King Saud University

Faculty of Sciences

Department of Mathematics

First Examination Math 132 Semester I 1439-1440
Time: 1H30

Exercise 1 : (4+3+(4+2))

1. Use a truth table to verify that the following conditional statement is a tautology:
$$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$$
2. Without using truth tables, prove that the following statement is a contradiction:
$$p \wedge (p \to q) \wedge (p \to \neg q)$$
3. (a) Without using truth tables, prove the following logical equivalence:
$$\neg[p \vee (\neg p \wedge q)] \equiv (\neg p \wedge \neg q)$$
   (b) Deduce from a) the following logical equivalence:
$$\neg[(r \to w) \vee ((r \wedge \neg w) \wedge q)] \equiv ((r \wedge \neg w) \wedge \neg q)$$

Exercise 2 : (3+3+3+3)

1. Given an integer n, prove by contraposition the following statement:
   if the integer $5n + 7$ is even, then the integer n is odd.
2. Given real numbers x, y, and z, prove by contradiction the following statement:
   if $(x + y + z = 58)$, then ($x \geq 16$ or $y \geq 38$ or $z \geq 4$).
3. Use mathematical induction to prove the following statement:
$$\sum_{k=0}^{n-1} (2k+1) = n^2, \quad \text{for each integer } n \text{ with } n \geq 1.$$
4. Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined as follows:
   $a_0 = 6, a_1 = 9$, and $a_n = 2a_{n-1} - a_{n-2}; \forall n \geq 2$.
   Use mathematical induction to prove the following statement:
$$a_n = 3n + 6, \quad \text{for each integer } n, \text{ with } n \geq 0.$$