King Saud University	College of Sciences		Department of Mathematics
Final Examination	Math 132	Semester I (1446)	Time:3 Hours

Question 1: (8 marks)

1. Without using truth tables, prove that the following conditional statement is a Tautology:
$$[(p \to q) \wedge p] \vee (q \to \neg p). \quad \text{(3 marks)}$$

2. Let $n \in \mathbb{N}$. Show that: if n^2 is odd, then $1 - n$ is even. (2 marks)

3. Use mathematical induction to prove the following statement:

$9+13+17+\cdots+(4n+5) = n(2n+7),$ for each integer n, with $n \geq 1$. (3 marks)

Question 2: (15 marks)

1. Let R be the relation on the set $\mathbb{Z}$ defined by

$$a, b \in \mathbb{Z};\ aRb \Longleftrightarrow b = -a$$

   Decide whether the relation R is reflexive, symmetric, antisymmetric or transitive. Justify your answers. (3 marks)

2. Let E be the relation on the set $\mathbb{Z} - \{0\}$ defined by

$$m, n \in \mathbb{Z} - \{0\};\ mEn \Longleftrightarrow 3mn > 0$$

(a) Show that E is an equivalence relation. (3 marks)

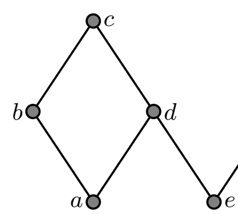
(b) Find $[1]$ and $[-1]$. (2 marks)

3. Let T be the equivalence relation on $A := \{1, 2, 3, 4, 5, 6\}$ with equivalence classes $\{1, 5, 6\}$, $\{2, 4\}$ and $\{3\}$.

   (a) Draw the digraph of T. (1 marks)

   (b) List all ordered pairs of T. (2 marks)

4. Let P be the partial ordering relation on the set $B := \{a, b, c, d, e, f\}$ represented by the following Hasse diagram.



   (a) List all ordered pairs of P. (2 marks)

   (b) Is P a total order. Justify your answer. (2 marks)

Question 3: (14 marks)

1. Consider the sets $X := \{a, b, c\}$, $Y := \{0, 1, 2\}$, and $Z := \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 0)\}$. Find the following sets.

   (a) $(X \times Y) - Z$. (1 mark)

   (b) $(X \cap Y) \times X$. (1 mark)

   (c) $\{\emptyset\} \times Z$. (1 mark)

2. Let f be the function from $C := \{a, b, c, d\}$ to $D := \{0, 1, 2, 3, 4\}$ defined by $f(a) = 0$, $f(b) = 4$, and $f(c) = f(d) = 1$.

   (a) Find $f(\{a, b\})$ and $f(\{a, c, d\})$. (1 mark)

   (b) Find $f^{-1}(\{0, 4\})$ and $f^{-1}(\{1\})$. (1 mark)

   (c) Decide whether f is **one to one** or **onto**. Justify your answers. (2 marks)

3. Let g and h be two function from $\mathbb{R}$ to $\mathbb{R}$ defined by $g(x) = 2x - 1$ and $h(x) = 3 - 3x$.

   (a) Find $g \circ h$ and $h \circ g$. (2 marks)

   (b) Prove that g is a one to one correspondence function. (2 marks)

   (c) Find $g^{-1}(x)$, for all $x \in \mathbb{R}$. (1 mark)

   (d) Decide whether $h \circ g$ is one to one or onto. Justify your answers. (2 marks)

Question 4: (3 marks)

1. Give the cardinal of each of the following sets.
   (a) $A_1 := \{k \in \mathbb{Z}; k \text{ is odd}\}$. (1 mark)
   (b) $A_2 := [0, \infty) \cap \mathbb{Q}^+$. (1 mark)
2. Show that the set $O := \{a \in \mathbb{Z}; 3|a\}$ is countable. (1 mark)