King Saud University	College of Sciences	Department of Mathematics	
First Examination	Math 132	Semester I (1446)	Time:1H30

Question 1: (10 marks)

1. Decide whether the following propositions is a tautology or a contradiction or a contingency?

$$(p \to \neg q) \to (r \wedge \neg p). \quad \text{(3 marks)}$$

2. Without using truth tables, prove that the following conditional statement is a Tautology:

$$[(p \vee q) \wedge \neg p] \to q. \quad \text{(3 marks)}$$

3. Without using truth tables, prove the following logical equivalence:

$$(p \to q) \to r \equiv (\neg r \to p) \land (q \to r). \quad \text{(3 marks)}$$

4. Determine the truth value of each of the following statements. (Justify your answer) (1 mark)

   (a) $\forall x \in \mathbb{R}; (x^2 < x^4)$.

   (b) $\exists x \in \mathbb{R}; (x^2 + 1 = 0)$.

Question 2: (10 marks)

1. Use a proof by contradiction to show that $\frac{\sqrt{5}-5}{3}$ is irrational. (Hint use the fact that $\sqrt{5}$ is irrational). (2 marks)

2. Let x, y and z be three real numbers. Use a proof by contraposition to show that: if $(2x - 4y + 5z = 8)$ then, $(x \leq 5$ or $y \geq 3$ or $z \leq 2)$. (2 marks)

3. Use mathematical induction to prove the following statement:

$$8 + 20 + 32 + \cdots + (12n - 4) = 6n^2 + 2n, \quad \text{for each integer } n, \text{ with } n \geq 1. \quad \text{(3 marks)}$$

4. Consider the sequence $\{u_n\}_{n=0}^{\infty}$ defined as follows: $\begin{cases} u_1 = 3 \\ u_2 = 6 \\ u_{n+1} = 2u_n - u_{n-1} + 2; \quad n \geq 2 \end{cases}$

Use mathematical induction to prove the following statement:

$$u_n = n^2 + 2, \qquad \text{for each integer } n, \text{ with } n \geq 1. \quad \text{(3 marks)}$$

Question 3: (5 marks)

1. Consider the set $A := \{1, 2, \{1\}, \{2\}, \{1, 2, \emptyset\}, \{1, \{1\}\}, \{2, \{2\}\}, \emptyset, \{\emptyset\}\}$.
   Determine whether each of the following four statements is true or false.
   (Justify your answer). (2 marks)
   (a) S_1: "$\{1, 2, \emptyset\} \subseteq A$".

   (b) S_2: "$\{1, \{1\}\} \subseteq A$".

   (c) S_3: "$\{1, \{\emptyset\}\} \subseteq A$".

   (d) S_4: "$A \cap \{1, 2, \emptyset, \{\{1\}, \{2\}\}\} = \{1, 2\}$".

2. Consider the following three sets $C := \{1, 2, 3, 4\}, D := \{2, 3\}$, and
   $E := \{(1, 2), (1, 4), (2, 2), (2, 4), (4, 4), (2, 3)\}$. Find the following sets: (3 marks)
   (i) $(C \cap D) \times C$.

(ii) $E \setminus (C \times D)$.

(iii) $\{\emptyset\} \times E$.