جامعة الملك سعود King Saud University — كلية العلوم – قسم الرياضيات	**الاختبارالشهري الثاني** **للمقرر ريض 111** **للفصل الصيفي** **1439-1440 هـ**
	الزمن: ساعة ونصف.

ملحوظة: ممنوع استخدام الالة الحاسبة

السؤال الأول (4 درجات): احسب $\frac{dy}{dx}$ فيما يلي :

(1) $y = \sinh(e^{2x}) + \operatorname{sech}^{-1}\left(\sqrt{x}\right)$. (درجتان)

(2) $y = x\tanh^2\left(\ln(x)\right)$. (درجتان)

السؤال الثاني (21 درجة): احسب التكاملات التالية :

(1) $\int e^{-x}\cosh(x)dx$ (درجتان)

(2) $\int \frac{dx}{\sqrt{1-e^{2x}}}$ (درجتان)

(3) $\int xe^{x}dx$ (درجتان)

(4) $\int \sin^2 x\ \cos^5 x\,dx$ (درجتان)

(5) $\int \sin(5x)\cos(3x)\,dx$ مع العلم أن : $\sin a\cos b = \frac{1}{2}[\sin(a-b)+\sin(a+b)]$ (درجتان)

(6) $\int \frac{2xdx}{x^2+6x+10}$ (3 درجات)

(7) $\int \sqrt{4-x^2}\,dx$ (3 درجات)

(8) $\int \frac{x+4}{x^3+4x}dx$ (3 درجات)

(9) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ (درجتان)

(1)

السؤال الأول:

1) $y = \sinh(e^{2x}) + \operatorname{sech}^{-1}(\sqrt{x})$

$$\frac{dy}{dx} = 2e^{2x}\cosh(e^{2x}) + \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}\sqrt{1-(\sqrt{x})^2}}$$ (1)

$$= 2e^{2x}\cosh(e^{2x}) - \frac{1}{2x\sqrt{1-x}}$$ (1)

2) $y = x\tanh^2(\ln(x))$

$$\frac{dy}{dx} = \tanh^2(\ln(x)) + 2x\cdot\frac{1}{x}\tanh(\ln(x))\operatorname{sech}^2(\ln(x))$$ (1)

$$= \tanh^2(\ln(x)) + 2\tanh(\ln(x))\operatorname{sech}^2(\ln(x))$$ (1)

السؤال الثاني:

1) $\int e^x \cosh(x)\,dx = \int e^{-x}\frac{e^x+e^{-x}}{2}\,dx$ $\frac{1}{2}$

$$= \int \frac{1+e^{-2x}}{2}\,dx = \int \frac{1}{2}\,dx + \frac{1}{2}\int e^{-2x}\,dx$$ $\frac{1}{2}$ $\frac{1}{2}$

$$= \frac{1}{2}x - \frac{1}{4}e^{-2x} + c$$ $\frac{1}{2}$

2) $\int \frac{dx}{\sqrt{1-e^{2x}}}$, $u = e^x \Rightarrow du = e^x dx \Rightarrow \frac{du}{u} = dx$ (1)

$$\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1}u + c = -\operatorname{sech}^{-1}e^x + c$$ (1)

3) $\int xe^x\,dx$

$u = x \Rightarrow du = dx$

$dx = e^x dx \Rightarrow v = e^x$ $\Big\} \Rightarrow \int xe^x dx = xe^x - \int e^x dx$ $\frac{1}{2}$

$$= xe^x - e^x + c$$ (1)

$$= e^x(x-1) + c$$

4) $\int \sin^2 x \cos^5 x \, dx = \int \sin^2 x (\cos^2 x)^2 \cos x \, dx$

$u = \sin x \Rightarrow du = \cos x \, dx$

$I = \int u^2 (1-u^2)^2 \, du = \int u^2 (1 - 2u^2 + u^4) \, du$

$= \int u^2 - 2u^4 + u^6 \, du = \frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} + c$

$= \frac{\sin^3 x}{3} - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + c$

5) $\int \sin(5x) \cdot \cos(3x) \, dx = \frac{1}{2}\int [\sin(2x) + \sin(8x)] \, dx$

$= \frac{1}{2}\left[-\frac{1}{2}\cos(2x) - \frac{1}{8}\cos(8x)\right] + c$

$= -\frac{1}{4}\left[\cos(2x) + \frac{1}{4}\cos(8x)\right] + c$

6) $\int \frac{2x \, dx}{x^2+6x+10} = \int \frac{2x+6-6}{x^2+6x+10} \, dx = \int \frac{2x+6}{x^2+6x+10} \, dx - \int \frac{6}{x^2+6x+10} \, dx$

$= \ln|x^2+6x+10| - 6\int \frac{dx}{(x+3)^2+1}$

$= \ln|x^2+6x+10| - 6\tan^{-1}(x+3) + c$

7) $\int \sqrt{4-x^2} \, dx =$

$x = 2\sin\theta \Rightarrow dx = 2\cos\theta \, d\theta$

$= 2\int \sqrt{4-4\sin^2\theta} \cos\theta \, d\theta = 4\int \cos^2\theta \, d\theta = 2\int 1+\cos 2\theta \, d\theta$

$= 2\theta + \frac{2}{2}\sin 2\theta + c$

$= 2\sin^{-1}\frac{x}{2} + 2\sin\theta\cos\theta + c$

$= 2\sin^{-1}\frac{x}{2} + 2x\sqrt{1-\frac{x^2}{4}} + c$

8) $\int \frac{x+4}{x^3+4x}\,dx = \int \frac{x+4}{x(x^2+4)}\,dx$

$$\frac{x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} \quad (1)$$

$$x+4 = Ax^2 + Bx^2 + 4A + Cx \Rightarrow$$

$$\left.\begin{aligned} A+B &= 0 \\ C &= 1 \\ 4A &= 4 \end{aligned}\right\} \Rightarrow \quad A=1,\ C=1,\ B=-1 \quad (1)$$

$$\int \frac{x+4}{x(x^2+4)}\,dx = \int \frac{dx}{x} + \int \frac{1-x}{x^2+4}\,dx \quad (1)$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

9) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = \int \frac{dx}{x^{1/2}+x^{1/3}}$ $\left(\frac{1}{2}\right)$

$u = x^{1/6} \Rightarrow u^6 = x \Rightarrow dx = 6u^5\,du$

$$\int \frac{6u^5}{u^3+u^2}\,du = 6\int \frac{u^3}{u+1}\,du \quad \left(\frac{1}{2}\right) = 6\int\left[(u^2-u+1) + \frac{1}{u+1}\right]du$$

$$= 6\left[\frac{u^3}{3} - \frac{u^2}{2} + u + \ln|u+1|\right] + C \quad \left(\frac{1}{2}+\frac{1}{2}\right)$$

$$= 6\left[\frac{x^{1/2}}{3} - \frac{x^{1/3}}{2} + x^{1/6} + \ln|x^{1/6}+1|\right] + C \quad \left(\frac{1}{2}\right)$$