ابرهان ٢٠

| جامعة الملك سعود King Saud University — كلية العلوم – قسم الرياضيات | الاختبار الشهري الثاني للمقرر 111 ريض للفصل الأول 1438-1437 هـ |
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| الإسم: ............................................<br>الرقم الجامعي:....................................<br>أستاذ المقرر:...................................... | الزمن: ساعة ونصف.<br>الدرجة: |

ملاحظات : 1. عدد الورقات 4    2. ممنوع استخدام الالة الحاسبة.

**السؤال الأول (4 درجات):** احسب $\frac{dy}{dx}$ فيما يلي :

**(1)** $y=\sinh\left(\frac{1}{x}\right)+\tanh\left(\sqrt{x}\right)$, $x>0$ . (درجتان)

$x>0$ ; $\frac{dy}{dx}=\left(\cosh\left(\frac{1}{x}\right)\right)\left(-\frac{1}{x^2}\right)+\operatorname{sech}^2(\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)$

1    1

**(2)** $y=(\ln x).\sinh^{-1}(x)$, $x>0$ . (درجتان)

$\frac{dy}{dx}=\left(\frac{1}{x}\right)\sinh^{-1}(x)+(\ln x).\left(\frac{1}{\sqrt{1+x^2}}\right)$

1    1

**السؤال الثاني (21 درجة):** احسب التكاملات التالية :

**(1)** $\int\frac{\tanh(\ln x)}{x}dx$ (درجتان)

$u=\ln x$ فان $du=\frac{dx}{x}$

1 $\int\frac{\tanh(\ln x)}{x}dx=\int\tanh(u)\,du=\int\frac{\sinh(u)}{\cosh(u)}du$

1 $=\ln(\cosh u)+cst=\ln(\cosh(\ln x))+cst$

$\int \frac{x}{\sqrt{x^4-16}}dx$ (2)

(درجتان)

$$\int \frac{x}{\sqrt{x^4-16}}dx = \int \frac{x}{\sqrt{(x^2)^2-4^2}}dx$$

نضع $u = x^2$ فان $du = 2x\,dx$

(0,5)

$$\int \frac{x}{\sqrt{x^4-16}}dx = \frac{1}{2}\int \frac{du}{\sqrt{u^2-4^2}} = \frac{1}{2}\cosh^{-1}\left(\frac{u}{4}\right) + cst$$

$$\int \frac{x}{\sqrt{x^4-16}}dx = \frac{1}{2}\cosh^{-1}\left(\frac{x^2}{4}\right) + cst \quad , \quad cst \in \mathbb{R}$$

(1,5)

$$= \frac{1}{2}\ln\left|\frac{x^2+\sqrt{x^4-16}}{4}\right| + cst$$

$\int \ln(1+x^2)\,dx$ (3)

(درجتان)

نستخدم التكامل بالتجزئة:

$u = \ln(1+x^2) \quad u' = \frac{2x}{1+x^2}$

$v' = 1 \Rightarrow v = x$

(1)

$$\int \ln(1+x^2)\,dx = x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}dx$$

(0,5)

$$\int \ln(1+x^2)\,dx = x\ln(1+x^2) - 2\int \frac{x^2+1-1}{1+x^2}dx$$

$$\int \ln(1+x^2)\,dx = x\ln(1+x^2) - 2\int \left[1 - \frac{1}{1+x^2}\right]dx$$

(0,5)

$$= x\ln(1+x^2) - 2(x - \tan^{-1}x) + cst$$

$$= x\ln(1+x^2) - 2x + 2\tan^{-1}x + cst$$

$\int \cos^3 x \sin^2 x\,dx$ (4)

(درجتان)

$$\int \cos^3 x \sin^2 x\,dx = \int \cos^2 x \sin^2 x \cos x\,dx$$

(1)

$$= \int (1-\sin^2 x)\sin^2 x \cos x\,dx$$

نضع $u = \sin x$ فان $du = \cos x\,dx$ وبالتالي

$$\int \cos^3 x \sin^2 x\,dx = \int (1-u^2)u^2\,du = \int [u^2 - u^4]\,du$$

(1)

$$= \frac{u^3}{3} - \frac{u^5}{5} + cst = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + cst$$

(5) $\int \sin(7x)\sin(4x)\,dx$ مع العلم ان : $\sin a \sin b = \frac{1}{2}[\cos(a-b)-\cos(a+b)]$ (درجتان)

نضع $a=7x$ و $b=4x$ فإن

(1) $\sin(7x)\sin(4x) = \frac{1}{2}[\cos(3x)-\cos(11x)]$

بتكامل طرفي المعادلة،

$$\int \sin(7x)\sin(4x)\,dx = \frac{1}{2}\int[\cos 3x - \cos(11x)]\,dx$$

(1) $$= \frac{1}{2}\left[\frac{\sin(3x)}{3} - \frac{\sin(11x)}{11}\right] + cst$$

(6) $\int x^3\sqrt{x^2+1}\,dx$ (3 درجات)

الطريقة الأولى : $u = x^2+1 \Leftrightarrow x^2 = u-1$

$du = 2x\,dx$

$$\int x^3\sqrt{x^2+1}\,dx = \int x^2 x\sqrt{x^2+1}\,dx$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\,du$$

$$= \frac{1}{2}\int [u^{3/2}-u^{1/2}]\,du$$

$$= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + cst$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + cst$$

طريقة ثانية : نضع $x = \tan\theta$ فإن ، $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$dx = \sec^2\theta\,d\theta$

$\sqrt{x^2+1} = \sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = \sec\theta$

(1) $\int x^3\sqrt{x^2+1}\,dx = \int \tan^3\theta\sec^3\theta\,d\theta$

$\int x^3\sqrt{x^2+1}\,dx = \int \tan^2\theta\sec^2\theta\sec\theta\tan\theta\,d\theta$

(1) $\int x^3\sqrt{x^2+1}\,dx = \int (\sec^2\theta-1)\sec^2\theta\sec\theta\tan\theta\,d\theta$

نضع $V=\sec\theta$

$$= \int (V^2-1)V^2\,dV$$

$$= \int (V^4-V^2)\,dV = \frac{V^5}{5} - \frac{V^3}{3} + cst$$

$$= \frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + cst = \frac{(x^2+1)^{5/2}}{5} - \frac{(x^2+1)^{3/2}}{3} + cst \quad (1)$$


$\sec\theta = \sqrt{1+x^2}$

(7) $\int \sqrt{x^2-4x}\,dx$ (3 درجات)

• أولا : إكمال المربع : $x^2-4x = (x-2)^2-4$

$$\int\sqrt{x^2-4x}\,dx = \int\sqrt{(x-2)^2-2^2}\,dx \quad (0,5)$$

• نضع $x-2 = 2\sec\theta$ فإن $dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{(x-2)^2-2^2} = \sqrt{(2\sec\theta)^2-2^2} = 2\tan\theta$

وبالتالي $\int\sqrt{x^2-4x}\,dx = 4\int\sec\theta\tan^2\theta\,d\theta$ (0,5)

$$\int\sqrt{x^2-4x}\,dx = 4\int\sec\theta(\sec^2\theta-1)\,d\theta$$

$$= 4\int(\sec^3\theta - \sec\theta)\,d\theta$$

$$= 2\sec\theta\tan\theta - 2\ln|\sec\theta+\tan\theta| + cst \quad (1)$$

$I = \int\sec^3\theta\,d\theta =$

$\int\sec\theta\sec^2\theta\,d\theta =$

$u = \sec\theta \Rightarrow u' = \sec\theta\tan\theta$

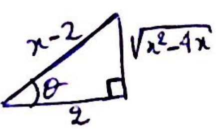
$v' = \sec^2\theta \quad v = \tan\theta$

$= \sec\theta\tan\theta - \int\sec\theta\tan^2\theta\,d\theta$

$= \sec\theta\tan\theta - \int\sec\theta(\sec^2\theta-1)\,d\theta$

$= \sec\theta\tan\theta - I + \ln|\sec\theta+\tan\theta|$

$I = \frac{1}{2}[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|] + cst$




$$\int\sqrt{x^2-4x}\,dx = 2\left(\frac{x-2}{2}\right)\frac{\sqrt{x^2-4x}}{2} - 2\ln\left|\frac{x-2}{2} + \frac{\sqrt{x^2-4x}}{2}\right| + cst$$

(1) $$\int\sqrt{x^2-4x}\,dx = \frac{1}{2}(x-2)\sqrt{x^2-4x} - 2\ln\left|\frac{(x-2)+\sqrt{x^2-4x}}{2}\right| + cst$$

(3 درجات) (تكامل دالة كسرية) $\int \frac{x-2}{x^3+x}dx$ **(8)**

(0,5) $x \neq 0$ , $f(x) = \frac{x-2}{x^3+x} = \frac{x-2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ .

حيث $A, B$ و $C$ ثوابت حقيقية .

- $A = \lim_{x\to 0} x f(x) = \lim_{x\to 0} \frac{x-2}{x^2+1} = -2$ ; $\boxed{A=-2}$

(1,5)

- $0 = \lim_{x\to\infty} x f(x) = A + B \Rightarrow \boxed{B=2}$
- $f(1) = \frac{-1}{2} = -2 + \frac{2+C}{2} = \frac{C-2}{2} \Rightarrow \boxed{C=1}$

$$\int \frac{x-2}{x^3+x}dx = -2\int \frac{1}{x}dx + \int \frac{2x+1}{x^2+1}dx$$

$$\int \frac{x-2}{x^3+x}dx = -2\ln|x| + \int \frac{2x}{x^2+1}dx + \int \frac{dx}{x^2+1}$$

(1) $$\int \frac{x-2}{x^3+x}dx = -2\ln|x| + \ln(x^2+1) + \tan^{-1}(x) + cst$$

(درجتان) $\int \frac{dx}{x^{1/2}+x^{1/6}}$ **(9)**

نضع . $u = x^{1/6}$ و بالتالي $u^6 = x$ فان $6u^5 du = dx$

(0,5) $x^{1/2} = (u^6)^{1/2} = u^3$

$$\int \frac{dx}{x^{1/2}+x^{1/6}} = \int \frac{6u^5}{u^3+u}du = 6\int \frac{u^5}{u(u^2+1)}du$$

$$= 6\int \frac{u^4}{u^2+1}du \quad \text{(تكامل دالة كسرية)}$$

(0,5)

$$\begin{array}{r|l} u^4 & u^2+1 \\ \hline -u^4-u^2 & u^2-1 \\ -u^2 & \\ u^2+1 & \\ \hline 1 & \end{array}$$

(1) لأن $u \in \mathbb{R}$ $\frac{u^4}{u^2+1} = u^2 - 1 + \frac{1}{1+u^2}$ بتكامل طرفي المعادلة:

$$\int \frac{u^4}{u^2+1}du = \int \left[u^2 - 1 + \frac{1}{1+u^2}\right]du$$

$$= \frac{u^3}{3} - u + \tan^{-1}(u) + cst$$

(1) $$\int \frac{dx}{x^{1/2}+x^{1/6}} = 2\sqrt{x} - 6x^{1/6} + 6\tan^{-1}(x^{1/6}) + c$$