KING SAUD UNIVERSITY
College of Computer & Information Sciences
Muzahmiyah Branch

(Math 151) **Discrete Mathematics**
First Semester 2019/2020

Score

Final Exam

Date: **17/12/2019** Time: 8:00 A.M – 10:00 A.M Time allowed: **2 hours**

STUDENT NAME	Answer
Registration Number	....
Section	....

- There are **6** multiple choice questions **in part A** and **11** questions **in part B**. The maximum score is **40** marks.
- Please do not forget to put your name and registration number on your paper.

Put your answers in the following table.

QUESTION	1	2	3	4	5	6
ANSWER	A	D	D	A	C	B

PART - A

1.5 $\times 6 = 9$

Q1.The proposition $(p \wedge q) \rightarrow (p \rightarrow q)$ is logically equivalent to

A. tautology B. contradiction C. contingency

Q2. The contrapositive of the statement "if a^2+b^2 is odd then a is even or b is even" is

A. "If a^2+b^2 even then a odd or b odd"
B. "If a^2+b^2 even then a odd and b odd"
C. "If a odd and b odd then a^2+b^2 odd"
D. "If a odd and b odd then a^2+b^2 even"

Q3. Let $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ be the matrix of the relation R. Then R is

A. Not reflexive and not transitive.

B. Not reflexive but transitive.

C. Reflexive but not transitive.

D. Reflexive and transitive.

Q4. The CSP form of $f(x, y, z) = x\left(y + \overline{z}\right)$ is

A. $x\,y\,z + x\,y\,\overline{z} + x\,\overline{y}\,\overline{z}$

B. $x\,y\,z + \overline{x}\,y\,\overline{z} + x\,\overline{y}\,\overline{z}$

C. $x\,y\,z + x\,y\,\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$

D. $x\,y\,z + x\,y\,\overline{z} + x\,\overline{y}\,\overline{z} + \overline{x}\,y\,\overline{z}$

Q5. The CPS form of $g(x, y, z) = x\,\overline{y} + z$ is

A. $(x + y + z)(\overline{x} + \overline{y} + z)$

B. $(x + y + z)(\overline{x} + y + z)(\overline{x} + \overline{y} + z)$

C. $(x + \overline{y} + z)(x + y + z)(\overline{x} + \overline{y} + z)$

D. $(\overline{x} + y + z)(\overline{x} + \overline{y} + z)(\overline{x} + \overline{y} + \overline{z})$

Q6. The adjacency matrix of a simple graph G is $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$. Then G is

A. tree

B. complete

C. bipartite

D. not connected

PART-B

Q1. Using logic laws show that $(p\to q)\wedge(p\to\neg r)\equiv p\to\neg(q\to r)$. **2 Marks**

$(p\to q)\wedge(p\to\neg r)\equiv(\neg p\vee q)\wedge(\neg p\vee\neg r)$ 0,5

$\equiv\neg p\vee(q\wedge\neg r)\equiv\neg p\vee\neg(\neg q\vee r)$ 0,5

$\equiv p\to\neg(\neg q\vee r)\equiv p\to\neg(q\to r)$ 0,5 0,5

Q2. Using proof by induction, show that: for all integer $n\geq 1$, $5|_{(7^n-2^n)}$. **3 Marks**

$P(n):\ 5\,|\,7^n-2^n$

0,5 • Basis Step : $n=1$ $5\,|\,7^1-2^1=5$ ✓ So $P(1)$ is true.

0,5 • Inductive step : let $k\geq 2$, we assume that $P(k)$ is true ($5\,|\,7^k-2^k \Leftrightarrow 7^k-2^k=5c,\ c\in\mathbb{Z}$)

Now we prove that $P(k+1)$ remains true $5\overset{?}{|}\,7^{k+1}-2^{k+1}$

2 $7^{k+1}-2^{k+1}=7.7^k-2^{k+1}=7(5c+2^k)-2.2^k$

$=35c+7.2^k-2.2^k=35c+5.2^k=5(\underbrace{7c+2^k}_{M})$

So $5\,|\,7^{k+1}-2^{k+1}$

Q3. Let R be the relation defined on the set $\mathbb{Z}$ as follows: $x\,R\,y\Leftrightarrow x+y\geq 12$
Determine whether the relation is reflexive, symmetric, antisymmetric, transitive. **4 Marks**

1 • R is not reflexive because $0\not R\,0$

1 • R is symmetric because if xRy then $x+y\geq 12$
Also $y+x\geq 12$ (addition is commutative) so yRx.

1 • R is not antisymmetric because $10\,R\,5$ & $5\,R\,10$
but $10\neq 5$

1 • R is not transitive because $10\,R\,3$ and $5R10$
but $5\not R\,3$.

5 R 10
10 R 3

Q4. The K-map of the Boolean function f is given by:

	zw	$z\bar{w}$	$\bar{z}\,\bar{w}$	$\bar{z}w$
xy	1	0	1	1
$x\bar{y}$	1	0	0	1
$\bar{x}\,\bar{y}$	0	0	0	0
$\bar{x}y$	1	1	0	0

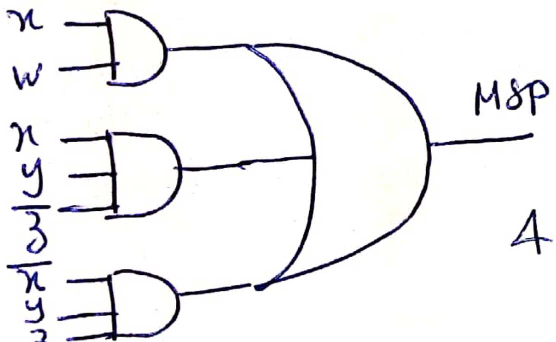
(a) Find MSP and MPS – form. **2 Marks**

1 → $MSP(f) = xw + xy\bar{z} + \bar{x}yz$

$MSP(\bar{f}) = \bar{x}\bar{y} + \bar{x}\bar{z} + \bar{y}\bar{w} + xz\bar{w}$

1 → $MPS(f) = (x+y)(x+z)(y+w)\cdot(\bar{x}+\bar{z}+w)$

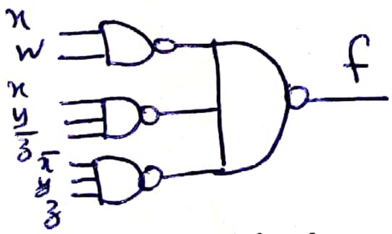
(b) Construct a *least And & Or logic network* for f. **1 Mark**



4 gates only because MPS has 5 gates

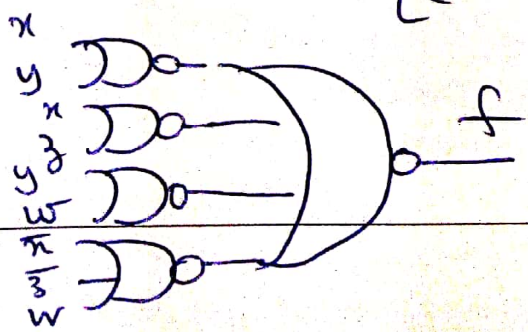
(c) Construct a logic network using *only Nand gates* for f. **1 Mark**

$MSP(f) = [\overline{\overline{xw + xy\bar{z} + \bar{x}yz}}]' = \left(\overline{(\overline{xw})\cdot(\overline{xy\bar{z}})(\overline{\bar{x}yz})}\right)$



(d) Construct a *least Nor logic network* for f. **1 Mark**

$MPS(f) = [\overline{\overline{(x+y)(x+z)(y+w)(\bar{x}+\bar{z}+w)}}]' = [\overline{(x+y)} + \overline{(x+z)} + \overline{(y+w)} + \overline{(\bar{x}+\bar{z}+w)}]'$

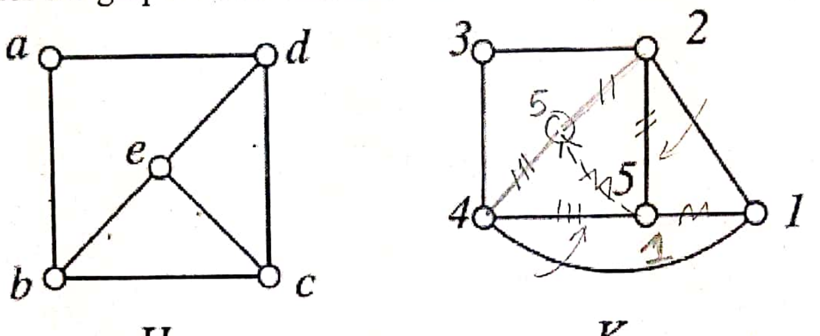


Q5. If G is a complete graph with 45 edges. How many vertices does G have? **2 Marks**

K_n has $\begin{cases} \frac{n(n-1)}{2} \text{ edges} \\ n \text{ vertices} \end{cases}$

So $\frac{n(n-1)}{2} = 45 \iff n(n-1) = 90$ So $\boxed{n = 10}$

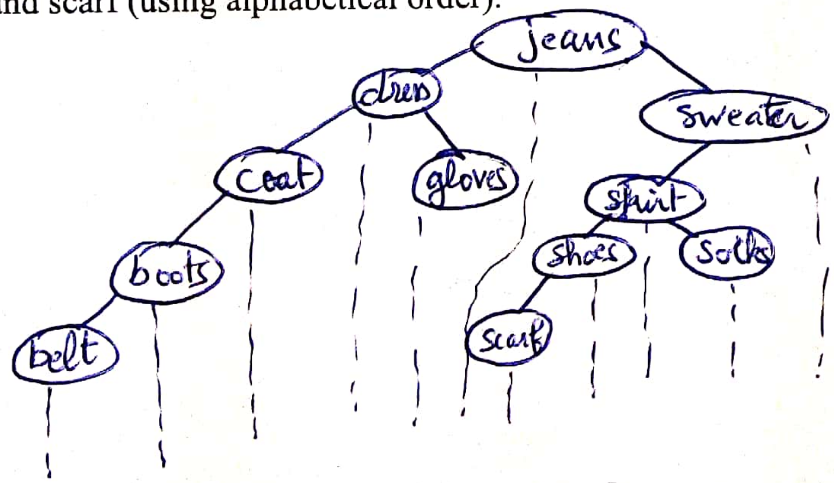
Q6. Determine whether the graphs H and K below are isomorphic. **2 Marks**



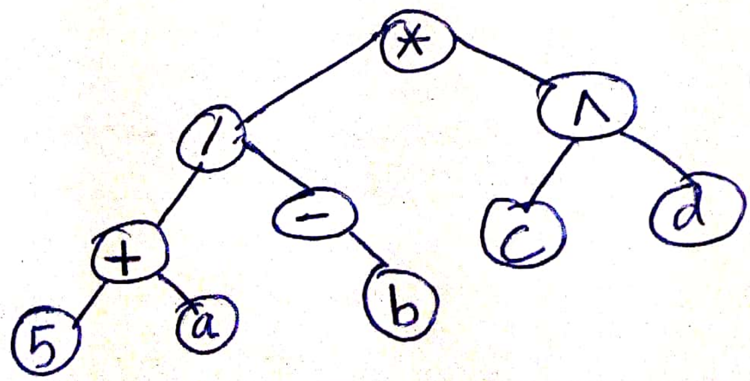
yes H is isomorphic to K because there exists an isomorphism

$x \in V(H)$	a	b	c	d	e
$f(x) \in V(K)$	3	4	1	2	5

Q7. Form a binary search tree for the words: jeans-*sweater-dress-skirt-socks-coat-gloves-shoes-boots-belt* and scarf (using alphabetical order). **3 Marks**



Q8. Represent the arithmetic expression $[(5+a)/-b]*(c\wedge d)$ by an ordered tree. **2 Marks**



Q9. (a) Evaluating a *Prefix* expression: **1 Mark**

*	+	2	-	2	1	/	-	4	2	+	-	5	3	1

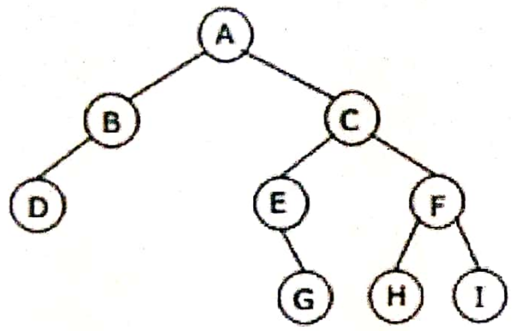
Value of expression:.. 2

(b) Evaluating a *Postfix* expression: **1 Mark**

5	3	+	6	2	/	*	3	5	*	+

Value of expression:.. 39

Q10. Find the preorder, the inorder and the postorder traversals of tree as below **3 Marks**



PREORDER:

A	B	D	C	E	G	F	H	I

1

INORDER:

D	B	A	E	G	C	H	F	I

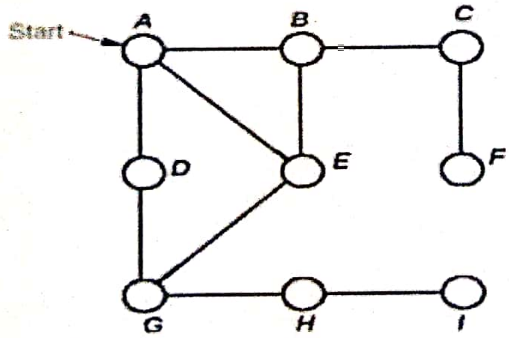
1

POSTORDER:

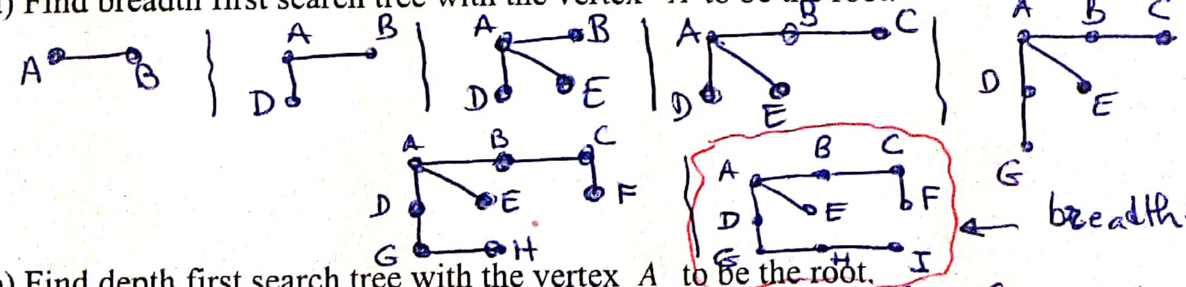
D	B	G	E	H	I	F	C	A

1

Q11. Let G be the following graph

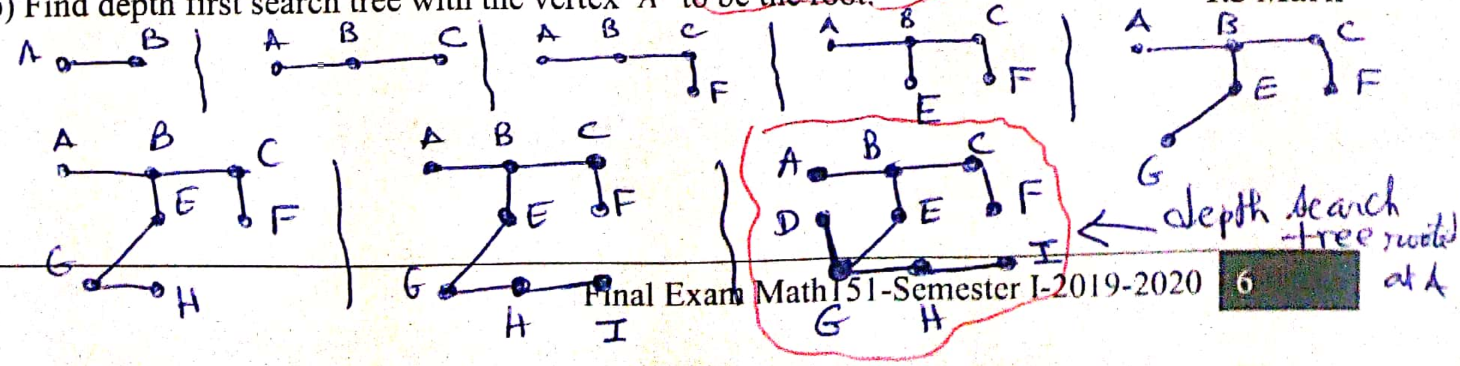


(a) Find breadth first search tree with the vertex A to be the root. **1.5 Mark**



breadth search tree rooted at A.

(b) Find depth first search tree with the vertex A to be the root. **1.5 Mark**



depth search tree rooted at A