSP** form. (2 pts)

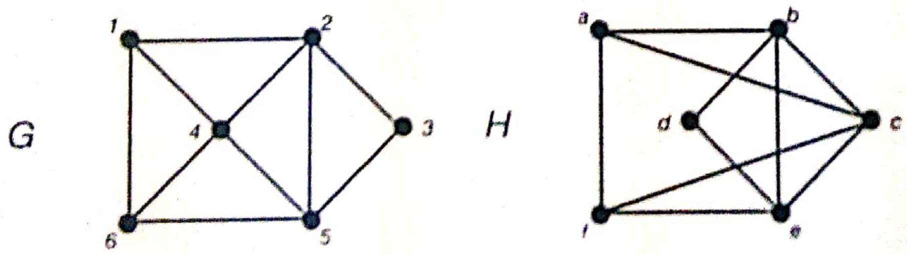
(ii) Write g in **MPS** form. (2 pts)

(iii) Construct a minimal "***AND-OR***" circuit for g. (1 pt)

(iv) Construct a circuit for g using ***NAND*** gates only. (1 pt)

(v) Construct a circuit for g using ***NOR*** gates only. (1 pt)

Q3. (a) Determine whether the following graphs G and H are isomorphic. (2 pts)



(b) Let J be the graph represented by the following adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

(i) Determine whether J is bipartite. (1 pt)

(ii) Determine whether J is a tree. (1 pt)

Turn the page.....

(c) Is there a tree with v vertices and e edges such that $3v = 5e$? (Justify your answer) (2 pts)

(d) Give an example of a graph K which is complete, complete bipartite and a tree. (1 pt)

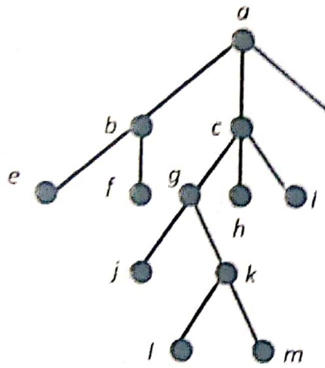
Q4. (a) Form a binary search tree for the words: *beetle, fly, ant, butterfly, bee, termite* (using alphabetical order). (2 pts)

(b) Let T be the ordered rooted tree below.

(i) Find the *inorder* traversal of T. (2 pts)

(ii) Find the *postorder* traversal of T. (2 pts)

T



(c) Let E be the arithmetic expression $((4 + y) * x)/((y - 3) \uparrow 4)$.

(i) Represent E by an ordered rooted tree. (2 pts)

(ii) Write E in *prefix* notation. (1 pt)

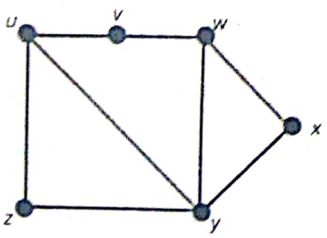
(ii) Write E in *postfix* notation. (1 pt)

(d) For the graph L below, find a spanning tree with root v,

(i) using *depth-first* search; (1 pt)

(ii) using *breadth-first* search. (1 pt)

L



Q1) (10pts)

a) (2pts)

$P \leftrightarrow q \equiv (P\wedge q)\vee(\neg P\wedge \neg q) \equiv \big(\neg(\neg P)\wedge \neg(\neg q)\big)\vee(\neg P \vee \neg q) \equiv \neg P \leftrightarrow \neg q.$

b) (4pts) $P(n): 3+\frac{3}{4}+\dots+\frac{3}{4^n}=\frac{4^{n+1}-1}{4^n}$. $\forall n\geq 0$.

BS: $P(0): 3=\frac{4-1}{1}=3$ True.

IS: we assume that $P(k)$ is true and we prove that $P(k+1)$ is true.
$k\geq 0$;

$P(k): 3+\frac{3}{4}+\dots+\frac{3}{4^k}=\frac{4^{k+1}-1}{4^k}$

$P(k+1): 3+\frac{3}{4}+\dots+\frac{3}{4^{k+1}}=\frac{4^{k+2}-1}{4^{k+1}}$

$3+\frac{3}{4}+\dots+\frac{3}{4^k}+\frac{3}{4^{k+1}}=\frac{4^{k+1}-1}{4^k}+\frac{3}{4^{k+1}}=\frac{4(4^{k+1}-1)+3}{4^{k+1}}=\frac{4^{k+2}-4+3}{4^{k+1}}=\frac{4^{k+2}-1}{4^{k+1}}.$

So $P(k+1)$ is true, thus; $\forall n\geq 0$ $P(n)$ is true.

c) (4pts)

- $(z,z)\notin R \Rightarrow R$ is not reflexive.
- $(y,x)\in R$ and $(x,y)\notin R \Rightarrow R$ not symmetric.
- $(y,z)\in R$ and $(z,y)\in R \Rightarrow R$ is not anti-symmetric.
- $(z,y)\in R$; $(y,x)\in R$ and $(z,x)\notin R \Rightarrow R$ is not transitive

Q2) (11 pts)

a) (2+2)

$f(x,y,z)=\overline{x+\bar x\bar y z}=\bar x\cdot(x+y+\bar z)=\bar x y+\bar x\bar z$

$=\bar x y(z+\bar z)+\bar x(y+\bar y)\bar z=\bar x yz+\bar x y\bar z+\bar x y\bar z+\bar x\bar y\bar z$

$\Rightarrow \boxed{CSP(f)=\bar x yz+\bar x y\bar z+\bar x\bar y\bar z}$

$\bar f(x,y,z)=\overline{\overline{x+\bar x\bar y z}}=x+\bar x\bar y z=x(y+\bar y)(z+\bar z)+\bar x\bar y z=xyz+xy\bar z+x\bar y z+x\bar y\bar z+\bar x\bar y z$

$\Rightarrow \boxed{CPS(f)=(\bar x+\bar y+\bar z)(\bar x+\bar y+z)(\bar x+y+\bar z)(\bar x+y+z)(x+y+\bar z)}$

b) i) (3pts)

	zw	$z\bar w$	$\bar z\bar w$	$\bar z w$
xy	1	1	1	0
$x\bar y$	1	0	0	1
$\bar x\bar y$	0	1	1	0
$\bar x y$	0	1	1	0

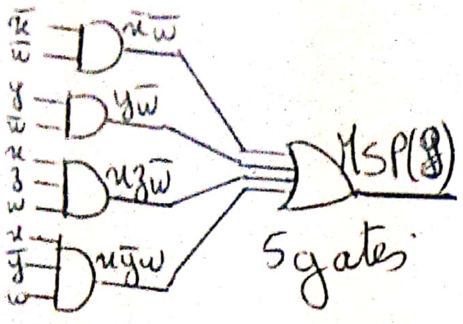
$\boxed{MSP(g)=\bar x\bar w+y\bar w+xzw+x\bar y w}$

ii) (2pts)

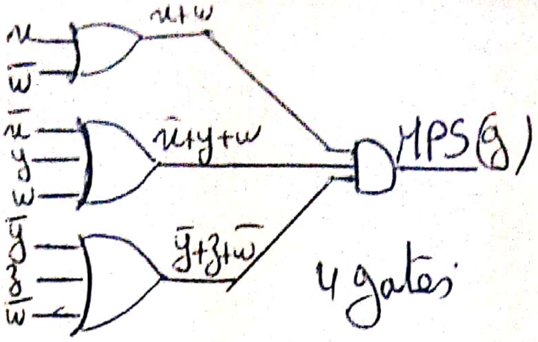
$MSP(\bar g)=\bar x w+x\bar y\bar w+y\bar z w$

$\Rightarrow \boxed{MPS(g)=(x+\bar w)(\bar x+y+w)(\bar y+z+\bar w)}$

iii) (1pts)



5 gates



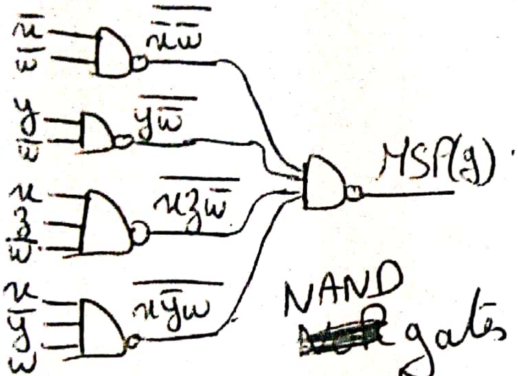
4 gates

A minimal "And-Or" circuit of g.

iv) (1pts)

$$MSP(g) = \overline{\overline{\bar{x}\bar{w}+y\bar{w}+xz\bar{w}+x\bar{y}w}}$$

$$= \overline{\overline{\bar{x}\bar{w}}\cdot\overline{y\bar{w}}\cdot\overline{xz\bar{w}}\cdot\overline{x\bar{y}w}}$$

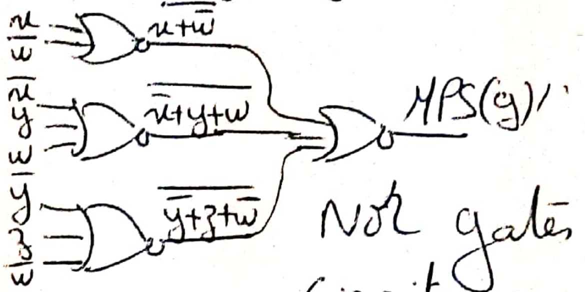


NAND ~~NOR~~ gates circuit

v) (1pts)

$$MPS(g) = \overline{\overline{(x+\bar{w})(\bar{x}+y+w)(\bar{y}+z+\bar{w})}}$$

$$= \overline{\overline{x+\bar{w}}+\overline{\bar{x}+y+w}+\overline{\bar{y}+z+\bar{w}}}$$



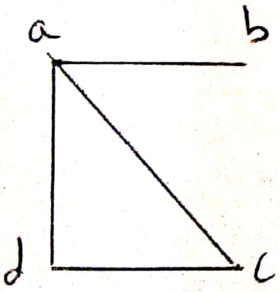
NOR gates circuit

Q3) (7pts)

a) G and H are isomorphic. (2pts)

1	2	3	4	5	6
a	b	d	c	e	f.

b) (2pts)



i) not bipartite, it contain a 3cycle in a, c, d, (1pts)

ii) (1pts) NO.

J have 4 vertices;

if J is tree then the number of edge is $4-1=3$

but there is 4

c) (2pts)

If this graph is tree then $e = v - 1$

$\Rightarrow 3v = 5(v-1)$

$\Rightarrow 3v - 5v = -5 \Rightarrow 2v = 5 \Rightarrow v = \frac{5}{2} \notin \mathbb{N}$

$\Rightarrow$ is not a graph so not a tree.

d) (1pts)

$K_{1,1} = K_2$.

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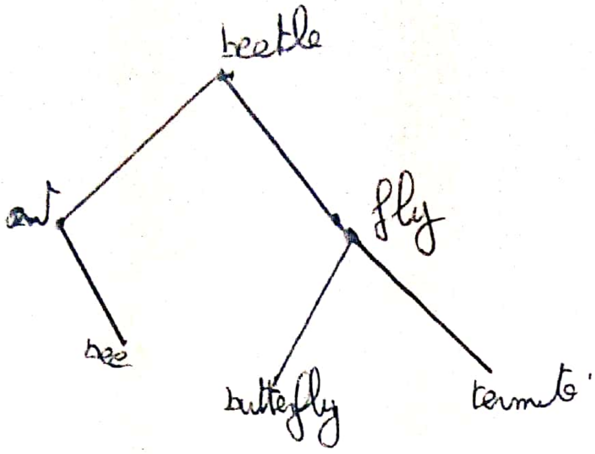
Q4) a) (2pts)



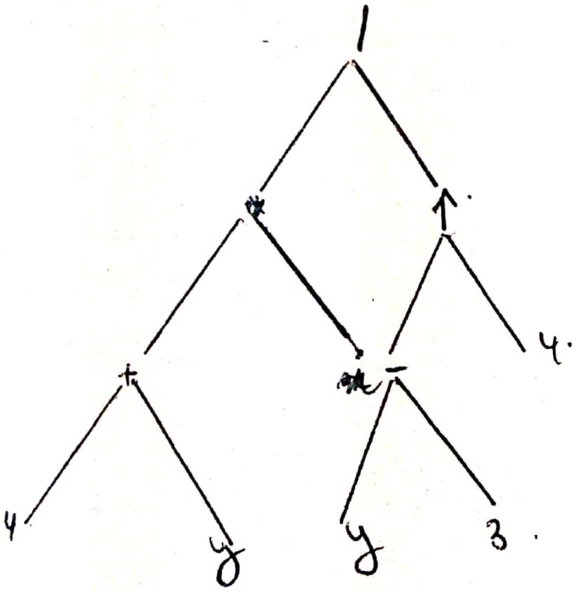
b) i)

inorder: e b f a j g l k m c h i d (2pts)

ii) postorder: e f b j l m k g h i c d a (2pts)

c)

i) (2pts)



ii) (1pt)

prefix: / * + 4 y x ↑ - y 3 4

iii) Postfix: (1pt)

y 4 + x * y 3 - 4 ↑ /

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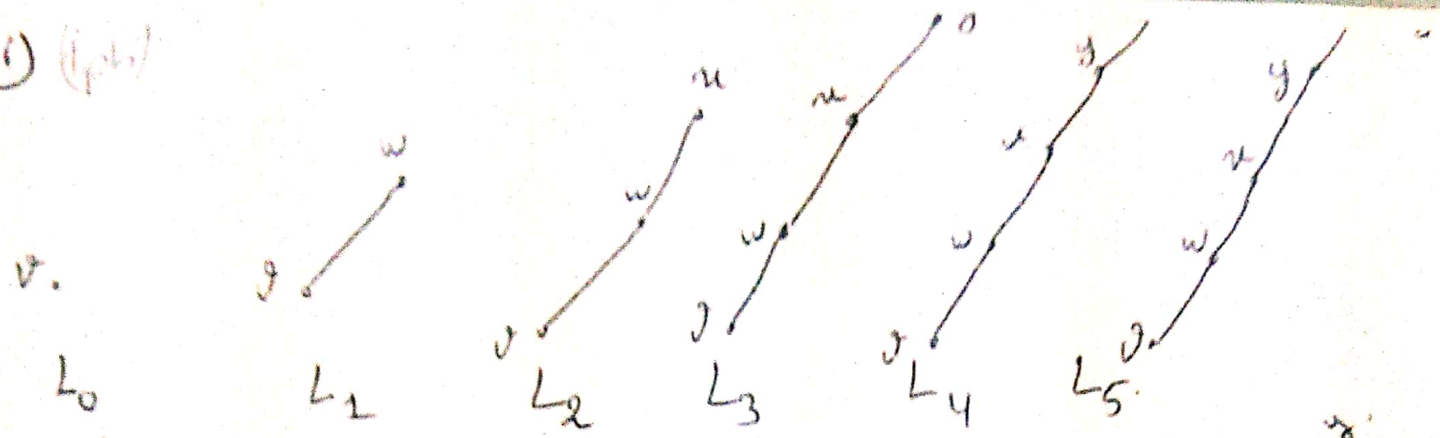
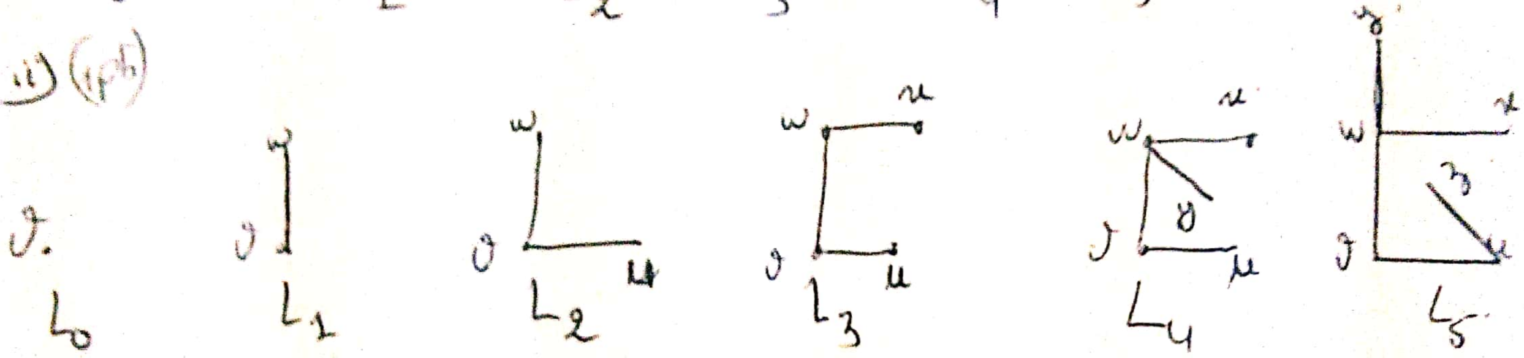
d) i) (1pt)


v.
L0
v w
L1
v w u
L2
v w u o
L3
v w x y
L4
v w x y
L5

ii) (1pt)


v.
L0
v y
L1
v w u
L2
w u v u
L3
w u v u y
L4
y w x v z x
L5