بسم الله الرحمن الرحيم

STAT 109

BIOSTATISTICS

Prepared by

Abdulrahman Alfaifi

King Saud University

Department of Statistics and Operation Research

alfaifi@ksu.edu.sa

@AlfaifiStat

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Chapter 1 Introduction

- **Some Basic Concepts:**

Descriptive Statistics: How to organize, summarize, and describe data.

Inferential Statistics: How to reach decisions about a large body of data by examine only a small part of the data.

Statistics:

1. Collection, organization, summarization, and analysis of data. (Descriptive Statistics).
2. Drawing of inferences and conclusions about a body of data (population) when only a part of the data (sample) is observed. (Inferential Statistics).

Biostatistics:

When the data is obtained from the biological sciences and medicine, we use the term "biostatistics".

Sources of Data:

1. Routinely kept records.
2. Surveys.
3. Experiments.
4. External sources. (published reports, data bank, …)

Population: the largest collection of entities (elements or individuals) in which we are interested at a particular time and about which we want to draw some conclusions.

Sample:

- A sample is a part of a population.
- We select various elements from the population on which we collect our data.

Variables:

The characteristic (خاصية أو ميزه) to be measured on the elements is called variable. The value of the variable varies (تتغير) from element to element.

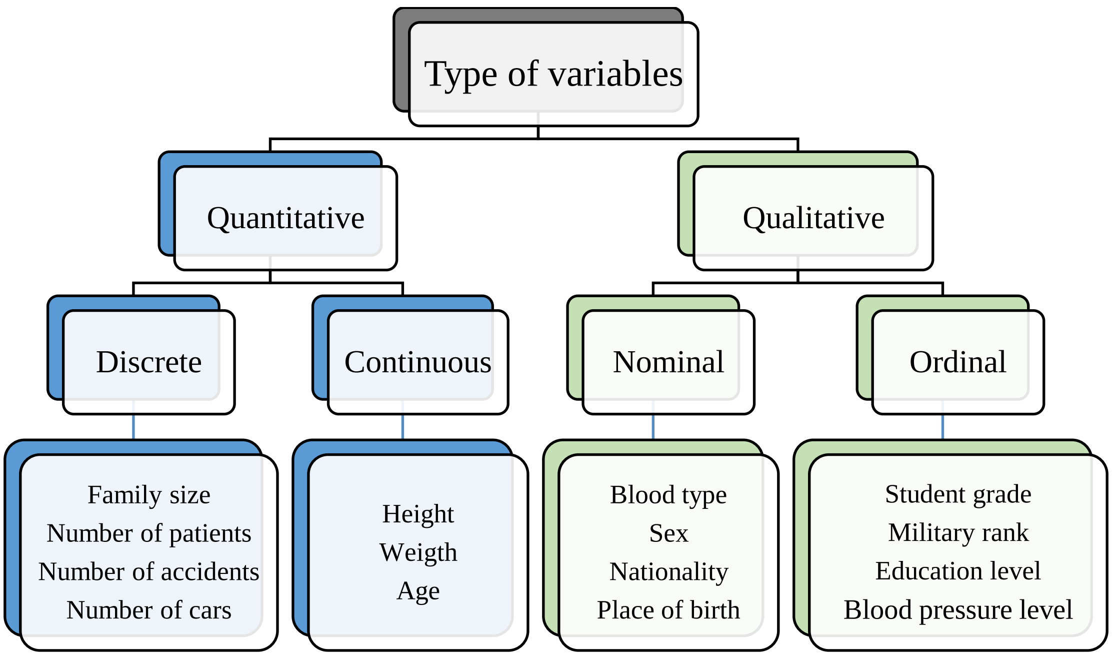
- <u>**Type of variables:**</u>

1) Quantitative Variables: is a characteristic that can be measured. The values of a quantitative variable are numbers indicating how much or how many of something.

Discrete variables	Continuous variables
There are jumps or gaps between the values.	There are no gaps between the values. A continuous variable can have any value within a certain interval of values.

2) Qualitative Variables: are words or attributes indicating to which category an element belongs.

Nominal variable	Ordinal variable
Classifies the observations into various mutually exclusive and collectively non-ranked categories.	Classifies the observations into various mutually exclusive and collectively ranked categories. The values of an ordinal variable are categories that can be ordered.



Explanation of "level" in the variables:

Blood pressure level (mmHg)	Quantitative continues.
Blood pressure level ($120 < X < 170$)	Quantitative continues.
Blood pressure level (Low, Normal, High)	Qualitative ordinal.
Blood pressure level	Qualitative ordinal.

Question 1:

1. For each of the following variables indicate whether it is quantitative or qualitative variable:

1.	The blood type of some patient in the hospital.	Qualitative nominal
2.	Blood pressure level of a patient.	Qualitative ordinal
3.	Weights of babies born in a hospital during a year.	Quantitative continues
4.	Gender of babies born in a hospital during a year.	Qualitative nominal
5.	The distance between the hospital to the house.	Quantitative continues
6.	Under-arm temperature of day-old infants born in a hospital.	Quantitative continues

2. Which of the following is an example of discrete variable

A	The number of students taking statistics in this term at KSU.
B	The time to exercise daily.
C	Whether or not someone has a disease.
D	Height of certain buildings.
E	Level of education.

3. Which of the following is not an example of discrete variable

A	The number of students at the class of statistics.
B	The number of times a child cry in a certain street.
C	The time to run a certain distance.
D	The number of buildings in a certain street.
E	The number of educated persons in a family.

4. Which of the following is an example of qualitative variable

A	The blood pressure in (mmHg).
B	The number of times a child brushes his/her teeth.
C	Whether or not someone fail in an exam.
D	Weight of babies at birth.
E	The time to run a certain distance.

5. The continuous variable is a

A	Variable with a specific number of values.
B	Variable which can’t be measured.
C	Variable takes on values within intervals.
D	Variable with no mode.
E	Qualitative variable.

6. Which of the following is an example of continuous variable

A	The number of visitors of the clinic yesterday.
B	The time to finish the exam.
C	The number of patients suffering from certain disease.
D	Whether or not the answer is true.

7. The discrete variable is

A	Qualitative variable.
B	Variable takes on values within interval.
C	Variable with a specific number of values.
D	Variable with no mode.

8. Which of the following is an example of nominal variable:

A	Age of visitors of a clinic.
B	The time to finish the exam.
C	Whether or not a person is infected by influenza.
D	Weight for a sample of girls.

9. The nominal variable is a

A	A variable with a specific number of values.
B	Qualitative variable that can’t be ordered.
C	variable takes on values within interval.
D	Quantitative variable.

10. Which of the following is an example of nominal variable:

A	The number of persons who are injured in accident.
B	The time to finish the exam.
C	Whether or not the medicine is effective.
D	Socio-economic level.

11. The ordinal variable is:

A	variable with a specific number of values.
B	variable takes on values within interval.
C	Qualitative variable that can be ordered.
D	Variable that has more than mode.

12. One of the following is an example of an ordinal variable:

A	Socio-economic level.
B	Blood type of a sample of patients.
C	The time of finish the exam.
D	The number of persons who are injured in accidents.

13. The number of students admitted in College of Medicine in King Saud University is a variable of type

A	Discrete.
B	Qualitative.
C	Continuous.
D	Nominal.

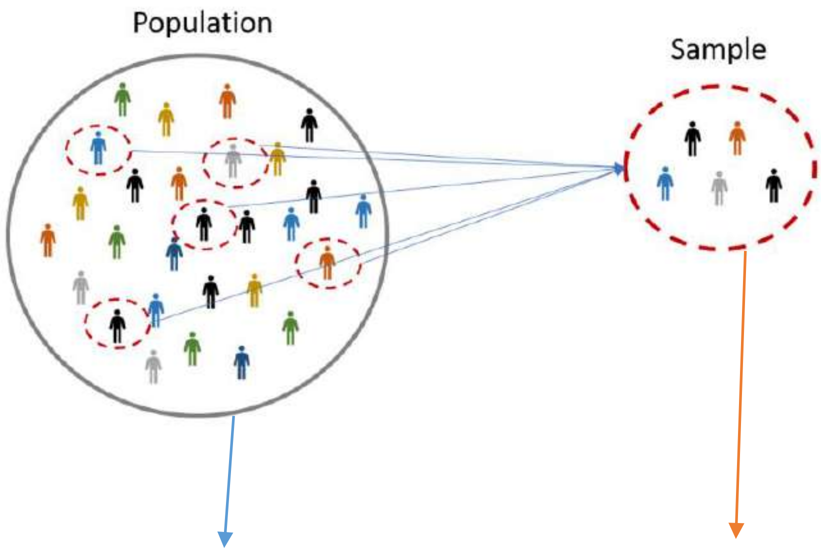
14. If a researcher interests to study the blood pressure level (High, Normal, Low) for 13 diabetics patients, what is the type of variables?

A	Qualitative nominal.
B	Quantitative nominal.
C	Qualitative ordinal.
D	Quantitative ordinal.

- **Statistical Inference:**

Parameter: is a measure obtained from the population.

Statistic: is a measure obtained from the sample. statistics are used to approximate (estimate) parameters.



Parameter	Statistic
Population mean μ Population variance σ^2	Sample mean $\overline{X}$ Sample variance S^2

Question 2:

1. A mean of a population is called:

A	Parameter	B	Statistic	C	Median	D	Mode

2. The measure that obtained from the population is called

A	Parameter	B	Sample	C	Population	D	Statistic

3. The measure that obtained from the sample is called

A	Parameter	B	Sample	C	Population	D	Statistic

4. Which of the following is an example of a statistic?

A	Population variance	B	Sample median	C	Population mean	D	Population mode

Question 3:

1. A sample is defined as:

A	The entire population of values.
B	A measure of reliability of the population.
C	A subset of data selected from a population.
D	Inferential statistics.

2. One of the following is an example of a statistic:

A	The sample mode.
B	The population median.
C	The population variance.
D	None of these.

3. One of the following is a part of a population:

A	Sample.
B	Statistic
C	Variable
D	None of these

4. The variable is a

A	Characteristic of the population to be measured.
B	Subset of the population.
C	Parameter of the population.
D	None of these.

Question 4:

From men with age more than 20 years living in Qaseem, we select 200 men. It was found that the average weight of the men was 76 kg.

1. The variable of interest is:

A	Age	B	Weight	C	200 men	D	76 kg

2. The sample size is:

A	76	B	20	C	200	D	1520

Question 5:

For each of the following situations, what is the population, the sample, the variables of interest and the type of variable:

Situation A: A study of 300 families in a small southern town to determine whether the family had school-age child.

The population	All families in a small southern town.
The sample	300 families in a small southern town.
The variable	whether the family had school-age child.
Type of variable	Qualitative nominal.

Situation B: A study was conducted on 250 patients over the past year to measure the distance a patient travels to reach a certain hospital.

The population	All patients admitted to a hospital during the past year.
The sample	250 patients admitted to a hospital during the past year.
The variable	The distance a patient travels to reach a certain hospital.
Type of variable	Quantitative continuous.

Question 6:

A study of 250 patients admitted to a hospital during the past year revealed that, on the average (mean), the patients lived 15 miles from the hospital.

1. The sample in the study is:

A	250 patients	B	250 hospitals	C	250 houses	D	15 miles

2. The population in this study is:

A	Some patients admitted to the hospital during the past year.
B	All patients admitted to the hospital during the past year.
C	250 patients admitted to the hospital during the past year.
D	500 patients admitted to the hospital during the past year.

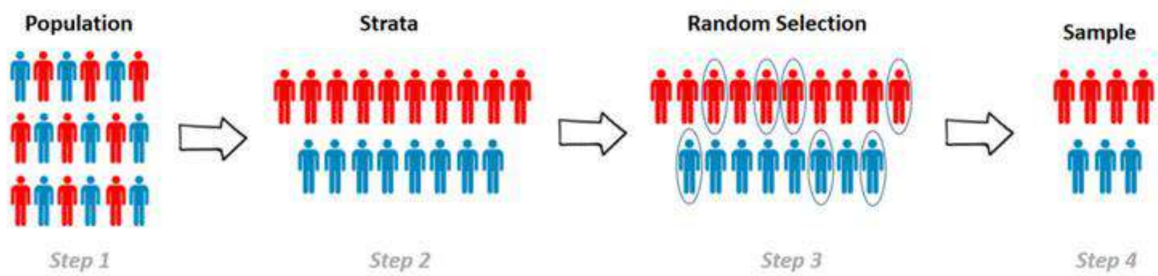
- **<u>Sampling and Statistical Inference:</u>**

(1) Simple Random Sampling:

If a sample of size (n) is selected from a population of size (N) in such a way that each element in the population has the same chance to be selected, the sample is called a simple random sample.

(2) Stratified Random Sampling:

In this type of sampling, the elements of the population are classified into several homogenous groups (strata). From each group, an independent simple random sample is drawn. The sample resulting from combining these samples is called a stratified random Sample.



<u>Question 7:</u>

A researcher was interested in estimating the mean of monthly salary of a certain city. There were 5000 employees in the city (2000 of which were female and 3000 of which were males). He selected a random sample of 40 female employees, and he independently selected a random sample of 60 male employees. Then, he combined these two random samples to obtain the random sample of his study.

1. The population size is:

A	100	B	3000	C	2000	D	5000

2. The sample size is:

A	40	B	60	C	100	D	1000

3. The variable of interested is:

A	Employee	B	City	C	Sex	D	Salary

4. The type of the random sample of this study is:

A	Stratified random sample	B	Simple random sample

5. If each element in the population has the same chance to be selected in the sample, then the sample called:

A	Simple random sample.
B	Sample space.
C	Stratified sample.
D	Complete sample.

Chapter 2 Strategies for Understanding the Meaning of Data

- **Frequency Tables:**

The Ordered Array:

An ordered array is a listing of the values in order of magnitude from the smallest to the largest value.

Grouped Data (The Frequency Distribution):

To group a set of observations (class interval), we select suitable class internals that is:

- Contiguous (متجاورة أو متلامسه)
- Non-overlapping intervals (فترات غير متداخله)
- Each value belongs to one, and only one, interval.

In Frequency Table:

True class intervals: indicate that there are no gaps between true class intervals (The end-point (true upper limit) of each true class interval equals to the start-point (true lower limit) of the following true class interval.

Mid-point of a class interval: is considered as a typical (approximated) value for all values in that class interval.

Cumulative frequency:

Cumulative frequency of the 1^{st} class interval = frequency
Cumulative frequency of other class intervals = frequency + cumulative frequency of the preceding class interval

Relative frequency and Percentage frequency:

Relative frequency = frequency / *n*
Percentage frequency = Relative frequency × 100%

Question 1:

The "life" of 40 similar car batteries recorded to the nearest tenth of a year.

The batteries are guaranteed to last 3 years.

Class Interval	True class Interval	Midpoint	Frequency	Relative Frequency
1.5–1.9	1.45–1.95	1.7	2	0.050
2.0–2.4	1.95–2.45	2.2	D	0.025
2.5–2.9	2.45–2.95	C	4	F
A	2.95–3.45	3.2	15	0.375
3.5–3.9	B	3.7	E	0.250
4.0–4.4	3.95–4.45	4.2	5	0.125
4.5–4.9	4.45–4.95	4.7	3	0.075

1. The value of A: $\boxed{3.0 - 3.4}$
2. The value of B: $\boxed{3.45 - 3.95}$
3. The value of C: $\boxed{C = \frac{2.45+2.95}{2} = 2.7}$
4. The value of D: $\boxed{\frac{D}{40} = 0.025 \Longrightarrow D = 40 \times 0.025 = 1}$
5. The value of E : $\boxed{\frac{E}{40} = 0.25 \Longrightarrow E = 40 \times 0.25 = 10}$
6. The value of F: $\boxed{F = \frac{4}{40} = 0.10}$

Question 2:

Fill in the table given below. Answer the following questions.

Class Interval	Frequency	Cumulative Frequency	Relative Frequency	Cumulative Relative Frequency
5 - 9	8			
10 - 14	15		C	
15 – 19	11	B		D
20 - 24	A	40	0.15	

1) The value of A is: $\boxed{A = 40 - (8 + 15 + 11) = 40 - 34 = 6}$

2) The value of B is: $\boxed{B = 8 + 15 + 11 = 34}$

3) The value of C is: $\boxed{C = \frac{15}{40} = 0.375}$

4) The value of D is: $\boxed{D = \frac{34}{40} = 0.85}$

5) The true class interval for the first class is: $\boxed{4.5 - 9.5}$

6) The number of observations less than 19.5 is: $\boxed{8 + 15 + 11 = 34}$

Question 3:

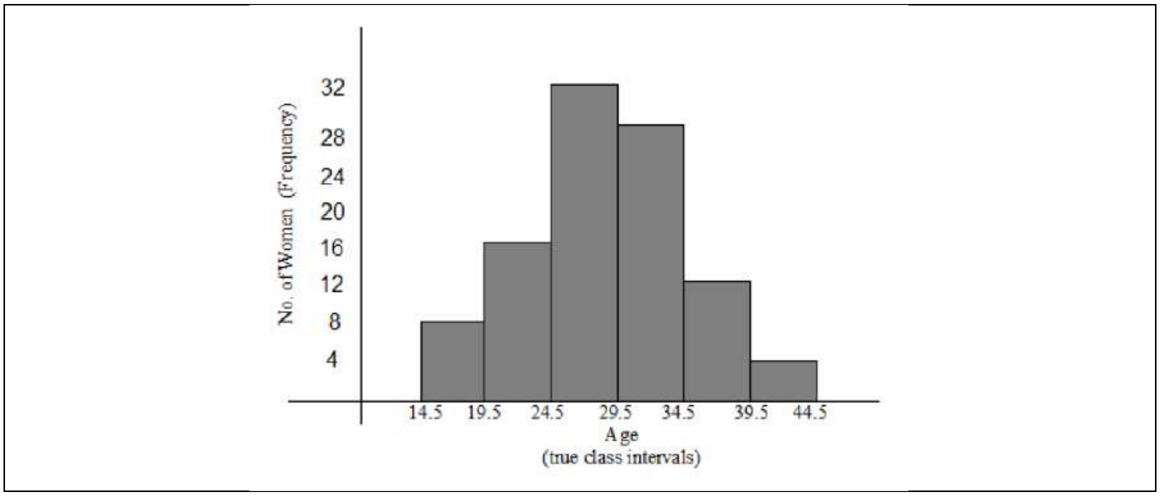
The table shows the weight loss (kg) of a sample of 40 healthy adults who fasted in Ramadan.

Class interval	Frequency	Cumulative Frequency
1.20 - 1.29	2	2
1.30 - 1.39	6	8
1.40 - 1.49	10	K
1.50 - 1.59	C	34
1.60 - 1.69	6	40

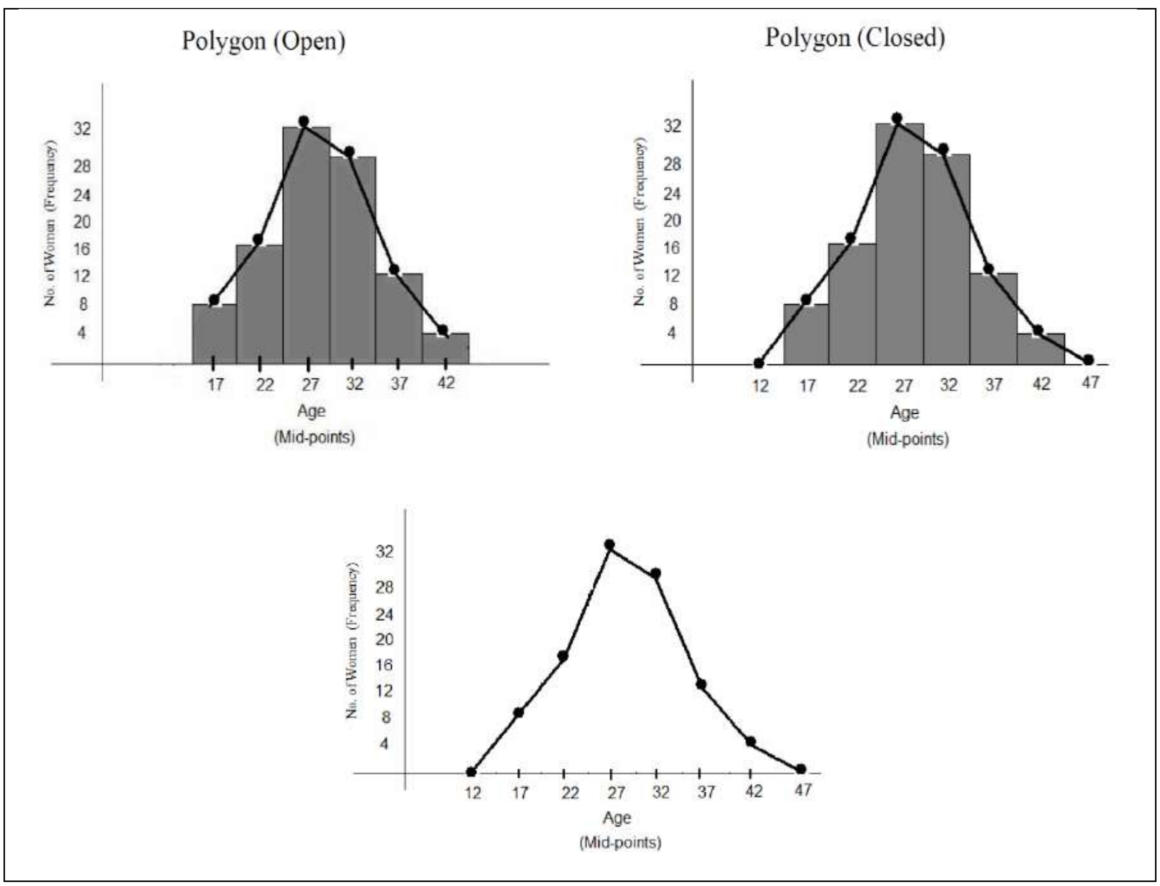
1) The value of the missing value K is $\boxed{18}$
2) The value of the missing value C is $\boxed{16}$

- **Displaying Grouped Frequency Distributions:**

1. Histogram:

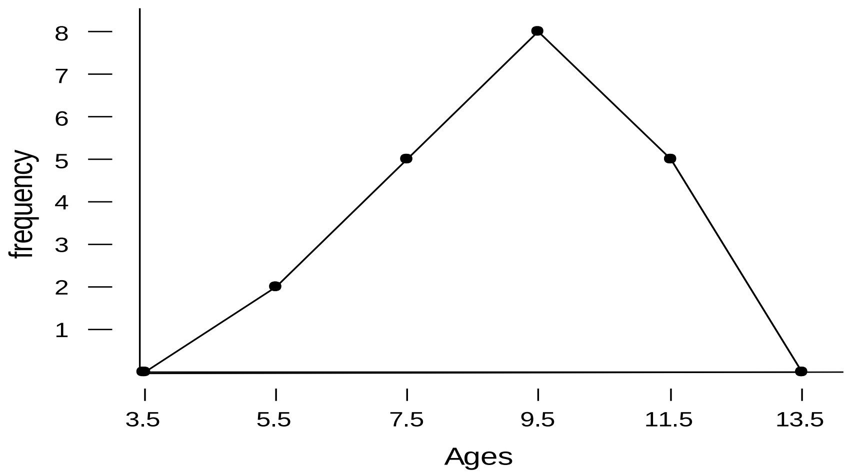


2. Polygon:



Question 4:

Consider the following frequency polygon of ages of 20 students in a certain school.



The frequency distribution of ages corresponding to above polygon is

(a)

True class limits	4.5- 6.5	6.5-8.5	8.5- 10.5	10.5 -12.5
frequency	2	5	8	5

(b)

True class limits	3.5- 5.5	5.5-7.5	7.5- 9.5	9.5 -11.5
frequency	2	5	8	4

(c)

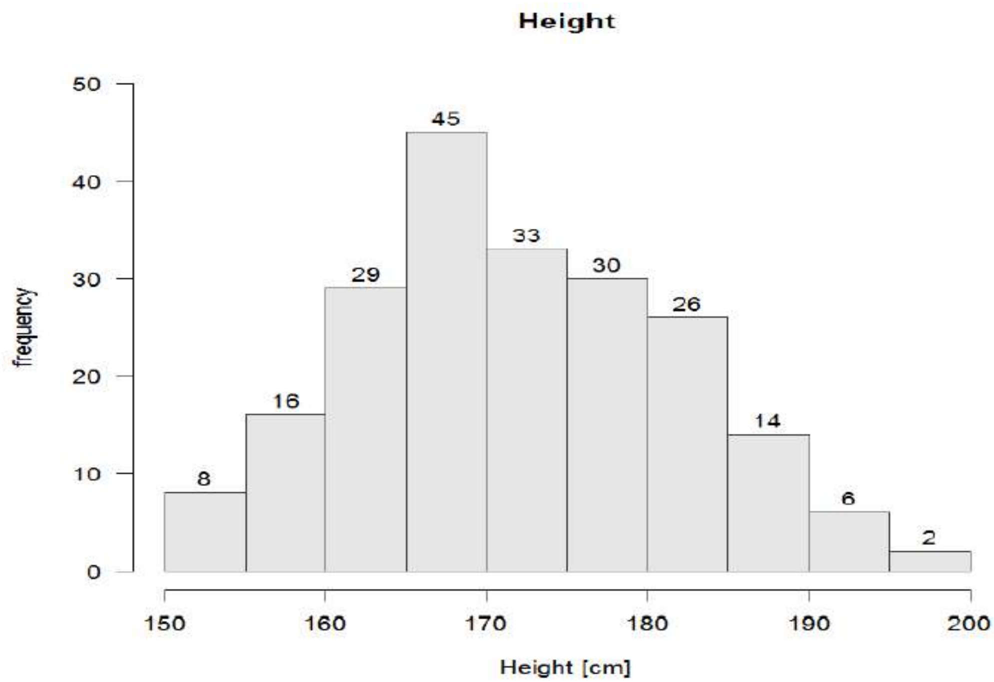
Class interval	5- 6	7-8	9- 10	11 -12
frequency	1	7	8	4

(d)

Class interval	5- 6	7-8	9- 10	11 -12
frequency	4	7	8	6

Question 5:

For a sample of students, we obtained the following graph for their height in (cm).



1. The variable under study is:

A	Patients	B	Graph	C	Height	D	Discrete

2. The type of variable:

A	Continuous	B	Discrete	C	Frequency	D	Height

3. The number of students with the lowest level height:

A	14	B	2	C	115	D	8

4. The sample size is:

A	28	B	209	C	156	D	130

5. The midpoint of the interval with highest frequency is:

A	182.5	B	130.5	C	167.5	D	30

6. The relative frequency of the interval with highest frequency is:

A	0.283	B	0.215	C	0.241	D	0.262

Question 6:

The following table gives the distribution of the ages of a sample of 50 patients who attend a dental clinic.

Age intervals (in years)	Frequency	Relative frequency
10 - 15	4	-
16 - 21	8	-
22 - 27	z	0.32
28 - 33	-	-
34 - 39	10	-

Less than	Cumulative Frequency
10	0
16	4
22	y
28	--
34	--
40	x

1. The class width is:

A	6	B	10	C	150	D	19

2. The value of x is:

A	22	B	28	C	50	D	10

3. The value of y is:

A	4	B	12	C	19	D	150

4. The value of z is:

A	14	B	12	C	50	D	16

5. Percent of the patients with age between 16 and 21 is:

A	16%	B	8%	C	20%	D	32%

6. The 5^{th} interval midpoint is:

A	38	B	52	C	27	D	36.5

Question 7:

Consider the following Table showing a frequency distribution of weights in a sample of 20 cans of fruits:

Class interval	True Class Limits	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
19.2 – 19.4			1		
19.5 – 19.7				0.10	
19.8 – 20.0			8		
			4		

1. The fifth-class interval is:

A	20.2 - 20.4	B	20.1-20.3	C	21.0 - 21.2	D	20.4 - 20.6

2. The second true class interval is:

A	19.45 - 19.75	B	19.5 – 19.7	C	19.25 -19.35	D	20.2 - 20.4

3. The midpoint of the fourth-class interval is:

A	20.5	B	20.2	C	19.9	D	20.1

4. The frequency of the second-class interval is:

A	10	B	4	C	2	D	3

5. The relative frequency of the fourth-class interval is:

A	0.20	B	0.15	C	0.13	D	0.40

6. The cumulative frequency of the final class interval is:

A	13	B	4	C	20	D	100

Question 8:

The following table gives the distribution of weight (in Kg) of a sample of 200 persons.

Weights	Frequency	Cumulative frequency	Relative frequency	Relative cumulative frequency
50-54	40			
56-60	A		0.15	
62-66		B		
68-72	60		C	
74-76	20			D

[1] The value of A is

A	20	B	30	C	40	D	50

[2] The class interval B is

A	90	B	100	C	110	D	120

[3] The value of C is

A	0.3	B	0.4	C	0.5	D	0.6

[4] The value of D is

A	0.70	B	0.85	C	0.90	D	1

Question 9: (اختر الإجابة بعد التقريب)

Consider the following table showing a frequency distribution of blood test of 52 diabetes patients.

Class interval	Frequency	Cumulative frequency	Relative frequency	Cumulative relative frequency
101 – 120	--	--	0.4423	--
121 – 140	--	--	--	D
B	--	C	0.2115	--
161 – 180	--	--	0.0577	--
Total	A	--	1	--

[1] The value of A is

A	1	B	3	C	52	D	80

[2] The class interval B is

A	122-140	B	161-180	C	131-140	D	141-160

[3] The value of C is

A	49	B	15	C	34	D	52

[4] The value of D is

A	0.5308	B	0.7308	C	0.4308	D	0.8308

[5] The true class intervals are

A	100 –120 120.5 –139.5 141 –160 161 –180	B	99.5 – 119.5 120.5 –140.5 140.5 –159.5 160.5 –179.5	C	100.5 – 120.5 120.5 –140.5 140.5 – 160.5 160.5 – 180.5	D	100.5 – 120.5 121.5 – 140.5 141.5 –160.5 161.5 – 180.5

[6] The midpoint of the first-class interval is

A	110.5	B	20	C	220	D	19

[7] Histogram of the frequency distribution is built based on

A	Frequency and cumulative.
B	Midpoints and cumulative.
C	True class interval and frequency.
D	None of them.

Question 10:

1. To group a set of observations in a frequency table, we should not do one of the following:

A	The intervals are overlapping.
B	The intervals are ordering from the smallest to the largest.
C	The minimum value of the observation belongs to the first interval.
D	The number of intervals should be no fewer than five class intervals.

2. If the lower limit of a class interval is 25 and the upper limit of this class interval is 30, the midpoint is equal to

A	27.5	B	2.5	C	27	D	5

3. If 7 out of 45 CEOs have a master's degree, then the relative frequency is equal to:

A	0.1556	B	7	C	45	D	15.56%

3. If 7 out of 45 CEOs have a master's degree, then the percentage of the relative frequency is equal to:

A	15.56%	B	0.1556	C	7	D	45

- **<u>Measures of Central tendency and Dispersion</u>**

A descriptive measure: is the concept of summarization of the data by means of a single number.

Measures of Central Tendency (Location):

• The values of a variable tend to be concentrated around the centre of the data.
• The centre of the data can be determined by the measures of central tendency.
• A measure of central tendency is considered to be a typical value of the set of data as a whole.

- **Advantages and disadvantages of the measures of central tendency:**

Measure of central tendency	Advantages	Disadvantages
Mean	Easily understood and easy to compute (Simplicity). There is one and only one mean for a given set of data (Uniqueness). Takes into account all values of all data.	Extreme values have an influence (effect) on the mean. Can only be found for quantitative variables.
Median	Easily understood and easy to compute (Simplicity). There is one and only one median for a given set of data (Uniqueness). Not affected by extreme values.	Does not take into account all values of the sample. In general, the median can only be found for quantitative variable. However, in some cases, the median can be found for ordinal qualitative variables.
Mode	Easily understood and easy to compute (Simplicity). Not affected by extreme values. The mode may be found for both quantitative and qualitative variables.	Not “good” measure, because it depends on few values of the data. Does not take into account all values of the sample. There might be no mode for a data set. There might be more than one mode for a data set.

- **For median:**

If the sample size (n) is an odd number, the rank of the median is

Ordered set	y_1	y_2	...	y_m	...	y_n
Rank	1	2	...	m	...	n

$$Rank = \frac{n+1}{2} = m$$

If the sample size (n) is an even number, the rank of the median is

Ordered set	y_1	y_2	...	y_m	y_{m+1}	...	y_n
Rank	1	2	...	m	$m+1$	...	n

$$Rank = \frac{n+1}{2} = m + 0.5$$

Find the median:

Student's ages: 4, 5, 2, 9, 10, 8, 4

عدد المشاهدات فردي

| 2 4 4 **[5]** 8 9 10 | $\Longrightarrow$ Median = 5

Student's ages: 10, 13, 9, 20, 11, 100

عدد المشاهدات زوجي

| 9 10 **[11 13]** 20 100 | $\Longrightarrow$ Median $= \frac{11+13}{2} = 12$

Student's grades: A, C, B, C, F, B, B

عدد المشاهدات فردي

| A B B **[B]** C C F | $\Longrightarrow$ Median = B

Student's grades: A, C, B, C, F, B, B, B

عدد المشاهدات زوجي

| A B B **[B B]** C C F | $\Longrightarrow$ Median = B

Student's grades: A, C, B, C, F, B, C, B

عدد المشاهدات زوجي

| A B B **[B C]** C C F | $\Longrightarrow$ No median

Measures of Dispersion (Variation or Shape):

The variation of a set of observations refers to the variety that they exhibit (يعرض). A measure of dispersion conveys information regarding the amount of variability present in a set of data. The variation or dispersion in a set of values refers to how spread out the values is from each other.

- The dispersion (variation) is small when the values are close together.
- There is no dispersion (no variation) if the values are the same.

The Variance:

- The variance is one of the most important measures of dispersion.
- The variance is a measure that uses the mean as a point of reference.
- The variance is small when the observations are close to the mean.
- The variance is large when the observations are spread out from the mean.
- The variance of the data is zero (no variation) when all observations have the same value (concentrated at the mean).

Standard Deviation:

The variance represents squared units, therefore, is not appropriate measure of dispersion when we wish to express the concept of dispersion in terms of the original unit. Standard deviation is better in this case.

Coefficient of Variation (C.V.):

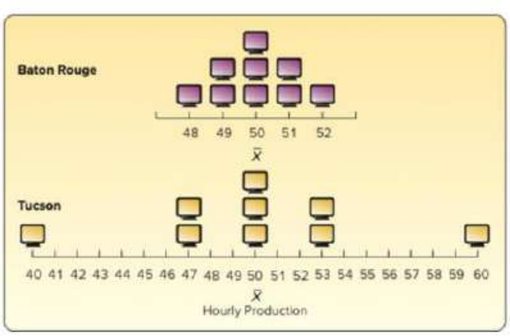
If we want to compare the variation of two variables, we cannot use the variance or the standard deviation because:

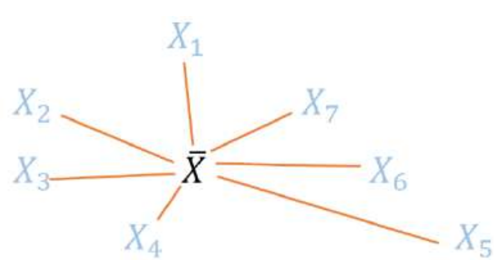
1. The variables might have different units.
2. The variables might have different means.

- We need a measure of the relative variation that will not depend on either the unit or on how large the values are. This measure is the coefficient of variation (C.V.)
- The C.V. is unit-less.
- The data set with larger value of C.V. has larger variation.

Measures of central tendency (Location)		
Mean	$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$	Unit
Median		Unit
Mode	The value with the highest frequency	Unit
Measures of dispersions (Shape)		
Range	$R = max - min$	Unit
Variance	$S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$ $S^2 = \frac{\sum_{i=1}^{n} {X_i}^2 - n\bar{X}^2}{n-1}$	Unit^2
Standard deviation	$S = \sqrt{S^2}$	Unit
Coefficient of variation	$C.V = \frac{S}{\bar{X}}$	Unit less

- Example: Hourly production at two computer monitor plants.





Question 11:

If the number of visits to the clinic made by 8 pregnant women in their pregnancy period is:

12 15 16 12 15 16 12 14

1. The type of the variable is:

   discrete

2. The sample mean is:

   $$\overline{X} = \frac{12+15+16+12+15+16+12+14}{8} = 14$$

3. The sample standard deviation is:

$$S^2 = \frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}$$

$$= \frac{(12-14)^2+(15-14)^2+(16-14)^2+(12-14)^2+(15-14)^2+(16-14)^2+(12-14)^2+(14-14)^2}{8-1}$$

$$= 3.14 \Longrightarrow S = 1.77$$

4. The sample median is:

   $$12 \;\; 12 \;\; 12 \;\; \boxed{14 \;\; 15} \;\; 15 \;\; 16 \;\; 16 \Longrightarrow \frac{14+15}{2} = 14.5$$

5. The coefficient of variation is:

   $$C.V = \frac{s}{\overline{X}} = \frac{1.77}{14} = 0.1266$$

6. The range is:

   $$16 - 12 = 4$$

Question 12:

Consider the following marks for a sample of students carried out on 10 quizzes:

6, 7, 6, 8, 5, 7, 6, 9, 10, 6

1. The mean mark is:

$$\boxed{\overline{X} = \frac{6+7+6+8+5+7+6+9+10+6}{10} = 7}$$

2. The median mark is:

$$\boxed{5\ 6\ 6\ 6\ \boxed{6\ 7}\ 7\ 8\ 9\ 10} \Longrightarrow \frac{6+7}{2} = 6.5$$

3. The mode for this data is: $\boxed{6}$
4. The range for this data is: $\boxed{5}$
5. The standard deviation for this data is:

$$\boxed{S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1} = \frac{(5-7)^2+(6-7)^2+\cdots+(10-7)^2}{10-1} = 2.434 \Longrightarrow S = 1.56}$$

6. The coefficient of variation for this data is:

$$\boxed{C.V = \frac{s}{\overline{X}} = \frac{1.56}{7} = 0.223}$$

Question 13:

Twenty adult males between the ages of 30 and 40 participated in a study to evaluate the effect of a specific health regimen involving diet and exercise on the blood cholesterol. Ten were randomly selected to be a control group, and ten others were assigned to take part in the regimen as the treatment group for a period of 6 months. The following data show the mean and the standard deviation of reduction in cholesterol experienced for the time period for the 20 subjects:

Control group: mean= 6.5, standard deviation=4.33

Treatment group: mean= 7.6, standard deviation=5.32.

By comparing the variability of the two data sets, we get

$$C.V_{Cont} = \frac{s_{Cont}}{\overline{X}_{Cont}} \times 100 = \frac{4.33}{6.5} \times 100 = 66.61\%$$

$$C.V_{Treat} = \frac{s_{Treat}}{\overline{X}_{Treat}} \times 100 = \frac{5.32}{7.6} \times 100 = 70\%$$

The relative variability of the control group is less than relative variability of the treatment group.

<u>Question 14:</u>

The data for measurements of the left ischia tuberosity (in mm Hg) for the SCI and control groups are shown below.

Control	131	115	124	131	122
SCI	60	150	130	180	163

1. The mean for the control group is:

$$\overline{X} = \frac{131+115+124+131+122}{5} = 124.60$$

2. The variance of the SCI group is:

$$\overline{X} = \frac{60+150+130+180+163}{5} = 136.6$$

$$S^2 = \frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1} = \frac{(60-136.6)^2+(150-136.6)^2+(130-136.6)^2+(180-136.6)^2+(163-136.6)^2}{5-1} = 2167.8$$

3. The unit of coefficient of variation for SCI group is

A	mm Hg	B	Hg	C	mm	D	Unit-less

4. Which group has more variation:

$$S^2 = \frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}$$

$$= \frac{(131-124.6)^2+(115-124.6)^2+(124-124.6)^2+(131-124.6)^2+(112-124.6)^2}{5-1} = 45.3 \Longrightarrow S = 6.7305$$

$$C.V_{Cont} = \frac{S_{Cont}}{\overline{X}_{Cont}} \times 100 = \frac{6.7305}{124.6} \times 100 = 5.4\%$$

$$C.V_{SCI} = \frac{S_{SCI}}{\overline{X}_{SCI}} \times 100 = \frac{\sqrt{2167.8}}{136.6} \times 100 = 34.08\%$$

A	Control group.
B	SCI group.
C	Both groups have the same variation.
D	Cannot compare between their variations.

Question 15:

Temperature (in Farahenient) recorded at 2 am in London on 8 days randomly chosen in a year were as follows: 40 -21 38 -9 26 -21 -49 44

1) The average temperature for the sample is:

A	248	B	1	C	6	D	48

2) The median temperature for the sample is:

A	8.5	B	-21	C	-8.5	D	17

3) The mode of temperature for the sample is:

A	-21	B	44	C	2	D	-49

4) The standard deviation for the sample data is:

A	35.319	B	30.904	C	1247.43	D	4

5) The coefficient of variation for the sample is:

A	17%	B	49%	C	4%	D	588.7%

6) The range of the sample is:

A	4	B	8	C	40	D	93

Question 16:

Consider the following weights for a sample of 6 babies: 5, 3, 5, 2, 5, 4

[1] The sample mean is

A	4	B	5	C	3	D	6

[2] The sample median is

A	4	B	5	C	4.5	D	3

[3] The sample mode is

A	4	B	3	C	4.5	D	5

[4] The sample standard deviation is

A	3.2649	B	8.2649	C	1.2649	D	2.2649

[5] The coefficient of variation for this sample is

A	40.00%	B	31.62%	C	200%	D	12.50%

Question 17:

Some families were selected and the number of children in each family were considered as follows: 5, 8, 0, 8, 3, 7, 8, 9 Then,

1) The sample size is:

A	9	B	6	C	8	D	5

2) The sample mode is:

A	9	B	0	C	8	D	No mode

3) The sample mean is:

A	48	B	6	C	8	D	0

4) The sample variance is:

A	2.915	B	8.5	C	9.714	D	3.117

5) The sample median is:

A	5.5	B	7.5	C	8	D	7

6) The range of data is:

A	8	B	0	C	3	D	9

7) The sample coefficient of variation is:

A	5.5	B	8	C	0.52	D	7

Question 18:

1. The …... is a measure of location?

A	Median	B	Range	C	Variance	D	CV

2.The …... is not affected by extreme values?

A	Range	B	Mean	C	Median	D	Variance

3.Which of the following measures can be used for the blood type in a given sample?

A	Median	B	Mean	C	Variance	D	Mode

Question 19:

The frequency table for daily number of car accidents during a month is:

Number of car accidents	Frequency
3	2
4	3
5	1
6	2
7	2
Total	10

3,3,4,4,4,5,6,6,7,7

1. The type of variable:

A	Nominal	B	Discrete	C	Ordinal	D	Continuous

2. The mean for the number of accidents is:

A	4.07	B	4.90	C	3.75	D	2.98

3. The median is:

A	5.5	B	5	C	4.5	D	4

4. The mode is:

A	4	B	5	C	6	D	3

5. The variance for the number of accidents is:

A	8.45	B	6.43	C	2.32	D	1.05

6. The coefficient of the variation is:

A	2%	B	31%	C	22%	D	12%

Question 20:

1. The biggest advantage of the standard deviation over the variance is:

A	The standard deviation is always greater than the variance.
B	The standard deviation is calculated with the median instead of the mean.
C	The standard deviation is better for describing the qualitative data.
D	The standard deviation has the same units as the original data.

2. Parameters and statistics:

A	Describe the same group of individuals.
B	Describe the population and the sample, respectively.
C	Describe the sample and the population, respectively.
D	None of these.

3. Which of the following location (central tendency) measures is affected by extreme values?

A	Median
B	Mean
C	Variance
D	Range

4. Which of the following measures can be used for the blood type in a given sample?

A	Mode
B	Mean
C	Variance
D	Range

5. If x_1, x_2 *and* x_3 has mean $\bar{x} = 4$, then x_1, x_2, x_3 and $x_4 = 4$ has mean:

A	equal 4
B	less than 4
C	greater than 4
D	None of this

6. The sample mean is a measure of

A	Relative position.
B	Dispersion.
C	Central tendency.
D	all of the above

7. The sample standard deviation is a measure of

A	Relative position.
B	Central tendency.
C	Dispersion.
D	all of the above.

8. Which of the following are examples of measures of dispersion?

A	The median and the mode.
B	The range and the variance.
C	The parameter and the statistic.
D	The mean and the variance.

9. If a researcher interests to study the blood pressure level (High, Normal, Low) for 13 diabetics patients, he may use:

A	Median and / or mode
B	Mean
C	Variance
D	Range

Question 21:

Find the mean and the variance for: 6 , 5 , 9 , 6 , 7 , 3

- $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{6+5+9+6+7+3}{6} = 6$

- $S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$

$$= \frac{(6-6)^2+(5-6)^2+(9-6)^2+(6-6)^2+(7-6)^2+(3-6)^2}{6-1} = 4$$

Question 22:

Find the mean and the variance: If $\sum_{i=1}^{6} X_i = 36$ and $\sum_{i=1}^{6} X_i^2 = 236$.

- $\overline{X} = \frac{\sum_{i=1}^{6} X_i}{6} = \frac{36}{6} = 6$

- $S^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2}{n-1} = \frac{236 - 6 \times 6^2}{6-1} = \frac{236-216}{5} = 4$

Question 23:

Suppose two samples of human males yield the following data (which is more variation)

	Sample 1 25 year	Sample 2 11 year
Mean weight	135 pound	60 pound
Standard deviation	10 pound	10 pound
Coefficient of variation (C.V)	$C.V_1 = \frac{S}{\overline{\overline{X}}} \times 10$ $= \frac{10}{135} \times 100$ $= 7.41\%$	$C.V_2 = \frac{S}{\overline{\overline{X}}} \times 100$ $= \frac{10}{60} \times 100$ $= 16.67\%$

Sample 2 has more variation tan sample 1

Question 24:

The following values are calculated in respect of heights and weights for sample of students, can we say that the weights shoe greater variation than the heights.

	Sample 1 height	Sample 2 weight
Mean	162.6 cm	52.36 kg
variance	127.69 cm^2	23.14 kg^2
Coefficient of variation (C.V)	$C.V_1 = \frac{S}{\overline{\overline{X}}} \times 10$ $= \frac{\sqrt{127.69}}{162.6} \times 100$ $= 6.95\%$	$C.V_2 = \frac{S}{\overline{\overline{X}}} \times 100$ $= \frac{\sqrt{23.14}}{52.36} \times 100$ $= 9.19\%$

Since CV_2 greater than CV_1, therefore we can say the weights show more variability than height

Chapter 3 Probability

Probability

Probability:

Probability is a measure (or number) used to measure the chance of the occurrence of some event. This number is between 0 and 1.

An experiment:

An experiment is some procedure (or process) that we do.

Sample space:

The sample space of an experiment is the set of all possible outcomes of an experiment. Also, it is called the universal set, and is denoted by Ω.

An event:

Any subset of the sample space Ω is called an event.

- $\emptyset \subseteq \Omega$ is an event (impossible event)
- $\Omega \subseteq \Omega$ is an event (sure event)

Equally likely outcomes:

The outcomes of an experiment are equally likely if the outcomes have the same chance of occurrence.

Probability of an event:

If the experiment has $n(\Omega)$ equally likely outcomes, then the probability of the event E is denoted by $P(E)$ and is defined by:

$$\boxed{P(E) = \frac{n(E)}{n(\Omega)} = \frac{\#\ of\ outcomes\ in\ E}{\#\ of\ outcomes\ in\ \Omega}}$$

Some Operations on Events:

Union of two events $\boxed{\cup}$:

The event $A \cup B$ consist of all outcomes in A or in B or in both A and B.

The event $A \cup B$ occurs if A occurs, or B occurs, or both A and B occur.

Intersection of two events $\boxed{\cap}$:

The event $A \cap B$ consist of all outcomes in both A and B.

The event $A \cap B$ occurs if both A and B occur.

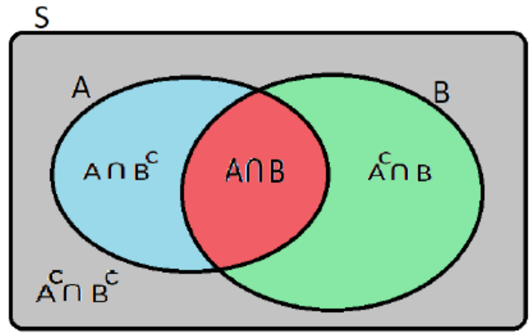
Conditional probability:

The conditional probability of the event A when we know that the event B has already occurred is defined by:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad ; \ P(B) \neq 0$$

Definitions and Theorems:

* $0 \le P(A) \le 1$
* $P(S) = 1$
* $P(\emptyset) = 0$



1- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2- $P(A \mid B) = P(A \cap B)/P(B)$
3- $P(A \cap B) = P(A) \times P(B)$ (if A & B are independent)
4- $P(A \cap B) = 0$ (if A & B are disjoint)
5- $P(A^c) = 1 - P(A)$; $P(A^c) = P(\bar{A}) = P(A')$

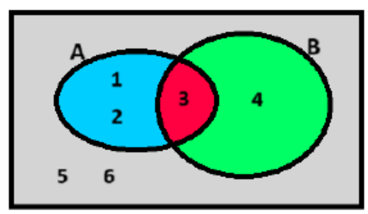
- $P(A \mid B) = P(A)$
- $P(B \mid A) = P(B)$

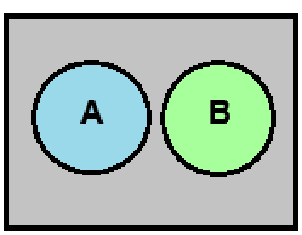
Suppose the sample space $S = \{1,2,3,4,5,6\}$

If $A = \{1,2,3\}$ and $B = \{3,4\}$

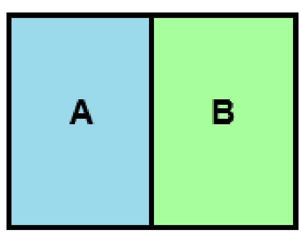
Then, $A \cap B = \{3\}$

$A \cup B = \{1,2,3,4\}$

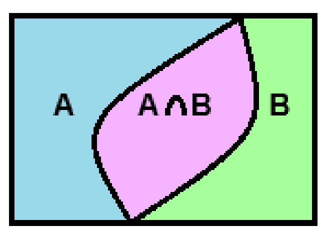




Disjoint
or
Mutually exclusive



Exhaustive
and
Disjoint



Exhaustive

Question 1:

Suppose that we have: $P(A) = 0.4\,, P(B) = 0.5\ \ and\ P(A \cap B) = 0.2$

1. The probability $P(A \cup B)$ is:

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7}$$

2. The probability $P(A \cap B^c)$ is:

$$\boxed{P(A \cap B^c) = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2}$$

3. The probability $P(A^c \cap B)$ is:

$$\boxed{P(A^c \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3}$$

4. The probability $P(A|B)$ is:

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = 0.4}$$

5. The events A and B are:

$$\boxed{P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Longrightarrow 0.2 = 0.4 \times 0.5}$$

A	Disjoint	B	Dependent	C	Equal	D	Independent

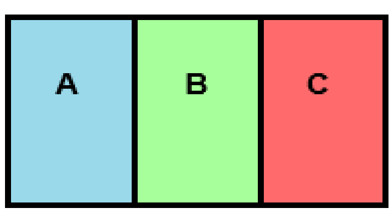
Question 2:

Consider three events A, B and C such that:

$$P(A) = 0.4, P(B) = 0.5 \text{ and } P(C) = 0.1$$

If A, B and C are disjoint events, then they are:

A	Symmetric	B	Exhaustive	C	Not exhaustive	D	None of them



Question 3:

If the events A, B we have: P(A) = 0.2, P(B) = 0.5 and P (A ∩ B) = 0.1, then:

1. The events A, B are:

$$P(A\cap B) \stackrel{?}{=} P(A)\times P(B) \Longrightarrow 0.1 = 0.2\times 0.5$$

A	Disjoint	B	Dependent	C	Both are empties	D	Independent

2. The probability of A or B is:

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.2 + 0.5 - 0.1 = 0.6$$

3. If P(A) = 0.3, P(B) = 0.4 and that A and B are disjoint, then P(A∪B) =

$$P(A\cup B) = P(A) + P(B) - 0 = 0.3 + 0.4 - 0 = 0.7$$

4. If P(A) = 0.2 and P(B | A) =0.4, then P(A ∩ B) =

$$P(B|A) = \frac{P(A\cap B)}{P(A)} \Longrightarrow 0.4 = \frac{P(A\cap B)}{0.2} \Longrightarrow P(A\cap B) = 0.2\times 0.4 = 0.08$$

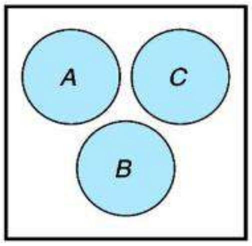
5. Suppose that the probability a patient smoke is 0.20. If the probability that the patient smokes and has a lung cancer is 0.15, then the probability that the patient has a lung cancer given that the patient smokes is

$$P(S) = 0.20 \quad P(S\cap C) = 0.15 \quad P(C|S) = ?$$
$$P(C|S) = \frac{P(C\cap S)}{P(S)} = \frac{0.15}{0.20} = 0.75$$

Question 4:

The probability of three mutually exclusive events A, B and C are given by $1/3$, $1/4$ and $5/12$ then $P(A\cup B\cup C)$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C)$$
$$= \frac{1}{3} + \frac{1}{4} + \frac{5}{12} = 1$$



A	0.57	B	0.43	C	0.58	D	1

<u>Question 5:</u>

Suppose that we have two events A and B such that,

$$P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.2.$$

[1] $P(A \cup B)$: $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7}$

[2] $P(A^c \cap B)$: $\boxed{P(A^c \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3}$

[3] $P(A^c \cap B^c)$: $\boxed{P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3}$

[4] $P(A^c)$: $\boxed{P(A^c) = 1 - P(A) = 1 - 0.4 = 0.6}$

[5] $P(A^c \mid B)$: $\boxed{P(A^c \mid B) = \frac{P(A^c \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6}$

[6] $P(B \mid A)$: $\boxed{P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.4} = 0.5}$

[7] The events A and B are …

$$\boxed{P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Longrightarrow 0.2 = 0.4 \times 0.5}$$

A	Exhaustive	B	Dependent	C	Equal	D	Independent

Question 6:

Let A and B two events defined on the same sample space.

If $P(A) = 0.7, P(B) = 0.3$

1. If the events A and B are mutually exclusive (disjoint) then, the value of $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - 0 = 0.7 + 0.3 - 0 = 1$$

A	0.21	B	0.52	C	0.79	D	1

2. If the events A and B are independent, then the value of $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) = 0.7$$

A	0.3	B	0.5	C	0.7	D	0.9

3. If the events A and B are independent, then the value of $P(A \cap \bar{B})$.

$$P(A \cap \bar{B}) = P(A)P(\bar{B}) = (0.7)(0.7)$$

A	0.09	B	0.21	C	0.49	D	0.54

4. If the events A and B are independent, then the value of $P(\overline{A \cup B})$.

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - [0.7 + 0.3 - (0.7)(0.3)] = 0.21$$

A	0.21	B	0.39	C	0.49	D	0.54

Question 7:

Following table shows 80 patients classified by sex and blood group.

Sex	Blood group		
	A	B	O
Male (M)	25	17	15
Female (F)	11	9	3

1) The probability that a patient selected randomly is a male and has blood group A is

A	25/36	B	25/80	C	25/57	D	52/80

2) The probability that a patient selected randomly is a female is

A	6/80	B	40/80	C	23/80	D	None

3) In a certain population, 4% have cancer, 20% are smokers and 2% are both smokers and have cancer. If a person is chosen at random from the population, find the probability that the person chosen is a smoker or has cancer.

$$P(C) = 0.04 \quad P(S) = 0.20 \quad P(S \cap C) = 0.02 \quad P(S \cup C) = ?$$
$$P(S \cup C) = P(C) + P(S) - P(S \cap C)$$
$$P(S \cup C) = 0.04 + 0.20 - 0.02 = 0.22$$

<u>Question 8:</u>

Gender	Diabetics (D)	Not Diabetic (D^c)	TOTAL
Male (M)	72	288	360
Female (F)	48	192	240
Total	120	480	600

Consider the information given in the table above. A person is selected randomly

1. The probability that the person found is male and diabetic is:

$$P(M \cap D) = \frac{72}{600} = 0.12$$

2. The probability that the person found is male or diabetic is:

$$P(M \cup D) = P(M) + P(D) - P(M \cap D) = \frac{360}{600} + \frac{120}{600} - \frac{72}{600} = \frac{408}{600}$$

3. The probability that the person found is female is:

$$P(F) = \frac{240}{600} = 0.4$$

4. Suppose we know the person found is a male, the probability that he is diabetic, is:

$$P(D|M) = \frac{P(M \cap D)}{P(M)} = \frac{72/600}{360/600} = \frac{72}{360} = 0.2$$

5. The events M and D are:

$$P(M \cap D) = P(M) \times P(D) \Longrightarrow \frac{72}{600} = \frac{360}{600} \times \frac{120}{600}$$

A	Mutually exclusive	B	Dependent	C	Equal	D	Independent

Question 9:

A group of people is classified by the number of fruits eaten and the health status:

Health status	Fruits eaten			Total
	Few (F)	Some (S)	Many (M)	
Poor (B)	80	35	20	135
Good(G)	25	110	45	180
Excellent (E)	15	95	75	185
Total	120	240	140	500

If one of these people is randomly chosen give:

1. The event "(eats few fruits) and (has good health) ", is defined as.

A	$F \cup G^c$	B	$F \cap G$	C	$F \cup E$	D	$S \cup E$

2. $P(B \cup M) =$

A	0.51	B	0.28	C	0.27	D	0.04

3. $P(G \cap S) =$

A	0.48	B	0.36	C	0.22	D	0.62

4. $P(E^c) =$

A	0.63	B	0.37	C	0.50	D	1

5. $P(G|S) =$

A	0.6111	B	0.2200	C	0.4583	D	0.36

6. $P(M|E) =$

A	0.6111	B	0.2200	C	0.405	D	0.36

<u>Question 10:</u>

A study was conducted to determine if doctors' wiliness to accept a new proposed medical device is equally distributed in three American hospitals Minneapolis, Houston and Dallas. A random sample of 5000 respondents was generated for the purpose. The respondents were acquainted with the medical device and were asked if they recommended it. The following table classifies the answers of the respondents in Minneapolis, Houston and Dallas hospitals.

Attitude to the medical device	Hospital			Total
	Minneapolis	Houston	Dallas	
Recommend	142	108	135	385
Not recommend	43	42	30	115
Total	185	150	165	500

1. What is the probability that a randomly selected respondent will recommend the new medical device?

A	0.95	B	0.42	C	0.53	D	0.77

2. Given that the respondent is from Dallas, what is the probability that he will not recommend the new medical device?

A	0.182	B	0.325	C	0.435	D	0.546

3. What is the probability that a randomly selected respondent in the study will be from Houston hospital and will recommend the new medical device?

A	0.115	B	0.523	C	0.216	D	0.756

4. The events {the respondent recommend the medical device} and {the respondent not recommend the medical device} are:

A	Mutually exclusive	B	Symmetric	C	Equal	D	None of them

5. What is the probability that a subject chosen at random in the study is not from Minneapolis hospital is:

A	0.46	B	0.75	C	0.63	D	0.86

6. Given that the respondent does not recommend the medical device, what is the probability that he is from Minneapolis hospital?

A	0.115	B	0.643	C	0.374	D	0.756

Question 11:

The following table classifies a sample of individuals according to gender and period (in years) attendance in the college:

Collage attended	Gender		Total
	Male	Female	
None	12	41	53
Two years	14	63	77
Three years	9	49	58
Four years	7	50	57
Total	42	203	245

Suppose we select an individual at random, then:

1. The probability that the individual is male is:

A	0.8286	B	0.1714	C	0.0490	D	0.2857

2. The probability that the individual did not attend college (None) and female is:

A	0.0241	B	0.0490	C	0.1673	D	0.2163

3. The probability that the individual has three year or two-year college attendance is:

A	0.551	B	0.0939	C	0.4571	D	0

4. If we pick an individual at random and found that he had three-year college attendance, the probability that the individual is male is:

A	0.0367	B	0.2143	C	0.1552	D	0.1714

5. The probability that the individual is not a four-year college attendance is:

A	0.7673	B	0.2327	C	0.0286	D	0.1429

6. The probability that the individual is a two-year college attendance or male is:

A	0.0571	B	0.8858	C	0.2571	D	0.4286

7. The events: the individual is a four-year college attendance and male are:

A	Mutually exclusive	B	Independent	C	Dependent	D	None of these

Question 12:

Obesity	Blood pressure		
	Low (L)	Medium (M)	High (H)
Has obesity (B)	25	17	15
Dose not obesity (B')	11	9	3

If an individual is selected at random from this group, then the probability that he/she

1. has obesity or has medium blood pressure is equal to

A	0.442	B	0.50	C	0.725	D	0.673

2. has low blood pressure given that he/she has obesity is equal to

A	0.90	B	0.1	C	0.66	D	0.44

- Bayes' Theorem, Screening Tests, Sensitivity, Specificity, and Predictive Value Positive and Negative

	Disease		
Test Result	Present (D)	Absent ($\bar{D}$)	Total
Positive (T)	a	b	
Negative ($\bar{T}$)	c	d	
Total	a+c	b+d	n

Sensitivity $P(T\|D) = \frac{a}{a+c}$	Probability of false positive (f +) $P(T\|\bar{D}) = \frac{b}{b+d}$
Probability of false negative (f -) $P(\bar{T}\|D) = \frac{c}{a+c}$	Specificity $P(\bar{T}\|\bar{D}) = \frac{d}{b+d}$

Sensitivity + False negative = 1	Specificity + False positive =1

The predictive value positive:

$$P(D|T) = \frac{P(D\cap T)}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D)+P(T|\bar{D})P(\bar{D})}$$

$$= \frac{(Sen)\times P(D_{given})}{(Sen)\times P(D_{given})+(f+)\times P(\bar{D}_{given})}$$

$P(D \mid T) = P(\text{the subject has the disease} \mid \text{positive result})$
$= P(\text{presence of the disease} \mid \text{positive result})$

The predictive value negative:

$$P(\bar{D}|\bar{T}) = \frac{P(\bar{T}\cap\bar{D})}{P(\bar{T})} = \frac{P(\bar{T}|\bar{D})P(\bar{D})}{P(\bar{T}|\bar{D})P(\bar{D})+P(\bar{T}|D)P(D)}$$

$$= \frac{(Spe)\times P(\bar{D}_{given})}{(Spe)\times P(\bar{D}_{given})+(f-)\times P(D_{given})}$$

$P(\bar{T} \mid \bar{D}) = P(\text{the subject dose not have the disease} \mid \text{negative result})$
$= P(\text{presence of the disease} \mid \text{negative result})$

Question 1:

The following table shows the results of a screening test:

	Disease confirmed (D)	Disease not confirmed ($\overline{D}$)
Positive test (T)	38	10
Negative test ($\overline{T}$)	5	18

1. The probability of false positive of the test is: $\frac{10}{28} = 0.3571$

2. The probability of false negative of the test is: $\frac{5}{43} = 0.1163$

3. The sensitivity value of the test is: $\frac{38}{43} = 0.8837$

4. The specificity value of the test is: $\frac{18}{28} = 0.6429$

Suppose it is known that the rate of the disease is 0.113,

$1 - 0.113 = 0.887$

5. The predictive value positive of a symptom is:

$$= \frac{(\text{Sen})\times P(D_{\text{given}})}{(\text{Sen})\times P(D_{\text{given}}) + (f+)\times P(\bar{D}_{\text{given}})} = \frac{0.8837\times 0.113}{0.8837\times 0.113 + 0.3571\times 0.887} = 0.2397$$

6. The predictive value negative of a symptom is:

$$= \frac{(\text{Spe})\times P(\bar{D}_{\text{given}})}{(\text{Spe})\times P(\bar{D}_{\text{given}}) + (f-)\times P(D_{\text{given}})} = \frac{0.6429\times 0.887}{0.6429\times 0.887 + 0.1163\times 0.113} = 0.9772$$

Question 2:

It is known that 40% of the population is diabetic. 330 persons who were diabetics went through a test where the test confirmed the disease for 288 persons. Among 270 healthy persons, test showed high sugar level for 72 persons. The information obtained is given in the table below.

Test	Diabetics (D)	Not Diabetic (D^c)	TOTAL
Positive (T)	288	72	360
Negative ($\bar{T}$)	42	198	240
TOTAL	330	270	600

1. The sensitivity of the test is: $\frac{288}{330} = 0.873$
2. The specificity of the test is: $\frac{198}{270} = 0.733$
3. The probability of false positive is: $\frac{72}{270} = 0.267$
4. The predictive probability positive for the disease is:

$$= \frac{(\text{Sen})\times P(D_{\text{given}})}{(\text{Sen})\times P(D_{\text{given}}) + (f+)\times P(\bar{D}_{\text{given}})} = \frac{0.873 \times 0.40}{0.873 \times 0.40 + 0.267 \times 0.60} = 0.686$$

Question 3:

The following table shows the results of a screening test evaluation in which a random sample of 700 subjects with the disease and an independent random sample of 1300 subjects without the disease participated:

Disease Test result	Present	Absent
Positive	500	100
Negative	200	1200

1. The sensitivity value of the test is: $\frac{500}{700} = 0.7143$
2. The specificity value of the test is: $\frac{1200}{1300} = 0.923$
3. The probability of false positive of the test is: $\frac{100}{1300} = 0.0769$
4. If the rate of the disease in the general population is 0.002, then the predictive value positive of the test is:

$$= \frac{(\text{Sen})\times P(D_{\text{given}})}{(\text{Sen})\times P(D_{\text{given}}) + (f+)\times P(\bar{D}_{\text{given}})} = \frac{0.7143 \times 0.002}{0.7143 \times 0.002 + 0.0769 \times 0.998} = 0.01827$$

Question 4:

In a study of high blood pressure, 188 persons found positive, of a sample of 200 persons with the disease subjected to a screening test. While, 27 persons found positive, of an independent sample of 300 persons without the disease subjected to the same screening test. That is,

	High Blood Pressure		
Test Result	Yes D	No $\bar{D}$	Total
Positive T	188	27	215
Negative $\bar{T}$	12	273	285
Total	200	300	500

[1] Given that a person has the disease, the probability of a positive test result, that is, the "sensitivity" of this test is:

A	0.49	B	**0.94**	C	0.35	D	0.55

[2] Given that a person does not have the disease, the probability of a negative test result, that is, the "specificity" of this test is:

A	**0.91**	B	0.75	C	0.63	D	0.49

[3] The "false negative" results when a test indicates a negative status given that the true status is positive is:

A	0.01	B	0.15	C	0.21	D	**0.06**

[4] The "false positive" results when a test indicates a positive status given that the true status is negative is:

A	0.16	B	0.31	C	**0.09**	D	0.02

Assuming that 15% of the population under study is known to be with high blood pressure.

[5] Given a positive screening test, what is the probability that the person has the disease? That is, the "predictive value positive" is:

A	0.22	B	**0.65**	C	0.93	D	0.70

[6] Given a negative screening test result, what is the probability that the person does not have the disease? That is, the "predictive value negative" is:

A	0.258	B	0.778	C	**0.988**	D	0.338

Question 5:

Suppose that the ministry of health intends to check the reliability of the central Diabetic Lab in Riyadh. A sample person with Diabetic disease (D) and another without the disease ($\bar{D}$) had the Lab tests and the results are given below:

	Present (D)	Absence ($\bar{D}$)
Positive (T)	950	40
Negative ($\bar{T}$)	25	640

Then:

1. The probability of false positive of the test is: $\frac{40}{680} = 0.0588$
2. The probability of false negative of the test is: $\frac{25}{975} = 0.0256$
3. The sensitivity value of the test is: $\frac{950}{975} = 0.9744$
4. The specificity value of the test is: $\frac{640}{680} = 0.9412$

Assume that the true percentage of Diabetic patients in Riyadh is 25%. Then

5. The predictive value positive of the test is:

$$= \frac{(\text{Sen}) \times P(D_{\text{given}})}{(\text{Sen}) \times P(D_{\text{given}}) + (f+) \times P(\bar{D}_{\text{given}})} = \frac{0.9744 \times 0.25}{0.9744 \times 0.25 + 0.0588 \times 0.75} = 0.8467$$

6. The predictive value negative of the test is:

$$= \frac{(\text{Spe}) \times P(\bar{D}_{\text{given}})}{(\text{Spe}) \times P(\bar{D}_{\text{given}}) + (f-) \times P(D_{\text{given}})} = \frac{0.9412 \times 0.75}{0.9412 \times 0.75 + 0.0256 \times 0.25} = 0.9910$$

Question 6:

A Fecal Occult Blood Screen Outcome Test is applied for 875 patients with bowel cancer. The same test was applied for another sample of 925 without bowel cancer. Obtained results are shown in the following table:

	Present Disease (D)	*Absent Disease* ($\bar{D}$)
Test Positive (T)	*850*	*10*
Test Negative ($\bar{T}$)	*25*	*915*

1. The probability of false positive of the test is: $\frac{10}{925} = 0.0108$

2. The probability of false negative of the test is: $\frac{25}{875} = 0.0286$

3. The sensitivity value of the test is: $\frac{850}{875} = 0.9714$

4. The specificity value of the test is: $\frac{915}{925} = 0.9892$

5. If the rate of the disease in the general population is equal to 15% then the predictive value positive of the test is

$$= \frac{(\text{Sen}) \times P(D_{\text{given}})}{(\text{Sen}) \times P(D_{\text{given}}) + (f+) \times P(\bar{D}_{\text{given}})} = \frac{0.9714 \times 0.15}{0.9714 \times 0.15 + 0.0108 \times 0.85} = 0.9407$$

More Exercises

Question 1:

Givens:

$$\boldsymbol{P(A) = 0.5,\quad P(B) = 0.4,\quad P(C \cap A^c) = 0.6,}$$
$$\boldsymbol{P(C \cap A) = 0.2,\quad P(A \cup B) = 0.9}$$

(a) ***What is the probability of $P(C)$:***

$$P(C) = P(C \cap A^c) + P(C \cap A) = 0.6 + 0.2 = 0.8$$

(b) ***What is the probability of $P(A \cap B)$:***

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow \quad 0.9 = 0.5 + 0.4 - P(A \cap B)$$
$$P(A \cap B) = 0$$

(c) ***What is the probability of $P(C \mid A)$:***

$$P(C \mid A) = \frac{P(C \cap A)}{P(A)} = \frac{0.2}{0.5} = 0.4$$

(d) ***What is the probability of $P(B^c \cap A^c)$***:

$$P(B^c \cap A^c) = 1 - P(B \cup A) = 1 - 0.9 = 0.1$$

Question 2:

Givens:

$$\boldsymbol{P(B) = 0.3,\quad P(A \mid B) = 0.4}$$

$\boldsymbol{Then\ find\ P(A \cap B)} = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$\Rightarrow 0.4 = \frac{P(A \cap B)}{0.3}$$
$$\Rightarrow P(A \cap B) = 0.4 \times 0.3 = 0.12$$

Question 3:

Givens:

$$\boldsymbol{P(A) = 0.3,\quad P(B) = 0.4,\quad P(A \cap B \cap C) = 0.03,\quad P(\overline{A \cap B}) = 0.88}$$

(1) Are the event A and b independent?

$$P(A \cap B) = 1 - P(\overline{A \cap B}) = 1 - 0.88 = 0.12$$

$$P(A) \times P(B) = 0.3 \times 0.4 = 0.12$$
$$\Rightarrow P(A \cap B) = P(A) \times P(B)$$
Therefore, A and B are independent.

(2) ***What is the probability of*** $\boldsymbol{P(C \mid A \cap B)}$:

$$P(C \mid A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{0.03}{0.12} = 0.25$$

Question 4:

Givens:

$$\boldsymbol{P(A_1) = 0.4,\ P(A_1 \cap A_2) = 0.2,\ P(A_3 \mid A_1 \cap A_2) = 0.75}$$

(1) ***Find the*** $\boldsymbol{P(A_2|A_1)}$:

$$P(A_2|A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.2}{0.4} = 0.5$$

(2) ***Find the*** $\boldsymbol{P(A_1 \cap A_2 \cap A_3)}$:

$$P(A_3 \mid A_1 \cap A_2) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)}$$

$$0.75 = \frac{P(A_1 \cap A_2 \cap A_3)}{0.2}$$

$$P(A_1 \cap A_2 \cap A_3) = 0.75 \times 0.2 = 0.15$$

Exercise 1:

A group of 400 people are classified according to their nationality as (250 Saudi and 150 non-Saudi), and they are classified according to their gender (100 Male and 300 female). The number of Saudi males is 60. Suppose that the experiment is to select a person at random from this group.

1. Summarizing the information in a table:

		Gender		Total
		Male (M)	Female (F)	
Nationality	Saudi (S)	**60**	190	**250**
	Non-Saudi (N)	40	110	**150**
Total		**100**	**300**	**400**

2. The probability that the selected person is Saudi is:
(A) 0.6
(B) 0.15
(C) 0.3158
(D) 0.625 (P(S) = 250/400)

3. The probability that the selected person is female is:
(A) 0.375
(B) 0.75 (P(F) = 300/400)
(C) 0.3667
(D) 0.6333

4. The probability that the selected person is female given that the selected person is Saudi is:
(A) 0.6333
(B) 0.3667
(C) 0.76 (P(F | S) = 190/250)
(D) 0.475

5. The events "S"={Selecting a Saudi} and "F"={Selecting a female} are:
(A) Not independent events (Because: P(F) ≠ P(F | S))
(B) Complement of each other
(C) Independent events
(D) Disjoint (mutually exclusive) events

Exercise 2:

A new test is being considered for diagnosis of leukemia. To evaluate this test, the researcher has applied this test on 30 leukemia patient persons and 50 non-patient persons. The results of the screen test applied to those people are as follows: positive results for 27 of the patient persons, and positive results for 4 of the non-patient persons, and negative results for the rest of persons.

1. Summarizing the information in a table:

		Leukemia Disease		Total
		D^+	D^-	
Result of the Test	T^+	**27**	**4**	31
	T^-	3	46	49
Total		**30**	**50**	80

2. The sensitivity of the test is:
(A) 0.3375
(B) 0.9 (P(T^+ | D^+) = 27/30)
(C) 0.88
(D) 0.1111

3. The specificity of the test is:
(A) 0.92 (P(T^- | D^-) = 46/50)
(B) 0.575
(C) 0.087
(D) 0.9388

4. The probability of false positive result is: (FP)
(A) 0.05
(B) 0.1481
(C) 0.1290
(D) 0.08 (P(T^+ | D^-) = 4/50) { Note: P(FP) = 1 – Specificity }

5. The probability of false negative result is: (FN)
(A) 0.0652
(B) 0.1111
(C) 0.1 (P(T^- | D^+) = 3/30) { Note: P(FN) = 1 – Sensitivity }
(D) 0.0375

Exercise3:

A new test is being considered for diagnosis of leukemia. To evaluate this test, the researcher has applied this test on a group of people and found that the sensitivity of the test was 0.92 and the specificity of the test was 0.94. Based on another independent study, it is found that the percentage of infected people with leukemia in the population is 5% (the rate of prevalence of the disease).

Given information:

Sensitivity = $P(T^+ \mid D^+) = 0.92$

specificity = $P(T^- \mid D^-) = 0.94$

$P(D) = 0.05$

1. The predictive value positive is:

(A) 0.4466 ($P(D^+ \mid T^+)$ = Bayes rule)

(B) 0.3987

(C) 0.9328

(D) 0.6692

2. The predictive value negative is:

(A) 0.7841

(B) 0.9955 ($P(D^- \mid T^-)$ = Bayes rule)

(C) 0.8774

(D) 0.3496

<u>Exercise</u>: (Hypothetical Example)

A new proposed test is being considered for diagnosis of Corona (COVID-19) disease. To investigate the efficiency of this test, the researcher has applied this test on 80 infected patients and 900 non-infected persons. The results of the screen test are given in the following table:

		Nature of the Disease		
		(Present: D^+) Infected Patients	(Absent: D^-) Non-infected People	Total
Result of the Test	$+ve$ (T^+)	75 (TP)	10 (FP)	85
	$-ve$ (T^-)	5 (FN)	890 (TN)	895
Total		80	900	980

Based on another independent study, it is found that the percentage of infected people with Corona (COVID-19) in this city is 4% (the rate of prevalence of the disease).

<u>1. Before-Test Questions:</u>

a) If a person was infected (D^+), what is the probability that the result of the test will be $+ve$ (T^+)?

$$\boldsymbol{P(T^+|D^+) = \textbf{Sensitivity of the Test}}$$

b) If a person was infected (D^+), what is the probability that the result of the test will be $-ve$ (T^-)?

$$\boldsymbol{P(T^-|D^+)} = \textbf{False Negative Result (FNR)}$$
$$= \boldsymbol{1 - P(T^+|D^+)}$$
$$= \boldsymbol{1 -} \textbf{ Sensitivity of the Test}$$

c) If a person was not infected (D^-), what is the probability that the result of the test will be $-ve$ (T^-)?

$$\boldsymbol{P(T^-|D^-)} = \textbf{Specificity of the Test}$$

d) If a person was not infected (D^-), what is the probability that the result of the test will be $+ve$ (T^+)?

$$\boldsymbol{P(T^+|D^-)} = \textbf{False Positive Result (FPR)}$$
$$= \boldsymbol{1 - P(T^-|D^-)}$$
$$= \boldsymbol{1 -} \textbf{ Specificity of the Test}$$

2. After-Test Questions:

a) If the result of the test was $+ve$ (T^+), what is the probability that the person is infected (D^+)?

$$\boldsymbol{P(D^+|T^+)} = \textbf{Predictive Value Positive (PVP)}$$

b) If the result of the test was $-ve$ (T^-), what is the probability that the person is not infected (D^-)?

$$\boldsymbol{P(D^-|T^-)} = \textbf{Predictive Value Negative (PVN)}$$

3. Efficiency of the Test:

$$\textbf{Efficiency} = \frac{\boldsymbol{True\ Positives + True\ Negatives}}{\boldsymbol{Total}} = \frac{\boldsymbol{TP + TN}}{\boldsymbol{n}}$$

Solution:

1. Before-Test Questions:

(a) The probability that the result of the test will be $+ve$ given that the person was infected is: (Sensitivity of the Test)

$$P(T^+|D^+) = \frac{P(T^+ \cap D^+)}{P(D^+)} = \frac{n(T^+ \cap D^+)}{n(D^+)} = \frac{75}{80} = 0.9375$$

(b) The probability that the result of the test will be $-ve$ given that the person was infected is: (False Negative Result =FNR)

$$P(T^-|D^+) = 1 - P(T^+|D^+) = 1 - 0.9375 = 0.0625$$

(c) The probability that the result of the test will be $-ve$ given that the person was not infected is: (Specificity of the test)

$$P(T^-|D^-) = \frac{P(T^- \cap D^-)}{P(D^-)} = \frac{n(T^- \cap D^-)}{n(D^-)} = \frac{890}{900} = 0.9889$$

(d) The probability that the result of the test will be $+ve$ given that the person was not infected is: (False Positive Result = FPR)

$$P(T^+|D^-) = 1 - P(T^-|D^-) = 1 - 0.9889 = 0.0111$$

2. After-Test Questions:
Define the following events:

D = {A randomly chosen person from the city is infected} → 4%
$P(D) = \frac{4}{100} = 0.04$

$\overline{D}$ = {A randomly chosen person from the city is not infected}
$P(\overline{D}) = 1 - P(D) = 1 - 0.04 = 0.96$

(a) The probability that the person is infected (D), given that the result was $+ve$ (T^+) is: (Predictive Value Positive = PVP)

$$P(D|T^+) = \frac{P(D \cap T^+)}{P(T^+)}$$
$$= \frac{P(T^+|D)P(D)}{P(T^+|D)P(D) + P(T^+|\overline{D})P(\overline{D})}$$
$$= \frac{0.9375 \times 0.04}{0.9375 \times 0.04 + 0.0111 \times 0.96}$$
$$= \frac{0.0375}{0.0375 + 0.010656}$$
$$= \frac{0.0375}{0.048156}$$
$$= 0.7787$$

(b) The probability that the person is not infected ($\overline{D}$), given that the result was $-ve$ (T^-) is: (Predictive Values Negative = PVN)

$$P(\overline{D}|T^-) = \frac{P(\overline{D} \cap T^-)}{P(T^-)}$$
$$= \frac{P(T^-|\overline{D})P(\overline{D})}{P(T^-|\overline{D})P(\overline{D}) + P(T^-|D)P(D)}$$
$$= \frac{0.9889 \times 0.96}{0.9889 \times 0.96 + 0.0625 \times 0.04}$$
$$= \frac{0.949344}{0.949344 + 0.00254}$$
$$= \frac{0.949344}{0.951884} = 0.9974$$

3. Efficiency of the Test:

$$\text{Efficiency} = \frac{True\ Positives + True\ Negatives}{Total}$$
$$= \frac{TP + TN}{n}$$
$$= \frac{75 + 890}{980}$$
$$= \frac{965}{980}$$
$$= 0.9847$$

Chapter 4 Probability Distribution

Random Variables

$$\text{Random Variables} \begin{cases} \text{Discrete Random Variables} \\ \text{Continuous Random Variables} \end{cases}$$

• Probability distribution function (PDF) $f(x) = P(X = x)$	• Cumulative distribution function (CDF) $F(x) = P(X \le x)$

• Properties of discreate random variable $0 \le f(x) \le 1$ $\sum f(x) = 1$	• The expected value (the mean) of discreate random variables $E(X) = \mu = \sum x\, f(x)$ • The variance of discreate random variable $Var(X) = \sigma^2 = \sum (X - \mu)^2\, f(x)$ $= E(X^2) - E(X)^2$

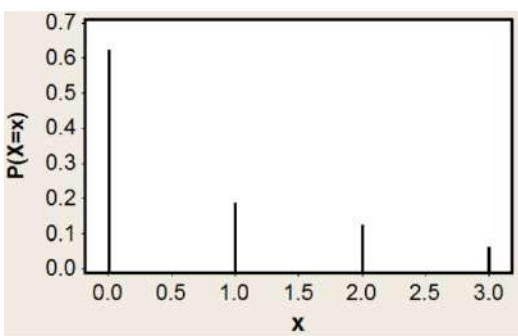
Probability distribution of discrete random variables:
Is a table, graph, formula and other device used to specify all possible value of the random variable along with their respective probabilities.

Example:
- The number of patients visiting KKUH in the week.
- The number of times a person had a cold in last year.

Graphical Presentation:

x	$P(X = x)$
0	0.6250
1	0.1875
2	0.1250
3	0.0625

The graphical presentation of this probability distribution is given by the following figure:



Question 1:

Given the following discrete probability distribution:

x	5	6	7	8
f(x)=P(X=x)	0.35	0.45	0.15	k

Find:

1. The value of k.

$$0.35 + 0.45 + 0.15 + k = 1 \implies \boxed{k = 0.05}$$

x	5	6	7	8
f(x)=P(X=x)	0.35	0.45	0.15	0.05

2. $P(X > 6) = 0.05 + 0.15 = 0.20$
3. $P(X \geq 6) = 0.45 + 0.15 + 0.05 = 0.65 \quad or \quad (1 - 0.35 = 0.65)$
4. $P(X < 4) = 0$
5. $P(X > 3) = 1$

Question 2:

Which of the following functions can be a probability distribution of a discrete random variable?

(a)

x	g(x)
0	0.6
1	-0.2
2	0.5
3	0.1
✘	

(b)

x	g(x)
0	0.4
1	0.1
2	0.5
3	0.2
✘	

(c)

x	g(x)
0	0.1
1	1.2
2	-0.6
3	0.3
✘	

(d)

x	g(x)
0	0.3
1	0.1
2	0.5
3	0.1
✓	

(e)

x	g(x)
0	0.2
1	0.4
2	0.3
3	0.4
✘	

(f)

x	g(x)
0	0.1
1	0.2
2	0.3
3	0.1
✘	

<u>Question 3</u>:
Which of the following is a probability distribution function:

$a.\ f(x) = \dfrac{x+1}{10}\ ;\ x = 0,1,2,3,4$

$b.\ f(x) = \frac{x-1}{5}\ ;\ x = 0,1,2,3,4$

$c.\ f(x) = \dfrac{1}{5}\ ;\ x = 0,1,2,3,4$

$d.\ f(x) = \dfrac{5-x^2}{6}\ ;\ x = 0,1,2,3$

a.

$$f(x) = \frac{x+1}{10};\ \ x = 0,1,2,3,4$$

x	0	1	2	3	4
$f(x)$	$^1/_{10}$	$^2/_{10}$	$^3/_{10}$	$^4/_{10}$	$^5/_{10}$

$f(x)$ *is not a P.D.F because* $\sum f(x) \neq 1$

b.

$$f(x) = \frac{x-1}{5};\ \ x = 0,1,2,3,4$$

x	0	1	2	3	4
$f(x)$	$-^1/_5$				

$f(x)$ *is not a P.D.F because every* $f(x)$ *shoud be* $0 \leq f(x) \leq 1$

c.

$$f(x) = \frac{1}{5};\ \ x = 0,1,2,3,4$$

x	0	1	2	3	4
$f(x)$	$^1/_5$	$^1/_5$	$^1/_5$	$^1/_5$	$^1/_5$

$f(x)$ *is a P.D.F*

d.

$$f(x) = \frac{5-x^2}{6};\ \ x = 0,1,2,3$$

x	0	1	2	3
$f(x)$				$-^4/_6$

$f(x)$ *is not a P.D.F because every* $f(x)$ *shoud be* $0 \leq f(x) \leq 1$

<u>Question 4</u>*:*

Given the following discrete probability distribution:

x	5	6	7	8
f(x)=P(X=x)	2k	3k	4k	k

Find the value of k.

$$2k + 3k + 4k + k = 1$$

$$10k = 1 \implies \boxed{k = 0.1}$$

x	5	6	7	8
f(x)=P(X=x)	0.2	0.3	0.4	0.1

<u>Question 5</u>:

Let X be a discrete random variable with probability mass function:
$f(x) = cx$; $x = 1,2,3,4$ What is the value of c?

x	1	2	3	4
$P(X = x)$	c	2c	3c	4c

$$c + 2c + 3c + 4c = 1 \quad \implies c = \frac{1}{10}$$

Then probability mass function:

x	1	2	3	4
$P(X = x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

<u>Question 6:</u>

Let X be a discrete random variable with probability function given by:

$$f(x) = c(x^2 + 2) \;\; ; \quad x = 0,1,2,3$$

$f(0) = c(0^2 + 2) = 2c$

$f(1) = c(1^2 + 2) = 3c$

$f(2) = c(2^2 + 2) = 6c$

$f(3) = c(3^2 + 2) = 11c$

x	0	1	2	3
f(x)	2c	3c	6c	11c

$$2c + 3c + 6c + 11c = 1 \qquad c = \frac{1}{22} = 0.04545$$

x	0	1	2	3
f(x)	$\frac{2}{22}$	$\frac{3}{22}$	$\frac{6}{22}$	$\frac{11}{22}$

<u>Question 7</u>:

Given the following discrete probability distribution:

x	5	6	7	8
f(x)=P(X=x)	0.2	0.4	0.3	0.1

Find:

1. Find the mean of the distribution $\mu = \mu_X = E(X)$.

$$E(X) = \mu = \mu_X = \sum_{x=5}^{8} x\, P(X = x)$$
$$= (5)(0.2) + (6)(0.4) + (7)(0.3) + (8)(0.1) = 6.3$$

2. Find the variance of the distribution $\sigma^2 = \sigma_X^2 = Var(X)$.

$$E(X^2) = (5^2 \times 0.2) + (6^2 \times 0.4) + (7^2 \times 0.3) + (8^2 \times 0.1) = 40.5$$

$$Var(X) = E(X^2) - E(X)^2$$
$$= 40.5 - 6.3^2 = 0.81$$

Or:

$$Var(X) = \sigma^2 = \sigma_X^2 = \sum (x - \mu)^2\, P(X = x)$$
$$= \sum_{x=5}^{8} (x - 6.3)^2\, P(X = x)$$

$$= (5 - 6.3)^2(0.2) + (6 - 6.3)^2(0.4) + (7 - 6.3)^2(0.3) + (8 - 6.3)^2(0.1) = 0.81$$

<u>Question 8:</u>

Given the following discrete distribution:

x	-1	0	1	2	3	4
P(X=x)	0.15	0.30	M	0.15	0.10	0.10

1. The value of M is equal to

$$M = 1 - (0.15 + 0.30 + 0.15 + 0.10 + 0.10) = 1 - 0.80 = 0.20$$

2. P (X $\leq$0.5) = $0.15 + 0.30 = 0.45$

3. P(X=0) = 0.30

4. The expected (mean) value E[X] is equal to

$$E(X) = (-1 \times 0.15) + (0 \times 0.30) + (1 \times 0.20) + (2 \times 0.15) + (3 \times 0.10) + (4 \times 0.10) = 1.05$$

Question 9:

The average length of stay in a hospital is useful for planning purposes. Suppose that the following is the probability distribution of the length of stay (X) in a hospital after a minor operation:

Length of stay (days)	3	4	5	6
Probability	0.4	0.2	0.1	k

(1) The value of k is

$$\boxed{k = 1 - (0.4 + 0.2 + 0.1) = 1 - 0.7 = 0.3}$$

(2) $P(X \leq 0) =$

$$\boxed{0}$$

(3) $P(0 < X \leq 5) =$

$$\boxed{0.4 + 0.2 + 0.1 = 0.7}$$

(4) $P(X \leq 5.5) =$

$$\boxed{0.4 + 0.2 + 0.1 = 0.7}$$

(5) The probability that the patient will stay at most 4 days in a hospital after a minor operation is equal to

$$\boxed{0.4 + 0.2 = 0.6}$$

(6) The average length of stay in a hospital is

$$\boxed{E(X) = (3 \times 0.4) + (4 \times 0.2) + (5 \times 0.1) + (6 \times 0.3) = 4.3}$$

Question 10:

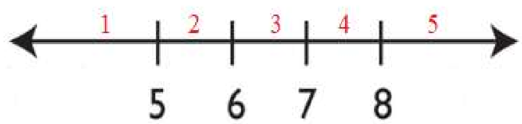
Given the following discrete probability distribution:

x	5	6	7	8
f(x)=P(X=x)	0.2	0.4	0.3	0.1

1. Find the cumulative distribution of X.

x	5	6	7	8
$F(x) = P(X \leq x)$	0.2	0.6	0.9	1

$$F(x) = \begin{cases} 0 & X < 5 \\ 0.2 & 5 \leq X < 6 \\ 0.6 & 6 \leq X < 7 \\ 0.9 & 7 \leq X < 8 \\ 1 & X \geq 8 \end{cases}$$



2. From the cumulative distribution of X, find:

a) $P(X \leq 7) \quad = 0.9$

b) $P(X \leq 6.5) = P(X \leq 6)$ =0.6

c) $P(X > 6) \quad = 1 - P(X \leq 6) = 1 - 0.6 = 0.4$

d) $P(X > 7) \quad = 1 - P(X \leq 7) = 1 - 0.9 = 0.1$

Question 11:

Given that the cumulative distribution of random variable T, is:

$$F(t) = P(T \leq t) = \begin{cases} 0 & t < 1 \\ 1/2 & 1 \leq t < 3 \\ 8/12 & 3 \leq t < 5 \\ 3/4 & 5 \leq t < 7 \\ 1 & t \geq 7 \end{cases}$$

1. Find $P(T = 5)$

T	1	3	5	7
f(t)	$\frac{1}{2} - 0 = \mathbf{0.5}$	$\frac{8}{12} - \frac{1}{2} = \mathbf{0.167}$	$\frac{3}{4} - \frac{8}{12} = \mathbf{0.083}$	$1 - \frac{3}{4} = \mathbf{0.25}$

$$P(T = 5) = 0.083$$

2. Find $P(1.4 < T < 6) = 0.167 + 0.083 = 0.25$

Binomial Distribution

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \;\; ; \quad x = 0,1 \dots ., n$$
$$* \, E(X) = np \qquad * \, Var(X) = npq$$
$$q = 1 - p$$

Combinations:

The number of different ways for selecting *r* objects from *n* distinct objects is denoted by $\binom{n}{r}$ or nCr and is given by: $\binom{n}{r} = \frac{n!}{(n-r)!}$

Bernoulli trail:

is an experiment with only two possible outcomes S = success and F = failure.

Binomial distribution is used to model an experiment for which:

1. The experiment has a sequence of n Bernoulli trails
2. The probability of success is $P(S) = p$,
   the probability of failure is $P(F) = 1 - p$.
3. The probability of success is $P(S) = p$ is constant for each trail.
4. The trails are independent; that is the outcomes of one trail has no effect on the outcome of any other trail.

The parameters of Binomial distribution are n and p and we write
$X \sim Binomial\ (n, p)$

$$P(X = x) = \binom{n}{x} (p)^x (1 - p)^{n-x} \;\; ; \;\; X = 0,1,2, \dots , n$$

Question 1:

Suppose that 25% of the people in a certain large population have high blood pressure. A Sample of 7 people is selected at random from this population. Let X be the number of people in the sample who have high blood pressure, follows a binomial distribution then

1) The values of the parameters of the distribution are:

$$p = 0.25 \quad , \quad n = 7$$

A	7 , 0.75	B	7 , 0.25	C	0.25 , 0.75	D	25 , 7

2) The probability that we find exactly one person with high blood pressure, is:

X	0	1	2	3	4	5	6	7
$P(X=x)$		*						

$$P(X=1) = \binom{7}{1}(0.25)^1(0.75)^6 = 0.31146$$

3) The probability that there will be at most one person with high blood pressure, is:

X	0	1	2	3	4	5	6	7
$P(X=x)$	*	*						

$$P(X \le 1) = \binom{7}{0}(0.25)^0(0.75)^7 + \binom{7}{1}(0.25)^1(0.75)^6 = 0.4449$$

4) The probability that we find more than one person with high blood pressure, is:

X	0	1	2	3	4	5	6	7
$P(X=x)$			*	*	*	*	*	*

$$P(X > 1) = 1 - P(X \le 1) = 1 - 0.4449 = 0.5551$$

Question 2:

In some population it was found that the percentage of adults who have hypertension is 24 percent. Suppose we select a simple random sample of five adults from this population. Then the probability that the number of people who have hypertension in this sample, will be:

$$p = 0.24 \quad , \quad n = 5$$

1. Zero*:*

$$P(X = 0) = \binom{5}{0}(0.24)^0(0.76)^5 = 0.2536$$

2. Exactly one

$$P(X = 1) = \binom{5}{1}(0.24)^1(0.76)^4 = 0.4003$$

3. Between one and three, inclusive

$$P(1 \le X \le 3) = \binom{5}{1}(0.24)^1(0.76)^4 + \binom{5}{2}(0.24)^2(0.76)^3 + \binom{5}{3}(0.24)^3(0.76)^2 = 0.7330$$

4. Two or fewer (at most two):

$$P(X \le 2) = \binom{5}{0}(0.24)^0(0.76)^5 + \binom{5}{1}(0.24)^1(0.76)^4 + \binom{5}{2}(0.24)^2(0.76)^3 = 0.9067$$

5. Five:

$$P(X = 5) = \binom{5}{5}(0.24)^5(0.76)^0 = 0.0008$$

6. The mean of the number of people who have hypertension is equal to:

$$E(X) = np = 5 \times 0.24 = 1.2$$

7. The variance of the number of people who have hypertension is:

$$Var(X) = npq = 5 \times 0.24 \times 0.76 = 0.912$$

Question 3:

The proportion of students wearing glasses is 35%. Let X the number of students wearing glasses in a random sample of 10 students. Find the following.

1) The standard deviation of X:

$$\boxed{n = 10,\ \ p = 0.35}$$

$$SD(X) = \sqrt{Var(X)} = \sqrt{npq} = \sqrt{10 \times 0.35 \times 0.65} = 1.508$$

A	3.542	B	1.508	C	4.568	D	2.275

2) The probability that $P(3 < X < 6)$ is:

X	0	1	2	3	4	5	6	7	8	9	10
$P(X = x)$					*	*					

$$P(3 < X < 6) = \binom{10}{4}(0.35)^4(0.65)^6 + \binom{10}{5}(0.35)^5(0.65)^5 = 0.391$$

A	0.013	B	0.072	C	0.391	D	0.751

3) The probability that X is at most 2 is equal to:

X	0	1	2	3	4	5	6	7	8	9	10
$P(X = x)$	*	*	*								

$$P(X \leq 2) = \binom{10}{0}(0.35)^0(0.65)^{10} + \binom{10}{1}(0.35)^1(0.65)^9 + \binom{10}{2}(0.35)^2(0.65)^8 = 0.2616$$

A	0.752	B	0.995	C	0.854	D	0.262

More Exercises

Exercise 1:

Find:

1. 6! $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
2. ${}_{8}C_{3}$ $= \frac{8!}{3!\,(8-3)!} = \frac{8!}{3! \times 5!} = 56$
3. ${}_{8}C_{10}$ $= 0$
4. ${}_{8}C_{-5}$ $= 0$

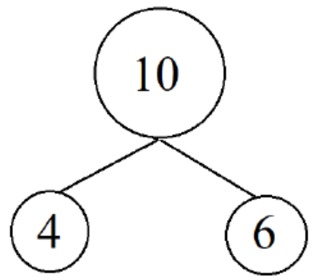
Exercise 2:

A box contains 10 cards numbered from 1 to 10. In how many ways can we select 4 cards out of this box?

Answer $= {}_{10}C_{4} = \frac{10!}{4!\,(10-4)!} = \frac{10!}{4! \times 6!}$

$= \frac{10\times9\times8\times7\times6!}{(4\times3\times2\times1)\,6!} = \frac{10\times9\times8\times7}{4\times3\times2\times1}$

$= 210$



Exercise 3:

The manager of a certain bank has recently examined the credit card account balances for the customers of his bank and found that 20% of the customers have excellent records. Suppose that the manager randomly selects a sample of 4 customers.

(A) Define the random variable X as:

X = The number of customers in the sample having excellent records.

Find the probability distribution of X.

$X \sim Binomial\ (n\ , p\ )$

$n = 4 \quad (Number\ of\ trials)$

$p = \frac{20}{100} = 0.2 \quad (Probability\ of\ success)$

$q = 1 - p = 1 - 0.2 = 0.8 \quad (Probability\ of\ failure)$

$x = 0, 1, 2, 3, 4 \quad (Possible\ values\ of\ X)$

(a) The probability function in a mathematical formula:

$$P(X=x)=\begin{cases}\dfrac{n!}{x!\,(n-x)!}\;p^{x}\,q^{n-x} & ;\quad x=0,1,2,\dots,n\\ 0 & ;Otherwise\end{cases}$$

$$P(X=x)=\begin{cases}\dfrac{4!}{x!\,(4-x)!}\;(0.2)^{x}\,(0.8)^{4-x} & ;\quad x=0,1,2,3,4\\ 0 & ;Otherwise\end{cases}$$

(b) The probability function in a table:

x	$P(X=x)$
0	$\frac{4!}{0!\,(4-0)!}\;(0.2)^{0}\,(0.8)^{4-0}=(1)(0.2)^{0}(0.8)^{4}=0.4096$
1	$\frac{4!}{1!\,(4-1)!}\;(0.2)^{1}\,(0.8)^{4-1}=(4)(0.2)^{1}(0.8)^{3}=0.4096$
2	$\frac{4!}{2!\,(4-2)!}\;(0.2)^{2}\,(0.8)^{4-2}=(6)(0.2)^{2}\,(0.8)^{2}=0.1536$
3	$\frac{4!}{3!\,(4-3)!}\;(0.2)^{3}\,(0.8)^{4-3}=(4)(0.2)^{3}\,(0.8)^{1}=0.0256$
4	$\frac{4!}{4!\,(4-4)!}\;(0.2)^{4}\,(0.8)^{4-4}=(1)(0.2)^{4}\,(0.8)^{0}=0.0016$
	Total = 1

x	$P(X=x)$
0	0.4096
1	0.4096
2	0.1536
3	0.0256
4	0.0016

(B) Find:

1. The probability that there will be 3 customers in the sample having excellent records.
$$P(X = 3) = 0.0256$$
2. The probability that there will be no customers in the sample having excellent records.
$$P(X = 0) = 0.4096$$
3. The probability that there will be at least 3 customers in the sample having excellent records.
$$P(X \geq 3) = P(x = 3) + P(X = 4) = 0.0256 + 0.0016$$
$$= 0.0272$$
4. The probability that there will be at most 2 customers in the sample having excellent records.
$$P(X \leq 2) = P(x = 0) + P(X = 1) + P(X = 2)$$
$$= 0.4096 + 0.4096 + 0.1536$$
$$= 0.9728$$
5. The expected number of customers having excellent records in the sample.
$$E(X) = \mu = \mu_X = np = 4 \times 0.2 = 0.8$$
6. The variance of the number of customers having excellent records in the sample.
$$Var(X) = \sigma^2 = \sigma_X^2 = npq = 4 \times 0.2 \times 0.8 = 0.64$$

Exercise 4: (Do it at home for yourself)
In a certain hospital, the medical records show that the percentage of lung cancer patients who smoke is 75%. Suppose that a doctor randomly selects a sample of 5 records of lung cancer patients from this hospital.
(A) Define the random variable X as:
X = The number of smokers in the sample.
Find the probability distribution of X.

(B) Find:

1. The probability that there will be 4 smokers in the sample.
2. The probability that there will be no smoker in the sample.
3. The probability that there will be at least 2 smokers in the sample.
4. The probability that there will be at most 3 smokers in the sample.
5. The expected number of smokers in the sample.
6. The variance of the number of smokers in the sample.

Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\ \lambda^{x}}{x!} \quad ; \quad x = 0,1,2,\ldots$$

$$E(X) = Var(X) = \lambda$$

The Poisson distribution:

1. It is discrete distribution.
2. The Poisson distribution is used to model a discrete random variable representing the number of occurrences of some random event in an interval of time or space (or some volume of matter).
3. The possible values of X are: 0,1,2,3, …
4. The discrete random variable X is said to have a Poison distribution with parameter (average or mean) λ if the probability distribution of X is given by: $P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad ; \quad X = 0,1,2,\ldots$

Example:

Some random quantities that can be modelled by Poisson distribution:

- No. of patients in a waiting room in an hour.
- No. of surgeries performed in a month.
- No. of rats in each house in a particular city.

<u>Question 1:</u>

The number of serious cases coming to a hospital during a night follows a Poisson distribution with an average of 10 persons per night, then:

1) The probability that 12 serious cases coming in the next night, is:

$$\lambda_{one\ night} = 10$$
$$\Downarrow$$
$$P(X = 12) = \frac{e^{-10}\ 10^{12}}{12!} = 0.09478$$

2) The average number of serious cases in a two nights' period is:

$$\lambda_{two\ nights} = 20$$

3) The probability that 20 serious cases coming in next two nights is:

$$\lambda_{two\ nights} = 20$$
$$\Downarrow$$
$$P(X = 20) = \frac{e^{-20}\ 20^{20}}{20!} = 0.0888$$

Question 2:

Given the mean number of serious accidents per year in a large factory is five. If the number of accidents follows a Poisson distribution, then the probability that in the next year there will be:

1. Exactly seven accidents:

$$\lambda_{one\ year} = 5$$

$$P(X = 7) = \frac{e^{-5}\ 5^7}{7!} = 0.1044$$

2. No accidents

$$P(X = 0) = \frac{e^{-5}\ 5^0}{0!} = 0.0067$$

3. one or more accidents

X	0	1	2	3	4	5	6	...
$P(X = x)$		*	*	*	*	*	*	*

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - 0.0067 = 0.9933$$

4. The expected number (mean) of serious accidents in the next two years is equal to

$$\lambda_{two\ years} = 10$$

5. The probability that in the next two years there will be three accidents

$$\lambda_{two\ years} = 10$$

$$P(X = 3) = \frac{e^{-10}\ 10^3}{3!} = 0.0076$$

Question 3:

The number of serious emergency cases in a small hospital during a day follow a Poisson distribution with an average of 5 cases per day, then:

1) The probability that exactly 2 emergency cases will come in a given day is:

$$\lambda_{one\ day} = 5$$

$$P(X = 2) = \frac{e^{-5}\ 5^{2}}{2!} = 0.0842$$

2) The average number of cases in 36 hours is:

$$\lambda_{36\ hours} = \lambda_{1.5\ day} = 1.5 \times 5 = 7.5$$

3) The variance of cases in 36 hours is:

$$\lambda_{36\ hours} = \lambda_{1.5\ day} = 1.5 \times 5 = 7.5$$

4) The standard deviation of cases in 36 hours is:

$$\sqrt{\lambda_{36\ hours}} = \sqrt{\lambda_{1.5\ day}} = \sqrt{1.5 \times 5} = \sqrt{7.5} = 2.74$$

More Exercise

Exercise 1:

Suppose that in a certain city, the weekly number of infected cases with Corona virus (COVID-19) has a Poisson distribution with an average (mean) of 5 cases per week.

(A) Find:

1. The probability distribution of the weekly number of infected cases (X).

$$P(X=x)=\begin{cases}\dfrac{e^{-\lambda}\,\lambda^{x}}{x!} & ;\quad x=0,1,2,3,\dots \\ 0 & ; Otherwise\end{cases}$$

$$\lambda=5$$

$$P(X=x)=\begin{cases}\dfrac{e^{-5}\,5^{x}}{x!} & ;\quad x=0,1,2,3,\dots \\ 0 & ; Otherwise\end{cases}$$

2. The probability that there will be 2 infected cases this week.

$$P(X=2)=\frac{e^{-5}\,5^{2}}{2!}=0.0842$$

3. The probability that there will be 1 infected case this week.

$$P(X=1)=\frac{e^{-5}\,5^{1}}{1!}=0.0337$$

4. The probability that there will be no infected cases this week.

$$P(X=0)=\frac{e^{-5}\,5^{0}}{0!}=0.0067$$

5. The probability that there will be at least 3 infected cases this week.

$$P(X\geq 3)=1-P(X<3)=1-P(X\leq 2)$$
$$=1-[P(X=0)+P(X=1)+P(X=2)]$$
$$=1-[\,0.0067+0.0337+0.0842\,]$$
$$=1-0.1246=0.8754$$

6. The probability that there will be at most 2 infected cases this week.

$$P(X\leq 2)=P(X=0)+P(X=1)+P(X=2)$$
$$=0.0067+0.0337+0.0842$$
$$=0.1246$$

7. The expected number (mean/average) of infected cases this week.

$$E(X) = \mu = \mu_X = \lambda = 5$$

8. The variance of the number of infected cases this week.

$$Var(X) = \sigma^2 = \sigma_X^2 = \lambda = 5$$

(B): Find:

1. The average (mean) of the number infected cases in a day.

$$\lambda = \frac{5}{7} = 0.7143$$

2. The probability distribution of the daily number of infected cases (X).

$$P(X = x) = \begin{cases} \dfrac{e^{-\lambda}\ \lambda^x}{x!} & ; \quad x = 0, 1, 2, 3, \ldots \\ 0 & ; Otherwise \end{cases}$$

$$\lambda = \frac{5}{7}$$

$$P(X = x) = \begin{cases} \dfrac{e^{-\frac{5}{7}}\ \left(\frac{5}{7}\right)^x}{x!} & ; \quad x = 0, 1, 2, 3, \ldots \\ 0 & ; Otherwise \end{cases}$$

3. The probability that there will be 2 infected cases tomorrow.

$$P(X = 2) = \frac{e^{-\frac{5}{7}}\ \left(\frac{5}{7}\right)^2}{2!} = 0.1249$$

4. The probability that there will be 1 infected case tomorrow.

$$P(X = 1) = \frac{e^{-\frac{5}{7}}\ \left(\frac{5}{7}\right)^1}{1!} = 0.3497$$

5. The probability that there will be no infected cases tomorrow.

$$P(X = 0) = \frac{e^{-\frac{5}{7}}\ \left(\frac{5}{7}\right)^0}{0!} = 0.4895$$

6. The probability that there will be at most 2 infected cases tomorrow.

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= 0.4895 + 0.3497 + 0.1249$$
$$= 0.9641$$

7. The probability that there will be at least 2 infected cases tomorrow.

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - [0.4895 + 0.3497]$$
$$= 1 - 0.8392 = 0.1608$$

8. The expected number (mean/average) of infected cases tomorrow.

$$E(X) = \mu = \mu_X = \lambda = \frac{5}{7} = 0.7143$$

9. The variance of the number of infected cases tomorrow.

$$Var(X) = \sigma^2 = \sigma_X^2 = \lambda = \frac{5}{7} = 0.7143$$

(C): Assuming that 4 weeks are in a month, find:

1. The average (mean) of the number infected cases per month.

$$E(X) = \mu = \mu_X = \lambda = 5 \times 4 = 20$$

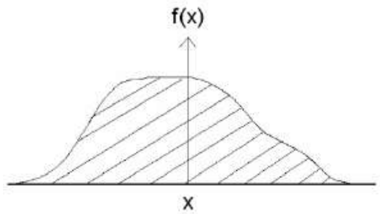
2. The variance of the number of infected cases per month.

$$Var(X) = \sigma^2 = \sigma_X^2 = \lambda = 5 \times 4 = 20$$

Continuous Probability Distributions

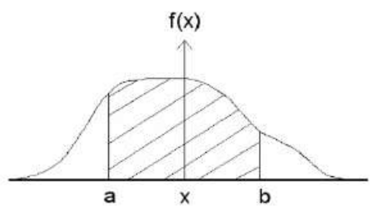
For any continuous random variable X, there exists a function $f(x)$, called the probability density function (pdf) of X, for which:

1. The total area under the curve of $f(x)$ equals to 1.



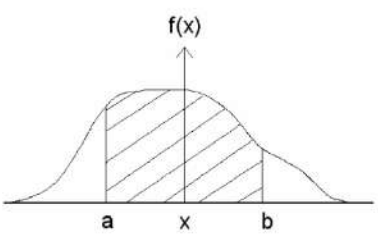
$$Total\ area = \int_{-\infty}^{\infty} f(x)\,dx = 1$$

2. The probability that X is between the points (a) and (b) equals to the area under the curve of $f(x)$ which is bounded by the point a and b.

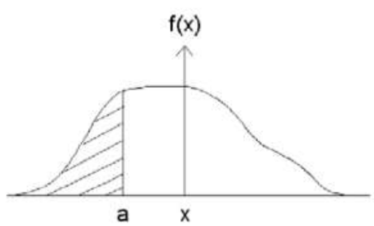


$$P(a \le X \le b) = \int_{a}^{b} f(x)dx = area$$

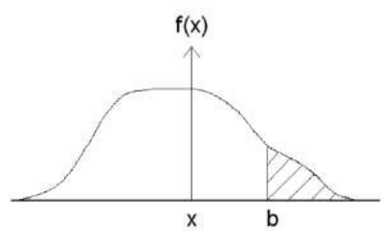
3. In general, the probability of an interval event is given by e area under the under the curve of $f(x)$ and above that interval.



$$P(a \le X \le b) = \int_{a}^{b} f(x)dx = area$$



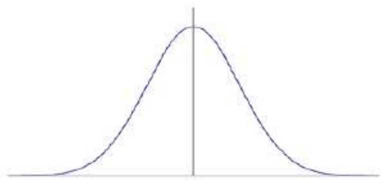
$$P(X \le a) = \int_{-\infty}^{a} f(x)dx = area$$



$$P(X \ge b) = \int_{b}^{\infty} f(x)\,dx = area$$

The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < X < \infty$$



Mean = Median = Mode

Normal distribution	$X \sim N(\mu, \sigma^2)$	
Standard normal	$Z \sim N(0, 1)$	$Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$

The continuous random variable X which has a normal distribution has several important characteristics:

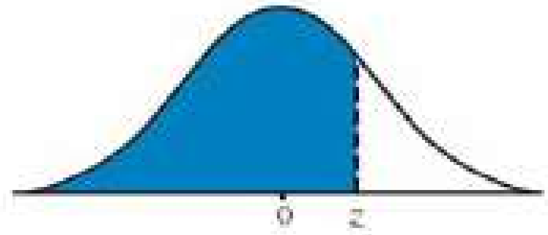
1. $-\infty < X < \infty$
2. The density function of X, $f(x)$, has a bell-shaped curve.
3. The highest point of the curve of $f(x)$ at the mean μ.
4. The curve of $f(x)$ is symmetric about the mean μ.
5. The normal distribution depends on two parameters:
   Mean = μ (determines the location)
   Standard deviation = σ (determines the shape)
6. If the random variable X is normally distributed with mean μ and standard deviation σ (variance σ^2), we write
   $$X \sim Normal\ (\mu, \sigma^2) \text{ or } X \sim N\ (\mu, \sigma^2)$$
7. The location of the normal distribution depends on μ. The shape of normal distribution depends on σ.

Question 1:

Given the standard normal distribution, Z~N (0, 1), find:

1. $P(Z < 1.43) = 0.92364$

z	0.00	0.01	0.02	0.03	0.04
0.00	0.50000	0.50399	0.50798	0.5[illegible]197	0.51595
0.10	0.53983	0.54380	0.54776	0.5[illegible]172	0.55567
0.20	0.57926	0.58317	0.58706	0.5[illegible]095	0.59483
0.30	0.61791	0.62172	0.62552	0.6[illegible]930	0.63307
0.40	0.65542	0.65910	0.66276	0.6[illegible]640	0.67003
0.50	0.69146	0.69497	0.69847	0.7[illegible]194	0.70540
0.60	0.72575	0.72907	0.73237	0.7[illegible]565	0.73891
0.70	0.75804	0.76115	0.76424	0.7[illegible]730	0.77035
0.80	0.78814	0.79103	0.79389	0.7[illegible]673	0.79955
0.90	0.81594	0.81859	0.82121	0.8[illegible]381	0.82639
1.00	0.84134	0.84375	0.84614	0.8[illegible]849	0.85083
1.10	0.86433	0.86650	0.86864	0.8[illegible]076	0.87286
1.20	0.88493	0.88686	0.88877	0.8[illegible]065	0.89251
1.30	0.90320	0.90490	0.90658	0.9[illegible]824	0.90988
1.40	[illegible]	[illegible]	[illegible]	0.92364	0.92507
1.50	0.93319	0.93448	0.93574	0.93699	0.93822
1.60	0.94520	0.94630	0.94738	0.94845	0.94950
1.70	0.95543	0.95637	0.95728	0.95818	0.95907
1.80	0.96407	0.96485	0.96562	0.96638	0.96712
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



$P(Z <$ أطراف الجدول $) =$ داخل الجدول

2. $P(Z > 1.67) = 1 - P(Z < 1.67) = 1 - 0.95254 = 0.04746$
3. $P(-2.16 < Z < -0.65)$

$$= P(Z < -0.65) - P(Z < -2.16)$$
$$= \quad 0.25785 \quad - \quad 0.01539 \quad = 0.24246$$

Question 2:

Given the standard normal distribution, Z~N(0,1), find:

1. $P(Z > 2.71) = 1 - P(Z < 2.71) = 1 - 0.99664 = 0.00336$

2. $P(-1.96 < Z < 1.96)$
$$= P(Z < 1.96) - P(Z < -1.96)$$
$$= \quad 0.9750 \quad - \quad 0.0250 \quad = 0.9500$$

3. $P(Z = 1.33) = 0$
4. $P(Z = 0.67) = 0$

5. If $P(Z < a) = 0.99290$, then the value of $a = 2.45$
6. If $P(Z < a) = 0.62930$, then the value of $a = 0.33$

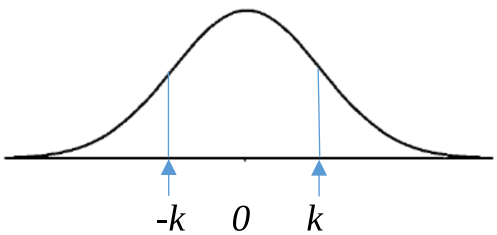
7. If $P(Z > a) = 0.63307 \Longrightarrow P(Z < a) = 1 - 0.63307$
$$\Longrightarrow P(Z < a) = 0.36693 \Longrightarrow a = -0.34$$

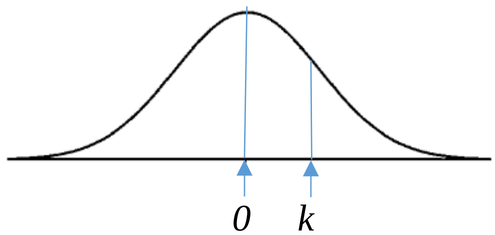
8. If $P(Z > a) = 0.02500 \Longrightarrow P(Z < a) = 1 - 0.02500$
$$\Longrightarrow P(Z < a) = 0.97500 \Longrightarrow a = 1.96$$

9. $Z_{0.9750} = 1.96$
10. $Z_{0.0392} = -1.76$
11. $Z_{0.01130} = -2.28$
12. $Z_{0.99940} = 3.24$
13. If $Z_{0.08} = -1.40$ then the value of $Z_{0.92}$ equals to:

A	-1.954	B	1	C	1.40	D	-1.40

14.If $P(-k < Z < k) = 0.8132$, then the value of k =





$$\Longrightarrow 2 \times P(0 < Z < k) = 0.8132$$

$$\Longrightarrow \quad P(0 < Z < k) = 0.4066$$

$$\Longrightarrow P(Z < k) - P(Z < 0) = 0.4066$$

$$\Longrightarrow P(Z < k) - \quad 0.5 \quad = 0.4066$$

$$\Longrightarrow P(Z < k) = 0.9066$$

$$\Longrightarrow k = 1.32$$

<u>Question 3:</u>

Given the standard normal distribution, then:

1) $P(-1.1 < Z < 1.1)$ *is:*

$$\Longrightarrow P(Z < 1.1) - P(Z < -1.1)$$

$$0.86433 - 0.13567 = 0.72866$$

2) $P(Z > -0.15)$ *is:*

$$= 1 - P(Z < -0.15)$$

$$= 1 - 0.44038 = 0.55962$$

3) The k value that has an area of 0.883 to its right, is:

Left	*Right*
<	>

$$P(Z > k) = 0.883$$
$$P(Z < k) = 1 - 0.883$$
$$P(Z < k) = 0.117$$
$$k = -1.19$$

Question 4:

The finished inside diameter of a piston ring is normally distributed with a mean 12 cm and standard deviation of 0.03 cm. Then,

1. The proportion of rings that will have inside dimeter less than 12.05.

$$X \sim N(\mu \ , \sigma^2)$$
$$X \sim N(12 \ , 0.03^2)$$

$$P(X < 12.05) = P\left(Z < \frac{12.05 - \mu}{\sigma}\right)$$

$$= P\left(Z < \frac{12.05 - 12}{0.03}\right) = P(Z < 1.67) = 0.9525$$

2. The proportion of rings that will have inside dimeter exceeding 11.97.

$$P(X > 11.97) = P\left(Z > \frac{11.97 - \mu}{\sigma}\right)$$

$$= P\left(Z > \frac{11.97 - 12}{0.03}\right) = P(Z > -1)$$
$$= 1 - P(Z < -1)$$
$$= 1 - 0.1587 = 0.8413$$

3. The proportion of rings that will have inside dimeter between 11.95 and 12.05.

$$P(11.95 < X < 12.05) = P\left(\frac{11.95 - 12}{0.03} < Z < \frac{12.05 - 12}{0.03}\right)$$

$$= P(\ -1.67 \ < Z < \ 1.67)$$

$$= P(Z < 1.67) - P(Z < -1.67)$$

$$= \ 0.9525 \ - \ 0.0475 = 0.905$$

Question 5:

The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128kg and a standard deviation of 9 kg

1. The probability of fat persons with weight at most 110 kg is:

$$X \sim N(\mu \ , \sigma^2)$$
$$X \sim N(128, 9^2)$$

$$P(X \leq 110) = P\left(Z < \frac{110 - 128}{9}\right) = P(Z < -2) = 0.0228$$

2. The probability of fat persons with weight more than 149 kg is:

$$P(X > 149) = P\left(Z > \frac{149 - 128}{9}\right) = 1 - P(Z < 2.33) = 1 - 0.9901 = 0.0099$$

3. The weight x above which 86% of those persons will be:

$$P(X > x) = 0.86 \Rightarrow P(X < x) = 0.14 \Rightarrow P\left(Z < \frac{x - 128}{9}\right) = 0.14$$

by searching inside the table for 0.14, and transforming X to Z, we got:

$$\frac{x - 128}{9} = -1.08$$

$$x - 128 = -1.08 \times 9$$

$$x = (-1.08 \times 9) + 128$$

$$x = 118.28$$

4. The weight x below which 50% of those persons will be:

P(X < x) = 0.5, by searching inside the table for 0.5, and transforming X to Z

$$\frac{x - 128}{9} = 0 \Rightarrow x = 128$$

Question 6:

If the random variable X has a normal distribution with the mean μ and the variance σ^2, then $P(X < \mu + 2\sigma)$ equal to:

$$P(X < \mu + 2\sigma) = P\left(Z < \frac{(\mu + 2\sigma) - \mu}{\sigma}\right) = P(Z < 2) = 0.9772$$

Question 7:

If the random variable X has a normal distribution with the mean μ and the variance 1, and if then $P(X < 3) = 0.877$ then μ equal to

$$\textit{Given that } \sigma = 1$$

$$P(X < 3) = 0.877 \Rightarrow P\left(Z < \frac{3-\mu}{1}\right) = 0.877$$

$$3 - \mu = 1.16 \Rightarrow \mu = 1.84$$

Question 8:

The weight in gram of beans in a tin is normally distributed with μ and standard deviation 7.8. given that 10% of tins contain less than 200g, then the value of μ equals to

$$\textit{Given that } \sigma = 7.8$$

$$P(X < 200) = 0.10 \Rightarrow P\left(Z < \frac{200-\mu}{7.8}\right) = 0.10$$

$$\frac{200-\mu}{7.8} = -1.285 \Rightarrow \mu = 210.023$$

Question 9:

Suppose that the marks of students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark is:

$$X \sim N(70, 25)$$

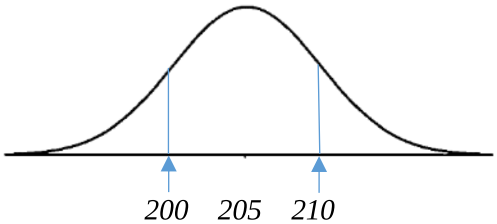
$$P(X < x) = 0.33 \Rightarrow P\left(Z < \frac{x - 70}{5}\right) = 0.33$$

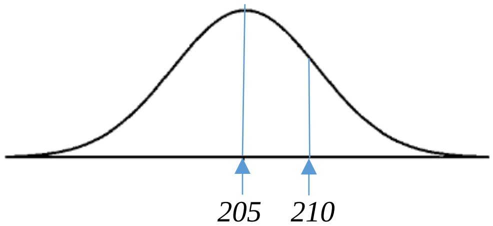
by searching inside the table for 0.33, and transforming X to Z, we got:

$$\frac{x - 70}{5} = -0.44 \Rightarrow x = 67.8$$

Question 10:

The weight in grams, of beans in a tin is normally distribution with mean 205g and standard deviation σ. If 98% of tins contains between 200g and 210g, then the value of σ equal to:





$$\Longrightarrow 2 \times P(205 < X < 210) = 0.98$$

$$\Longrightarrow \quad P(205 < X < 210) = 0.49$$

$$\Longrightarrow P(X < 210) - P(X < 205) = 0.49$$

$$\Longrightarrow P\left(Z < \frac{210-205}{\sigma}\right) - \quad 0.5 \quad = 0.49$$

$$\Longrightarrow P\left(Z < \frac{5}{\sigma}\right) = 0.99$$

$$\Longrightarrow \quad \frac{5}{\sigma} = 2.33 \quad \Longrightarrow \sigma = 2.15$$

Question 11:

What k value corresponds to 17% of the area between the mean and the z value?
$P(\mu < Z < k) = 0.17$

$P(\mu < Z < k) = 0.17$
$P(Z < k) - P(Z < \mu) = 0.17$
$P(Z < k) - \quad 0.5 \quad = 0.17$
$P(Z < k) = 0.67$
$k = 0.44$

Question 12:

A nurse supervisor has found that staff nurses complete a certain task in 10 minutes on average. If the times required to complete the task are approximately normally distributed with a standard deviation of 3 minutes, then:

1) The probability that a nurse will complete the task in less than 8 minutes is:

$$X \sim N(10, 3^2)$$

$$P(X < 8) = P\left(Z < \frac{8-10}{3}\right) = P(Z < -0.67) = 0.2514$$

2) The probability that a nurse will complete the task in more than 4 minutes is:

$$P(X > 4) = 1 - P\left(Z < \frac{4-10}{3}\right) = 1 - P(Z < -2) = 1 - 0.0228 = 0.9772$$

3) If eight nurses were assigned the task, the expected number of them who will complete it within 8 minutes is approximately equal to:

$$n \times P(0 < X < 8) = 8 \times P\left(\frac{0-10}{3} < Z < \frac{8-10}{3}\right)$$
$$= 8 \times P(-3.33 < Z < -0.67)$$
$$= 8 \times [P(Z < -0.67) - P(Z < -3.33)]$$
$$= 8 \times [0.2514 - 0.0004] = 2$$

4) If a certain nurse completes the task within k minutes with probability 0.6293; then k equals approximately:

$$P(0 < X < k) = 0.6293$$
$$\Longrightarrow P\left(\frac{0-10}{3} < Z < \frac{k-10}{3}\right) = 0.6293$$
$$\Longrightarrow P\left(-3.33 < Z < \frac{k-10}{3}\right) = 0.6293$$
$$\Longrightarrow P\left(Z < \frac{k-10}{3}\right) - P(Z < -3.33) = 0.6293$$
$$\Longrightarrow P\left(Z < \frac{k-10}{3}\right) - 0.0004 = 0.6293$$
$$\Longrightarrow P\left(Z < \frac{k-10}{3}\right) = 0.6297$$
$$\Longrightarrow \frac{k-10}{3} = 0.33 \Longrightarrow k = 11$$

<u>Question 13:</u>

Given the normally distributed random variable X with mean 491 and standard deviation 119,

1. If P (X < k) = 0.9082, the value of k is equal to

A	649.27	B	390.58	C	128.90	D	132.65

2. If P (292< X <M) = 0.8607, the value of M is equal to

A	766	B	649	C	108	D	136

<u>Question 14:</u>

The IQ (Intelligent Quotient) of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10, then:

1) The probability that an individual picked at random will have an IQ greater than 75 is:

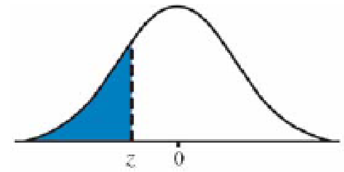
A	0.9332	B	0.8691	C	0.7286	D	0.0668

2) The probability that an individual picked at random will have an IQ between 55 and 75 is:

A	0.3085	B	0.6915	C	0.6247	D	0.9332

3) If the probability that an individual picked at random will have an IQ less than k is 0.1587. Then the value of k

A	50	B	45	C	51	D	40



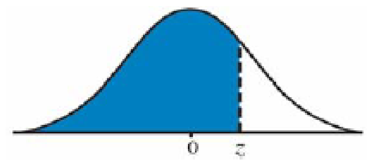
Standard Normal Table

Areas Under the Standard Normal Curve

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

Standard Normal Table (continued)

Areas Under the Standard Normal Curve



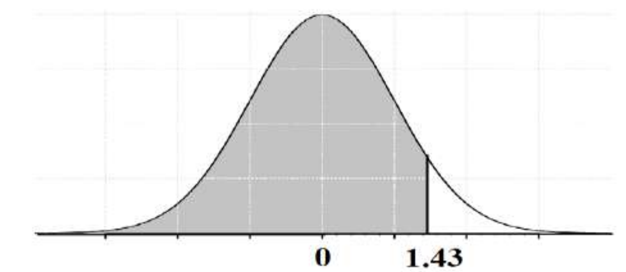
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	**0.00**
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	**0.10**
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	**0.20**
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	**0.30**
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	**0.40**
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	**0.50**
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	**0.60**
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	**0.70**
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	**0.80**
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	**0.90**
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	**1.00**
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	**1.10**
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	**1.20**
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	**1.30**
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	**1.40**
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	**1.50**
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	**1.60**
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	**1.70**
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	**1.80**
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	**1.90**
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	**2.00**
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	**2.10**
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	**2.20**
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	**2.30**
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	**2.40**
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	**2.50**
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	**2.60**
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	**2.70**
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	**2.80**
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	**2.90**
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.9990	**3.00**
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	**3.10**
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	**3.20**
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	**3.30**
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	**3.40**
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	**3.50**

More Exercises:

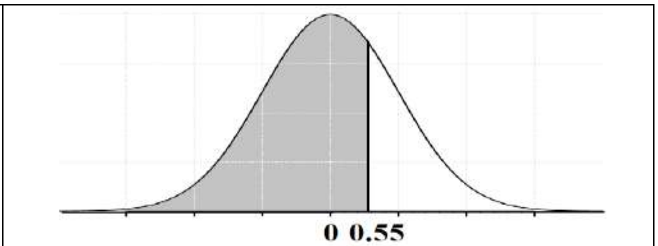
Exercise 1:

Suppose that the random variable Z has a standard normal distribution

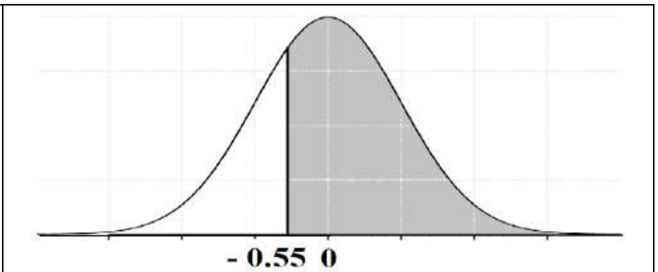
(a) Find the area to the left of Z =1.43.

$P(Z < 1.43) = 0.92364$	

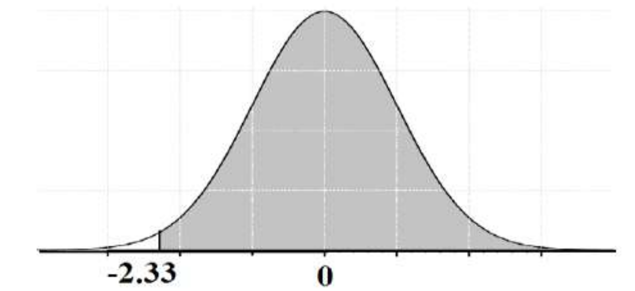
(b) Find P (Z < 0.55).

$P(Z < 0.55) = 0.70884$	

(c) Find P (Z > - 0.55).

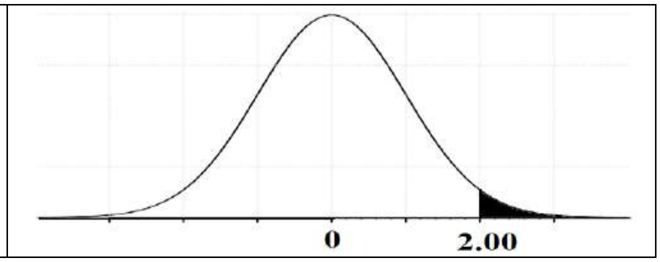
$P(Z > -0.55)$ $=1-P(Z<-0.55)$ $=1-0.29116$ $= 0.70884$	

(d) Find P (Z > - 2.33).

$P(Z > -2.33)$ $= 1-P(Z<-2.33) =1- 0.00990$ $= 0.9901$	

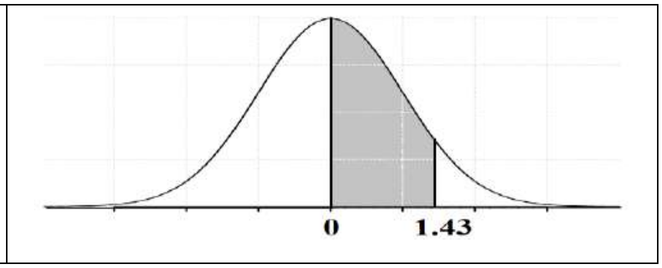
(e) Find the area to the right of z = 2.

P(Z > 2.00) =1-P(Z<2.00)
= 1- 0.97725
= 0.02275



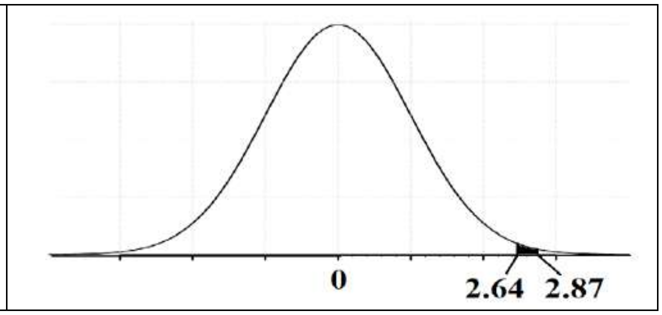
(f) Find the area under the curve between z = 0 and z = 1.43.

P(0 < Z <1.43)
= P(Z <1.43) – P(Z<0.00)
= 0.92364 - 0.5
= 0.42364



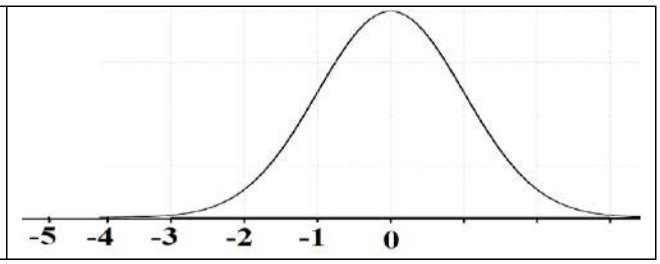
(g) Find the probability that Z will take a value between z = 2.64 and z = 2.87.

P(2.64 < Z <2.87)
= P(Z <2.87) – P(Z<2.64)
= 0.99795 -0.99585
= 0.0021



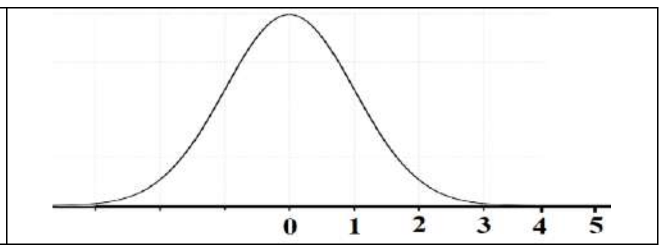
(h) Find P(Z < - 5).

P(Z < - 5) ≈ 0



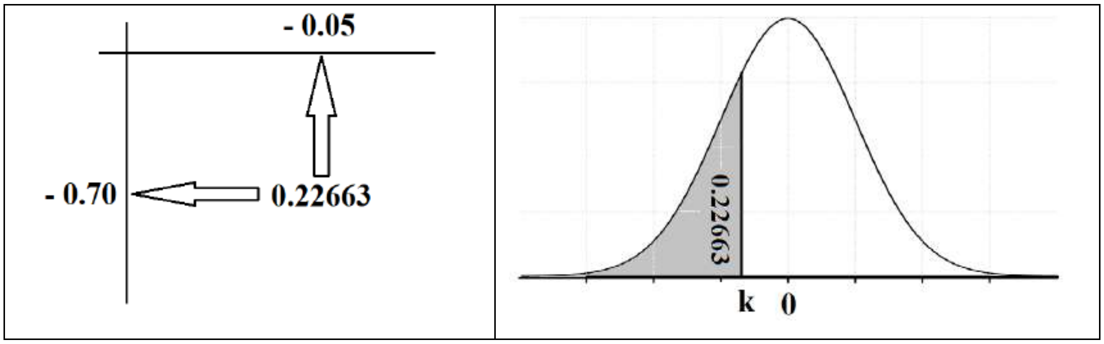
(i) Find P(Z > 5).

P(Z > 5) = 1-P(Z<5)
≈ 1 - 1
= 0



(j) If $P(Z \leq k) = 0.22663$, then find the value of k.

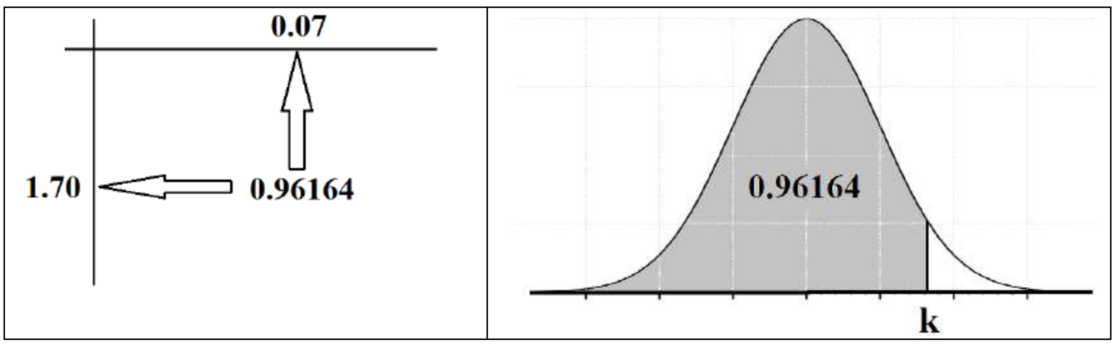
$k = -0.75$



(k) If $P(Z \geq k) = 0.03836$, then find the value of k.

$P(Z<k)=1- P(Z \geq k) =1-0.03836 = 0.96164$

$k = 1.77$



l) If $P(-2.67 < Z \leq k) = 0.97179$, then find the value of k.

$0.97179 = P(-2.67 < Z \leq k)$

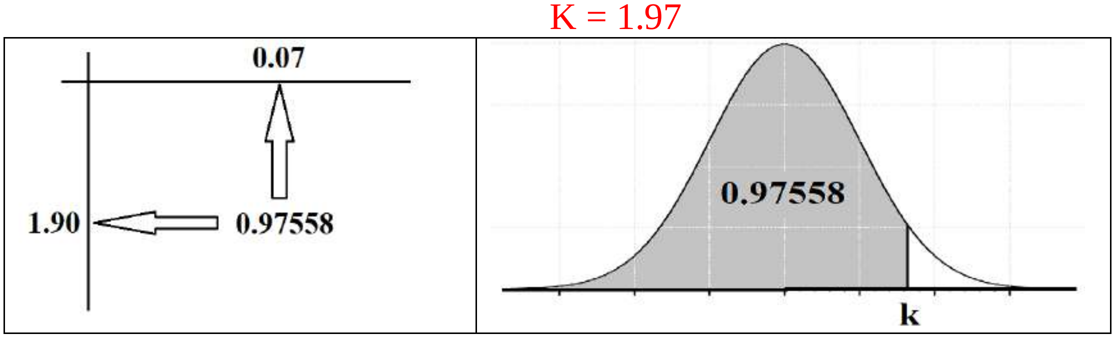
$= P(Z<K) - P(Z < -2.67)$

$P(Z<K) = 0.97179 + P(Z < -2.67)$

$P(Z<K) =0.97179 + 0.00379$

$= 0.97558$

$K = 1.97$



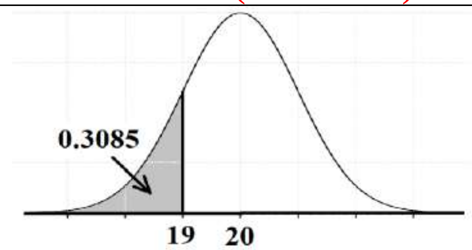
Exercise 2:

Suppose that the time for a person to be tested for corona virus (in minutes) has a normal distribution with mean $\mu = 20$ and variance $\sigma^2 = 4$.

(1) If we select a person at random, what is the probability that his examination period will be less than 19 minutes?

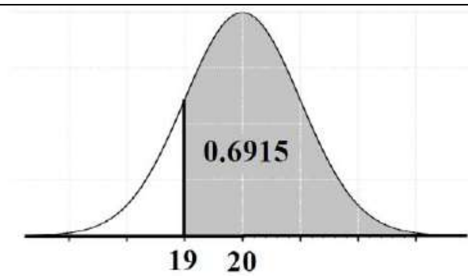
Let X = person's period of examination (in minutes)

$$P(X < 19) = P\left(\frac{X-\mu}{\sigma} < \frac{19-\mu}{\sigma}\right) = P\left(Z < \frac{19-20}{2}\right) = P(Z < -0.5) = 0.3085$$



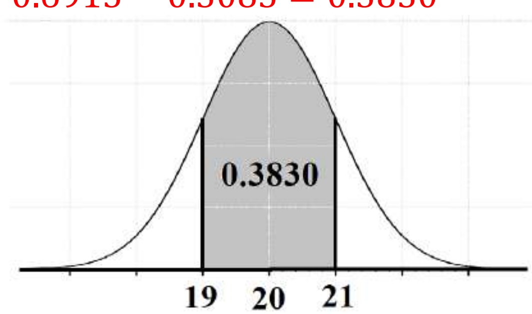
(2) If we select a person at random, what is the probability that his examination period will be more than 19 minutes?

$$P(X > 19) = 1 - P(X < 19) = 1 - 0.3085 = 0.6915$$



(3) If we select a person at random, what is the probability that his examination period will be between 19 and 21 minutes?

$$P(19 < X < 21) = P(X < 21) - P(X < 19)$$
$$= P\left(\frac{X-\mu}{\sigma} < \frac{21-\mu}{\sigma}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{19-\mu}{\sigma}\right)$$
$$= P\left(Z < \frac{21-20}{2}\right) - P\left(Z < \frac{19-20}{2}\right)$$
$$= P(Z < 0.5) - P(Z < -0.5)$$
$$= 0.6915 - 0.3085 = 0.3830$$



(4) What is the percentage of persons whose examination period are less than 19 minutes?

$$\% = P(X < 19) * 100\% = 0.3085 * 100\%$$
$$= 30.85\%$$

(5) If we select a sample of 2000 persons, how many persons would be expected to have examination periods that are less than 19 minutes?

$$\text{Expected number} = 2000 \times P(X < 19)$$
$$= 2000 \times 0.3085$$
$$= 617$$

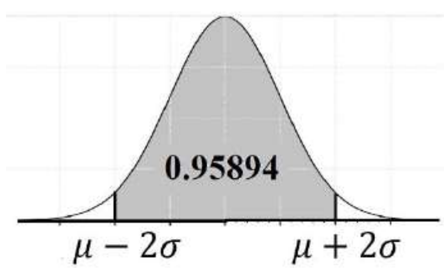
Exercise 3:

Suppose that we have a normal population with mean μ and standard deviation σ.

(1) Find the percentage of values which are between $\mu - 2\sigma$ and $\mu + 2\sigma$.

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(X < \mu + 2\sigma) - P(X < \mu - 2\sigma)$$

$$= P\left(\frac{X-\mu}{\sigma} < \frac{(\mu+2\sigma)-\mu}{\sigma}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{(\mu-2\sigma)-\mu}{\sigma}\right)$$
$$= P\left(Z < \frac{(\mu+2\sigma)-\mu}{\sigma}\right) - P\left(Z < \frac{(\mu-2\sigma)-\mu}{\sigma}\right)$$
$$= P\left(Z < \frac{2\sigma}{\sigma}\right) - P\left(Z < \frac{-2\sigma}{\sigma}\right)$$
$$= P(Z < 2.00) - P(Z < -2.00)$$
$$= 0.97725 - 0.02275$$
$$= 0.9545$$



(2) Find the percentage of values which are between $\mu - \sigma$ and $\mu + \sigma$.

Dot it yourself

(3) Find the percentage of values which are between $\mu - 3\sigma$ and $\mu + 3\sigma$.

Dot it yourself

<u>Exercise 4:</u> (Read it yourself)

In a study of fingerprints, an important quantitative characteristic is the total ridge count for the 10 fingers of an individual. Suppose that the total ridge counts of individuals in a certain population are approximately normally distributed with a mean of 140 and a standard deviation of 50. Then:

(1) The probability that an individual picked at random from this population will have a ridge count of 200 or more is:

$$\begin{aligned} P(X > 200) &= 1 - P(X < 200) \\ &= 1 - P\left(Z < \frac{200-\mu}{\sigma}\right) \\ &= 1 - P\left(Z < \frac{200-140}{50}\right) \\ &= 1 - P(Z < 1.2) \\ &= 1 - 0.88493 = 0.11507. \end{aligned}$$

(2) The probability that an individual picked at random from this population will have a ridge count of less than 100 is:

$$\begin{aligned} P(X < 100) &= P\left(Z < \frac{100-\mu}{\sigma}\right) \\ &= P\left(Z < \frac{100-140}{50}\right) \\ &= P(Z < -0.80) = 0.18673 \end{aligned}$$

(3) The probability that an individual picked at random from this population will have a ridge count between 100 and 200 is:

$$\begin{aligned} P(100 < X < 200) &= P(X < 200) - P(X < 100) \\ &= P(X < 200) - P(X < 100) \\ &= P\left(Z < \frac{200-140}{50}\right) - \left(Z < \frac{100-140}{50}\right) \\ &= P(Z < 1.20) - (Z < -0.80) \\ &= 0.88493 - 0.18673 = 0.6982 \end{aligned}$$

(4) The percentage of individuals whose ridge counts are between 100 and 200 is:

$$\begin{aligned} P(100 < X < 200) * 100\% &= 0.6982 * 100\% \\ &= 69.82\% \end{aligned}$$

(5) If we select a sample of 5,000 individuals from this population, how many individuals would be expected to have ridge counts that are between 100 and 200?

$$\begin{aligned} \text{Expected number} &= 5000 \times P(100 < X < 200) \\ &= 5000 \times 0.6982 = 3491 \end{aligned}$$

Chapter 5 Sampling Distribution

Sampling Distribution

Single Mean	Two Means
$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ $E(\bar{X}) = \bar{X} = \mu$ $Var(\bar{X}) = \frac{\sigma^2}{n}$	$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$ $E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$ $Var(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
Single Proportion	**Two Proportions**
For large sample size ($n \geq 30,\ np > 5,\ nq > 5$) $\hat{p} \sim N\left(p, \frac{pq}{n}\right)$ $E(\hat{p}) = p$ $Var(\hat{p}) = \frac{pq}{n}$	For large sample size ($n_1 \geq 30,\ n_1p_1 > 5,\ n_1q_1 > 5$) ($n_2 \geq 30,\ n_2p_2 > 5,\ n_2q_2 > 5$) $\hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}\right)$ $E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2$ $Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}$

	population normal or not normal n large ($n \geq 30$)		population normal n small ($n < 30$)	
	σ known	σ unknown	σ known	σ unknown
Sampling distribution	$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$	$Z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$	$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$	$T = \frac{\bar{X} - \mu}{s/\sqrt{n}}$

Some Results about Sampling Distribution of $\bar{X}$:

Result (1): mean and variance of $\bar{X}$

If $X_1, X_2, \ldots, X_n$ is a random sample of size n from any distribution with mean μ and variance σ^2, then:

1. The mean of $\bar{X}$ is : $\mu_{\bar{X}} = \mu$
2. The variance $\bar{X}$ is: $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$
3. The standard deviation of $\bar{X}$ is call <u>the standard error</u> and is defined by: $\sigma_{\bar{X}} = \sqrt{\sigma_{\bar{X}}^2} = \frac{\sigma}{\sqrt{n}}$

Result (2): Sampling from normal population:

If $X_1, X_2, \ldots, X_n$ is a random sample of size n from Normal (μ, σ^2), then the sample mean has a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$, that is:

1. $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$
2. $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$

We use this result when sampling from normal distribution with known variance σ^2.

Result (3): Central limit theorem (CLT): Sampling from Non-normal population:

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size n from non-normal population with mean μ and variance σ^2. If the sample size n is large $(n > 30)$, then:

1. $\bar{X} \approx N(\mu, \frac{\sigma^2}{n})$ (approximately)
2. $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \approx N(0, 1)$ (approximately)

We use this result when sampling distribution from non-normal distribution with known variance σ^2 and with large sample size.

Result (4): Normal population and unknown variance σ^2

If $X_1, X_2, \ldots, X_n$ is a random sample of size n from a normal distribution with mean μ and unknown variance σ^2, that is Normal (μ, σ^2), then the statistic:

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Has a t-distribution with $(n - 1)$ degrees of freedom, where S is the sample standard deviation

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

We write:

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{(v=n-1)} \text{ degrees of freedom } (df = v = n - 1).$$

Question 1:

The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

1. The sample mean $\bar{X}$ of a random sample of 5 batteries selected from this product has mean $E(\bar{X}) = \mu_{\bar{X}}$.

$$\boxed{\mu = 5 \ ; \ \sigma = 1 \ ; \ n = 5}$$

$$E(\bar{X}) = \mu = 5$$

2. The variance $Var(\bar{X}) = \sigma_{\bar{X}}^2$ of the sample mean $\bar{X}$ of a random sample of 5 batteries selected from this product is equal to:

$$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{1}{5} = 0.2$$

3. The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4.

$$n = 16 \rightarrow \frac{\sigma}{\sqrt{n}} = \frac{1}{4}$$

$$P(4.5 < \bar{X} < 5.4) = P\left(\frac{4.5-\mu}{\frac{\sigma}{\sqrt{n}}} < Z < \frac{5.4-\mu}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(\frac{4.5-5}{\frac{1}{4}} < Z < \frac{5.4-5}{\frac{1}{4}}\right) = P(-2 < Z < 1.6)$$

$$= P(Z < 1.6) - P(Z < -2)$$

$$= 0.9452 - 0.0228 = 0.9224$$

4. The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is:

$$P(\bar{X} < 5.5) = P\left(Z < \frac{5.5-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z < \frac{5.5-5}{1/4}\right) = P(Z < 2) = 0.9772$$

5. The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:

$$P(\bar{X} > 4.75) = P\left(Z > \frac{4.75-\mu}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(Z > \frac{4.75-5}{\frac{1}{4}}\right) = P(Z > -1)$$

$$= 1 - P(Z < -1) = 1 - 0.1587 = 0.841$$

6. If $P(\bar{X} > a) = 0.1492$ where $\bar{X}$ represent the sample mean for a random sample of size 9 of such batteries, then the numerical value of a is:

$P(\bar{X} > a) = 0.1492 \;\; ; \;\; n = 9$

$$P\left(Z > \frac{a-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = 0.1492$$

$$\Rightarrow P\left(Z < \frac{a-5}{\frac{1}{3}}\right) = 1 - 0.1492$$

$$\Rightarrow P\left(Z < \frac{a-5}{\frac{1}{3}}\right) = 0.8508$$

$$\frac{a-5}{\frac{1}{3}} = 1.04 \Longrightarrow a = 5.347$$

Question 2:

Suppose that you take a random sample of size $n = 64$ from a distribution with mean $\mu = 55$ and standard deviation $\sigma = 10$. Let $\bar{X} = \frac{1}{n}\sum x$ be the sample mean.

1. What is the approximate sampling distribution of $\bar{X}$.

$$\boxed{\mu = 55 \;\; ; \;\; \sigma = 10 \;\; ; \;\; n = 64}$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) = \bar{X} \sim N\left(55, \frac{100}{64}\right)$$

2. What is the mean of $\bar{X}$?

$$E(\bar{X}) = \mu = 55$$

3. What is the standard error (standard deviation) of $\bar{X}$?

$$S.D(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{64}} = \frac{10}{8}$$

4. Find the probability that the sample mean $\bar{X}$ exceeds 52.

$$P(\bar{X} > 52) = P\left(Z > \frac{52-55}{\frac{10}{8}}\right) = P(Z > -2.4)$$

$$= 1 - P(Z < -2.4)$$

$$= 1 - 0.0082 = 0.9918$$

<u>Question 3:</u>

Suppose that the hemoglobin levels (in g/dl) of healthy Saudi females are approximately normally distributed with mean of 13.5 and a standard deviation of 0.7. If 15 healthy adult Saudi female is randomly chosen, then:

1. The mean of $\overline{X}$ ($E(\overline{X})$ or $\mu_{\overline{X}}$)

A	0.7	B	13.5	C	15	D	3.48

2. The standard error of $\overline{X}$ ($\sigma_{\overline{X}}$)

A	0.181	B	0.0327	C	0.7	D	13.5

3. $P(\overline{X} < 14) =$

A	0.99720	B	0.99440	C	0.76115	D	0.9971

4. $P(\overline{X} > 13.5) =$

A	0.99	B	0.50	C	0.761	D	0.622

5. $P(13 < \overline{X} < 14) =$

A	0.9972	B	0.9944	C	0.7615	D	0.5231

<u>Question 4:</u>

If the uric acid value in normal adult males is approximately normally distributed with a mean and standard derivation of 5.7 and 1 mg percent, respectively, find the probability that a sample of size 9 will yield a mean

1. Greater than 6 is:

A	0.2109	B	0.1841	C	0.8001	D	0.8159

2. At most 5.2 is:

A	0.6915	B	0.9331	C	0.8251	D	0.0668

3. Between 5 and 6 is:

A	0.1662	B	0.7981	C	0.8791	D	0.9812

<u>Question 5:</u>

Medical research has concluded that people experience a common cold roughly two times per year. Assume that the time between colds is normally distributed with a mean 165 days and a standard deviation of 45 days. Consider the sampling distribution of the sample mean based on samples of size 36 drown from the population:

1. The mean of sampling distribution $\bar{X}$ is:

A	210	B	36	C	45	D	165

2. The distribution if the mean of $\bar{X}$ is:

A	$N(165,2025)$	B	$N(165,45)$	C	$T, with\ df = 30$	D	$N(165,7.5)$

3. $\mathrm{P}(\bar{\mathrm{X}} > 178) =$

A	0.0415	B	0.615	C	0.958	D	0.386

<u>Sampling Distribution: Two Means:</u>

$$* \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2\ , \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$* E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 \qquad * Var(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

<u>Question 6:</u>

A random sample of size $n_1 = 36$ is taken from normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let $\bar{X}_1$ and $\bar{X}_2$ be the average of the first and second sample, respectively.

1. Find $E(\bar{X}_1 - \bar{X}_2)$ and $Var(\bar{X}_1 - \bar{X}_2)$.

$$n_1 = 36\ , \mu_1 = 70\ , \sigma_1 = 4$$
$$n_2 = 49\ , \mu_2 = 85\ , \sigma_2 = 5$$

$$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 70 - 85 = -15$$

$$Var(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{16}{36} + \frac{25}{49} = 0.955$$

2. Find $P(\bar{X}_1 - \bar{X}_2 > -16)$.

$$P(\bar{X}_1 - \bar{X}_2 > -16) = P\left(Z > \frac{-16-(-15)}{\sqrt{0.955}}\right) = 1 - P\left(Z < \frac{-16-(-15)}{\sqrt{0.955}}\right)$$

$$= 1 - P(Z < -1.02) = 0.8461$$

Question 7:

A random sample of size 25 is taken from a normal population (1st population) having a mean of 100 and a standard of 6. A second random sample of size 36 is taken from a different normal population (2nd population) having a mean of 97 and a standard deviation of 5. Assume that these two samples are independent.

1. The probability that the sample mean of the first sample will exceed the sample mean of the second sample by at least 6 is:

$$n_1 = 25\,,\ \mu_1 = 100\,, \sigma_1 = 6$$
$$n_2 = 36\,,\ \mu_2 = 97\ \ , \sigma_2 = 5$$

$$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 100 - 97 = 3 \qquad Var(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{36}{25} + \frac{25}{36} = 2.134$$

$$P(\bar{X}_1 > \bar{X}_2 + 6) = P(\bar{X}_1 - \bar{X}_2 > 6)$$

$$= P\left(Z > \frac{6-(3)}{\sqrt{2.134}}\right) = P(Z > 2.05)$$
$$= 1 - P(Z < 2.05)$$
$$= 1 - 0.9798 = 0.0202$$

2. The probability that the difference between the two-sample means will be less than 2 is:

$$P(\bar{X}_1 - \bar{X}_2 < 2) = P\left(Z < \frac{2-(3)}{\sqrt{2.134}}\right)$$

$$= P(Z < -0.68) = 0.2483$$

Question 8:

Given two normally distributed population with equal means and variances $\sigma_1^2 = 100$, $\sigma_2^2 = 350$. Two random samples of size $n_1 = 40$, $n_2 = 35$ are drown and sample means $\bar{X}_1$ and $\bar{X}_2$ are calculated, respectively, then

$\mu_1 - \mu_2 = 0$

1. $P(\bar{X}_1 - \bar{X}_2 > 12)$ is

A	0.1499	B	0.8501	C	0.9997	D	0.0003

2. $P(5 < \bar{X}_1 - \bar{X}_2 < 12)$ is

A	0.0789	B	0.9217	C	0.8002	D	None of these

Sampling Distribution: Single Proportion:

For large sample size ($n \geq 30,\ np > 5,\ nq > 5$)

$$* \hat{p} \sim N\left(p, \frac{pq}{n}\right)$$

$$* E(\hat{p}) = p \qquad * Var(\hat{p}) = \frac{pq}{n}$$

Question 9:

Suppose that 10% of the students in a certain university smoke cigarette. A random sample of 30 student is taken from this university. Let $\hat{p}$ be the proportion of smokers in the sample.

1. Find $E(\hat{p}) = \mu_{\hat{p}}$ the mean of $\hat{p}$.

$$p = 0.1 \quad ; \quad n = 30 \quad ; \quad q = 1 - p = 0.9$$

$$E(\hat{p}) = p = 0.1$$

2. Find $Var(\hat{p}) = \sigma^2_{\hat{p}}$ the variance of $\hat{p}$.

$$Var(\hat{p}) = \frac{pq}{n} = \frac{0.1 \times 0.9}{30} = 0.003$$

3. Find an approximate distribution of $\hat{p}$

$$\hat{p} \sim N(0.1, 0.003)$$

4. Find $P(\hat{p} > 0.25)$.

$$P(\hat{p} > 0.25) = P\left(Z > \frac{0.25 - 0.1}{\sqrt{0.003}}\right) = P(Z > 2.74)$$

$$= 1 - P(Z < 2.74) = 1 - 0.99693 = 0.00307$$

Question 10:

Suppose that 15% of the patients visiting a certain clinic are females. If A random sample of 35 patients was selected, $\hat{p}$ represent the proportion of females in the sample. then find:

1. The expected value of $(\hat{p})$ is:

A	0.35	B	0.85	C	0.15	D	0.80

2. The standard deviation of $(\hat{p})$ is:

A	0.3214	B	0.0036	C	0.1275	D	0.0604

3. The approximate sampling distribution of $(\hat{p})$ is:

A	N(0.15,0.0604)	B	Binomial(0.15,35)	C	$N(0.15, 0.0604^2)$	D	$Binomial(0.15, 35^2)$

4. The $P(\hat{p} > 0.15)$ is:

A	0.35478	B	0.5	C	0.96242	D	0.46588

Question 11:

In a study, it was found that 31% of the adult population in a certain city has a diabetic disease. 100 people are randomly sampled from the population. Then

1. The mean for the sample proportion $\left(E(\hat{p}) \text{ or } \mu_{\hat{p}}\right)$ is:

A	0.40	B	0.31	C	0.69	D	0.10

2. $P(\hat{p} > 0.40) =$

A	0.02619	B	0.02442	C	0.0256	D	0.7054

Sampling Distribution: Two Proportions:

For large sample size $(n_1 \geq 30,\ n_1p_1 > 5,\ n_1q_1 > 5)$
$(n_2 \geq 30,\ n_2p_2 > 5,\ n_2q_2 > 5)$

$$* \hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}\right)$$

$$* E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 \qquad * Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}$$

Question 12:

Suppose that 25% of the male student and 20% of the female student in certain university smoke cigarettes. A random sample of 35 male students is taken. Another random sample of 30 female student is independently taken from this university. Let $\hat{p}_1$ and $\hat{p}_2$ be the proportions of smokers in the two sample, respectively.

1. Find $E(\hat{p}_1 - \hat{p}_2) = \mu_{\hat{p}_1 - \hat{p}_2}$, the mean of $\hat{p}_1 - \hat{p}_2$.

$$p_1 = 0.25 \ ; \quad n_1 = 35$$
$$p_2 = 0.20 \ ; \quad n_2 = 30$$

$$E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 = 0.25 - 0.20 = 0.05$$

2. Find $Var(\hat{p}_1 - \hat{p}_2) = \sigma^2_{\hat{p}_1 - \hat{p}_2}$, the variance of $\hat{p}_1 - \hat{p}_2$.

$$Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2} = \frac{0.25 \times 0.75}{35} + \frac{0.2 \times 0.8}{30} = 0.01069$$

3. Find an approximate distribution of $\hat{p}_1 - \hat{p}_2$.

$$\hat{p}_1 - \hat{p}_2 \sim N(0.05, 0.01069)$$

4. Find $P(0.1 < \hat{p}_1 - \hat{p}_2 < 0.2)$

$$P(0.1 < \hat{p}_1 - \hat{p}_2 < 0.2) = \left(\frac{0.1 - 0.05}{\sqrt{0.01069}} < Z < \frac{0.2 - 0.05}{\sqrt{0.01069}}\right)$$

$$= \quad (0.48 < Z < 1.45)$$
$$= P(Z < 1.45) - P(Z < 0.48)$$
$$= 0.92647 - 0.68439 = 0.24208$$

Question 13:
Suppose that 7 % of the pieces from a production process A are defective while that proportion of defective for another production process B is 5 %. A random sample of size 400 pieces is taken from the production process A while the sample size taken from the production process B is 300 pieces. If $\hat{p}_1$ and $\hat{p}_2$ be the proportions of defective pieces in the two samples, respectively, then:

1. The sampling distribution of $(\hat{p}_1 - \hat{p}_2)$ is:

A	N(0,1)	B	Normal	C	T	D	Unknown

2. The value of the standard error of the difference $(\hat{p}_1 - \hat{p}_2)$ is:

A	0.02	B	0.10	C	0	D	0.22

Question 14:
In a study to make an inference between the proportion of houses heated by gas in city A and city B, the following information was collected:

	Proportion of houses heated by gas	Sample size
City A	43%	90
City B	51%	150

Suppose p_A proportion of city A houses which are heated by gas, p_B proportion of city B houses which are heated by gas. The two sample are independent.

1. The sampling distribution for the sample proportion of city B which are heated by gas is:

A	$\hat{p}_B \sim N\left(p_B, \frac{p_B q_B}{n_B}\right)$	B	$\hat{p}_B \sim N\left(\hat{p}_B, \frac{\hat{p}_B \hat{q}_B}{n_B}\right)$	C	$\hat{p}_B \sim N(\hat{p}_B, \hat{p}_B \hat{q}_B)$	D	$\hat{p}_B \sim N(p_B, p_B q_B)$

2. The sampling distribution of $\hat{p}_A$ is (approximately) normal if:

A	$n_A \geq 30$ $n_A p_A > 5$	B	$n_A \geq 30$ $n_A p_A > 5$ $n_A q_A > 5$	C	$n_A p_A > 5$	D	$\frac{p_A}{n_A} > 5$

The student (t) Distribution:

- Student's t-distribution.
- t-distribution is a distribution of a continuous random variables.
- Recall that, if $X_1, X_2, \dots, X_n$ is a random sample of size n from $N(\mu, \sigma^2)$, then:

$$Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

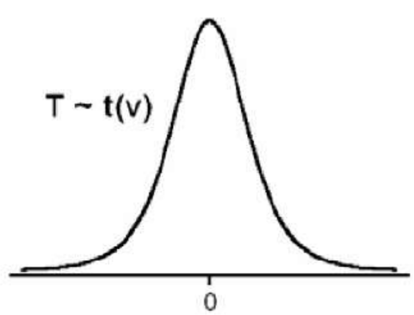
We can apply this result only when σ^2 is known.

- If σ^2 is unknown we replace the population variance σ^2 with the sample variance S^2 to have the following statistic:

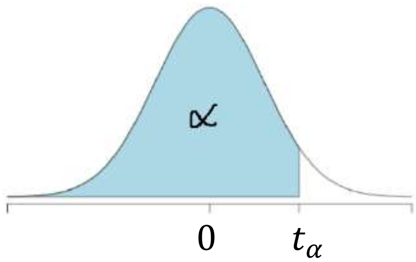
$$T = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$$

Note:

- T-distribution is a continuous distribution.
- The value of t random variable range is $-\infty < t < \infty$
- The mean of t distribution is 0
- Its symmetric about the mean 0
- The shape of t distribution is similar to $N(0,1)$
- When $n \to \infty$ then, t distribution $\to N(0,1)$



The student (t) Distribution:



يجب ان تكون اشارة الاحتمال أقل من (<) قبل البحث في جدول (t) :

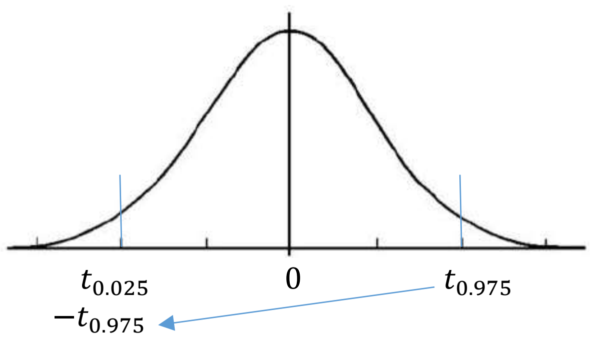
$$t_{\alpha,v} \Rightarrow P(T < t_{\alpha,v})$$
$$\Rightarrow P(T < -t_{1-\alpha,v}) \qquad ; v = n-1$$

- $If\ P(T < t_{0.99,22}) \Rightarrow t_{0.99,22} = 2.508$

- $If\ P(T > t_{0.975,18}) \Rightarrow P(T < t_{0.025,18})$

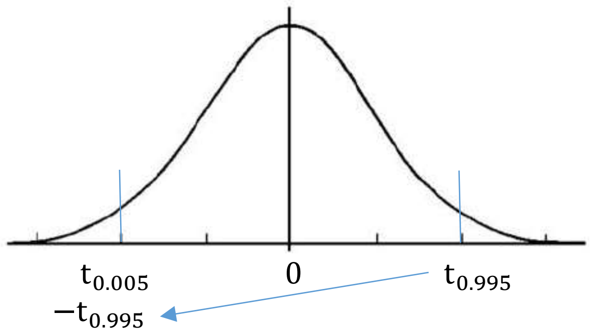
$$\Rightarrow P(T < -t_{0.975,18})$$

$$\Rightarrow -t_{0.975,18} = -2.101$$



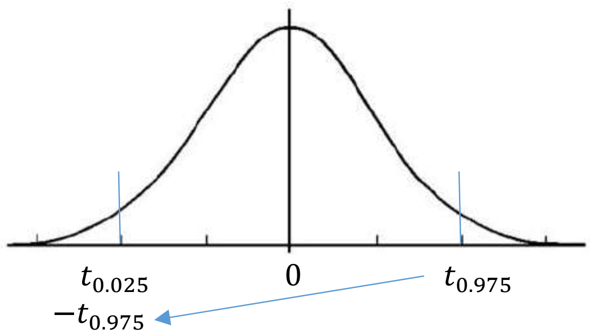
- If $P(T > t_{\alpha,v})$ where $v = 24, \alpha = 0.995$

$$\Longrightarrow P(T > t_{0.995,24})$$
$$\Longrightarrow P(T < t_{0.005,24})$$
$$\Longrightarrow P(T < -t_{0.995,24})$$
$$\Longrightarrow -t_{0.995,24} = -2.797$$



- If $P(T > t_{\alpha,v})$ where $v = 7, \alpha = 0.975$

$$\Longrightarrow P(T > t_{0.975\ 7})$$
$$\Longrightarrow P(T < t_{0.025\ 7})$$
$$\Longrightarrow P(T < -t_{0.975,7})$$
$$\Longrightarrow -t_{0.975,7} = -2.365$$



<u>Question 1:</u>

Let T follow the t distribution with 9 degrees of freedom, then
The probability $(T < 1.833)$ equal to:

بما ان الاشارة اقل من (<) اذن ننظر للجدول

v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055

$\alpha = 0.95$

- $The\ probabitity\ P(T < -1.833)\ equal\ to:$

v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
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10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055

$\alpha = 1 - 0.95 = 0.05$

<u>Question 2:</u>
Let T follow the t distribution with 9 degrees of freedom, then
The t-value that leaves an area of 0.10 to the right is:

$$P(T > t_{0.10,9})$$
$$P(T < t_{0.90,9}) \implies t_{0.90,9} = 1.383$$

ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055

<u>Question 3:</u>
Given the t-distribution with 12 degrees of freedom, then
The t-value that leaves an area of 0.025 to the left is:

$$P(T < t_{0.025,12})$$
$$P(T < -t_{0.975,12})$$

ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055

$$-t_{0.975,12} = -2.179$$

Question 4:

Consider the student t distribution:

Find the t-value with $n = 17$ the leaves an area of 0.01 to the left:

df = n − 1

= 17 − 1 = 16

$$P(T < t_{0.01,16})$$
$$P(T < -t_{0.99,16})$$

v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
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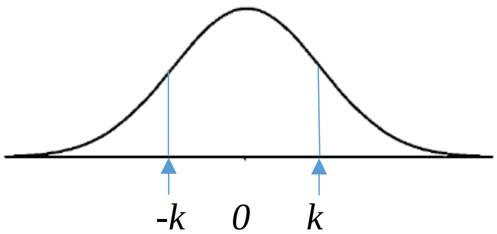
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861

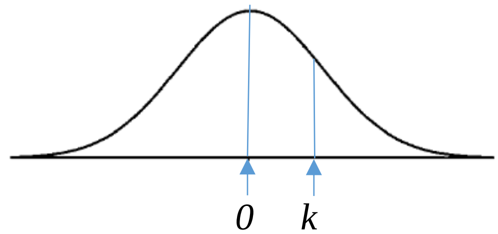
$$t_{0.99,16} = 2.583$$
$$-t_{0.99,16} = -2.583$$

A	−2.58	B	2.567	C	2.58	D	−2.567

<u>Question 5:</u>

Let T follows t-distribution 13 degrees of freedom. If $P(-k < T < k) = 0.9$, then the value of k is:





$$\Longrightarrow 2 \times P(0 < T < k) = 0.9$$

$$\Longrightarrow \quad P(0 < T < k) = 0.45$$

$$\Longrightarrow P(T < k) - P(T < 0) = 0.45$$

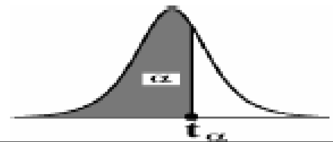
$$\Longrightarrow P(T < k) - \quad 0.5 \quad = 0.45$$

$$\Longrightarrow P(T < k) = 0.95 \qquad \text{df} = 13$$

$$\Longrightarrow k = 1.771$$

ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921

Critical Values of the t-distribution (t_α)



v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
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5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387

- **Question from previous midterms and finals:**

<u>Question:</u>
Given two normally distributed populations with a mean $\mu_1 = 10$ and $\mu_2 = 20$, and variances of $\sigma_1^2 = 100$ and $\sigma_2^2 = 80$. If two samples are taken from the populations of size $n_1 = 25$ and $n_1 = 16$ are taken from the populations.
Let $\overline{X}_1$ and $\overline{X}_2$ be the average of the first and second sample, respectively.

$$n_1 = 25 \, , \mu_1 = 10 \, , \sigma_1^2 = 100$$
$$n_2 = 16 \, , \mu_2 = 20 \, , \sigma_2^2 = 80$$

1. **Find the sampling distribution for $\overline{\mathbf{X}}_1$.**

$$\bar{X}_1 \sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right)$$

$$\bar{X}_1 \sim N\left(10, \frac{100}{25}\right)$$

$$\bar{X}_1 \sim N(10, \ 4)$$

2. **Find the sampling distribution for $(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)$.**

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2 \ , \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$\bar{X}_1 - \bar{X}_2 \sim N\left(10 - 20 \ , \frac{100}{25} + \frac{80}{16}\right)$$

$$\bar{X}_1 - \bar{X}_2 \sim N(-10 \quad , \ 4 \quad + 5)$$

$$\bar{X}_1 - \bar{X}_2 \sim N(-10, 9)$$

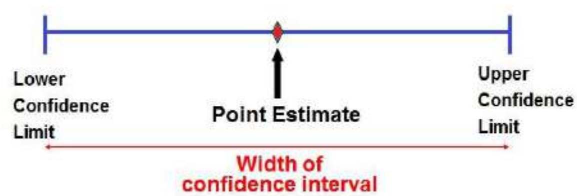
Chapter 6 Estimation and Confidence Interval

Estimation and Confidence Interval

Single Mean	**Two Means**
$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ σ known	$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ σ_1 and σ_2 known
$\bar{X} \pm t_{1-\frac{\alpha}{2},(n-1)} \frac{S}{\sqrt{n}}$ σ unknown	$(\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2},\ (n_1+n_2-2)}\ S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ σ_1 and σ_2 unknown
Single Proportion	**Two Proportions**
For large sample size ($n \geq 30,\ np > 5,\ nq > 5$) $\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$	For large sample size ($n_1 \geq 30,\ n_1p_1 > 5,\ n_1q_1 > 5$) ($n_2 \geq 30,\ n_2p_2 > 5,\ n_2q_2 > 5$) $(\hat{p}_1 - \hat{p}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

$$\bar{X} \pm \underbrace{\left(\overbrace{Z_{1-\frac{\alpha}{2}}}^{\text{Reliability coefficeint}} \frac{\sigma}{\sqrt{n}} \right)}_{\substack{\text{margin of error} \\ or \\ \text{precision of the estimate}}}$$



	99% confidence interval
	95% confidence interval
	90% confidence interval

*** Statistical Inferences:** (Estimation and Hypotheses Testing) It is the procedure by which we reach a conclusion about a population on the basis of the information contained in a sample drawn from that population.

Estimation:

Approximating (or estimating) the actual values of the unknown parameters:

- **Point Estimate:** A point estimate is single value used to estimate the corresponding population parameter.

- **Interval Estimate (Confidence Interval):** An interval estimate consists of two numerical values defining a range of values that most likely includes the parameter being estimated with a specified degree of confidence.

Hypothesis Testing:

Answering research questions about the unknown parameters of the population (confirming or denying (نقبل أو نرفض) some conjectures or statements about the unknown parameters).

Confident interval for population proportion ($\boldsymbol{p}$)

Result: for large sample size ($n \geq 30,\ np > 5\ , nq > 5$), an approximate $(1-\alpha)100\%$ confidence interval for (p) is:

$$\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

Estimator ± (Reliability Coefficient) × (Standard Error)

Confident interval for ($\hat{\boldsymbol{p}}_1 - \hat{\boldsymbol{p}}_2$)

Result: for large samples sizes $\begin{pmatrix} n_1 \geq 30 & n_2 \geq 30 \\ n_1p_1 > 5 & n_2q_2 > 5 \\ n_1q_1 > 5 & n_2q_2 > 5 \end{pmatrix}$ an approximate $(1-\alpha)100\%$ confidence interval for $(\hat{p}_1 - \hat{p}_2)$ is:

$$(\hat{p}_1 - \hat{p}_2)\ \pm \left(Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

Estimator ± (Reliability Coefficient) × (Standard Error)

Question 1:

Suppose we are interested in making some statistical inference about the mean μ, of a normal population with standard deviation $\sigma = 2$. Suppose that a random sample of size $n = 49$ from this population gave a sample mean $\bar{X} = 4.5$.

a. Find the upper limit of 95% of the confident interval for μ

$$\sigma = 2 \quad \bar{X} = 4.5 \quad n = 49$$

$$95\% \rightarrow \alpha = 0.05 \qquad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 4.5 + \left(1.96 \times \frac{2}{7}\right) = 5.06$$

b. Find the lower limit of 95% of the confident interval for μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 4.5 - \left(1.96 \times \frac{2}{7}\right) = 3.94$$

Question 2:

A researcher wants to estimate the mean of a life span a certain bulb. Suppose that the distribution is normal with standard deviation 5 hours. Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.

$$\sigma = 5 \;,\; \bar{X} = 390 \;,\; n = 49$$

a. find $Z_{0.975}$:

$$Z_{0.975} = 1.96$$

b. find a point estimate for μ

$$E(\bar{X}) = \hat{\mu} = \bar{X} = 390$$

c. Find the upper limit of 95% of the confident interval for μ

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 390 + \left(1.96 \times \frac{5}{\sqrt{49}}\right) = 391.4$$

d. Find the lower limit of 95% of the confident interval for μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 390 - \left(1.96 \times \frac{5}{\sqrt{49}}\right) = 388.6$$

Question 3:

A sample of 16 college students were asked about time they spent doing their homework. It was found that the average to be 4.5 hours. Assuming normal population with standard deviation 0.5 hours.

$\sigma = 0.5 \quad \bar{X} = 4.5 \quad n = 16$

1. The point estimate for μ is:

A	0 hours	B	10 hours	C	0.5 hours	D	4.5 hours

2. The standard error of $\bar{X}$ is:

$$S.E(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{16}}$$

A	0.125 hours	B	0.266 hours	C	0.206 hours	D	0.245hours

3. The correct formula for calculating $100\,(1-\alpha)\%$ confidence interval for μ is:

A	$\bar{X} \pm t_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	B	$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	C	$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$	D	$\bar{X} \pm t_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$

4. The upper limit of 95% confidence interval for μ is:

A	4.745	B	4.531	C	4.832	D	4.891

5. The lower limit of 95% confidence interval for μ is:

A	5.531	B	7.469	C	3.632	D	4.255

6. The length of the 95% confidence interval for μ is:

$$\text{Length} = 4.745 - 4.255 = 0.49$$

A	4.74	B	0.49	C	0.83	D	0.89

<u>Question 4:</u>
Let us consider a hypothetical study on the height of women in their adulthood. A sample of 24 women is drown from a normal distribution with population mean μ and variance σ^2. The sample mean and variance of height of the selected women are 151 cm and 18.65 cm^2 respectively. Using given data, we want to constract a 99% confidentce interval for the mean height of the adult women in the populatopn from which the sample was drawn randomly.

$$\overline{X} = 151 \quad ; \quad n = 24 \quad ; \quad S^2 = 18.65 \Longrightarrow S = 4.32$$

a. Point estimate for μ

$$\hat{\mu} = \overline{X} = 151$$

b. Find the upper limit of 99% of the confident interval for μ

$$\overline{X} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S}{\sqrt{n}}\right)$$

$$= 151 + \left(2.807 \times \frac{4.32}{\sqrt{24}}\right) = 153.4753$$

$$99\% \rightarrow \alpha = 0.01$$

$$t_{1-\frac{\alpha}{2},n-1} = t_{1-\frac{0.01}{2},24-1}$$

$$= t_{0.995,23} = 2.807$$

c. Find the lower limit of 99% of the confident interval for μ

$$\overline{X} - \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S}{\sqrt{n}}\right)$$

$$= 151 - \left(2.807 \times \frac{4.32}{\sqrt{24}}\right) = 148.5247$$

Estimation and Confidence Interval: Two Means

1- $(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$

2- $(\bar{X}_1 - \bar{X}_2) \pm \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2}\, Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Question 5:

The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kg. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kg. Another type of thread (type II) is approximately followed normal distribution with standard deviation 6.8 kg. A sample of 25 pieces of the thread has an average tensile strength pf 64.4 kg. then for 98% confidence interval of the difference in tensile strength means between type I and type II, we have:

$$\text{Theard 1}: \ n_1 = 20, \bar{X}_1 = 72.8, \sigma_1 = 6.8$$
$$\text{Thread 2}: \ n_2 = 25, \bar{X}_2 = 64.4, \sigma_2 = 6.8$$

$$98\% \rightarrow \alpha = 0.02 \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.325$$

$$(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$
$$(72.8 - 64.4) \pm \left(2.325 \times \sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}} \right)$$

$$(3.657, 13.143)$$

(1): The lower limit $= 3.657$
(2): The upper limit $= 13.143$

Question 6:

	First sample	Second sample
Sample size (n)	12	14
Sample mean ($\overline{X}$)	10.5	10
Sample variance (S^2)	4	5

1. Estimate the difference $\mu_1 - \mu_2$:

$$\hat{\mu}_1 - \hat{\mu}_2 = \overline{X}_1 - \overline{X}_2 = 10.5 - 10 = 0.5$$

2. Find the pooled standard deviation estimator Sp :

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2} = \frac{4(11) + 5(13)}{24} = 4.54 \Longrightarrow \boxed{Sp = 2.13}$$

3. The upper limit of 95% confidence interval for $(\mu_1 - \mu_2)$ is:

$$95\% \rightarrow \alpha = 0.05 \rightarrow t_{1-\frac{\alpha}{2}, n_1+n_2-2} = t_{0.975, 24} = 2.064,$$

$$(\overline{X}_1 - \overline{X}_2) + \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$(0.5) + \left(2.064 \times 2.13\sqrt{\frac{1}{12} + \frac{1}{14}}\right) = 2.23$$

4. The lower limit of 95% confidence interval for $(\mu_1 - \mu_2)$ is:

$$(\overline{X}_1 - \overline{X}_2) - \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$(0.5) - \left(2.064 \times 2.13\sqrt{\frac{1}{12} + \frac{1}{14}}\right) = -1.23$$

Question 7:

A researcher was interested in comparing the mean score of female students μ_1, with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	Male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\bar{X}_1 = 82.63$	$\bar{X}_2 = 80.04$
Variance	$S_1^2 = 15.05$	$S_2^2 = 20.79$

1. The point estimate of $\mu_1 - \mu_2$ is:

A	2.63	B	-2.37	C	2.59	D	0.59

2. The estimate of the pooled variance S_p^2 is:

A	17.994	B	18.494	C	17.794	D	18.094

3. The upper limit of the 95% confidence interval for $\mu_1 - \mu_2$ is :

A	26.717	B	7.525	C	7.153	D	8.2

4. The lower limit of the 95% confidence interval for $\mu_1 - \mu_2$ is :

A	-21.54	B	-2.345	C	-3.02	D	-1.973

<u>Estimation and Confidence Interval: Single Proportion</u>

For large sample size ($n \geq 30,\ np > 5,\ nq > 5$)

$$* \text{ Point estimate for } P \text{ is: } \frac{x}{n}$$

$$* \text{ Interval estimate for } P \text{ is: } \hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

Question 7:

A random sample of 200 students from a certain school showed that 15 student smoke. Let p be the proportion of smokers in the school.

1. Find a point estimate for p.

$$n = 200 \quad \& \quad x = 15$$

$$\hat{p} = \frac{x}{n} = \frac{15}{200} = 0.075 \rightarrow \hat{q} = 0.925$$

2. Find 95% confidence interval for p.

$$95\% \rightarrow \alpha = 0.05 \quad \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}}\right) = 0.075 \pm \left(1.96 \times \sqrt{\frac{0.075 \times 0.925}{200}}\right)$$

The 95% confidence interval is: $(0.038,\ 0.112)$

Question 8:

A researcher's group has perfected a new treatment of a disease which they claim is very efficient. As evidence, they say that they have used the new treatment on 50 patients with the disease and cured 25 of them. To calculate a 95% confidence interval for the proportion of the cured.

1. The point estimate of p is equal to:

A	0.25	B	0.50	C	0.01	D	0.33

2. The reliability coefficient $\left(z_{1-\frac{\alpha}{2}}\right)$ is equal is:

A	1.96	B	1.645	C	2.02	D	1.35

3. The 95% confidence interval is equal to:

A	(0.1114,0.3886)	B	(0.3837,0.6163)	C	(0.1614,0.6386)	D	(0.3614,0.6386)

Estimation and Confidence Interval: Two Proportions

For large sample size $(n_1 \geq 30,\ n_1p_1 > 5,\ n_1q_1 > 5)$
$(n_2 \geq 30,\ n_2p_2 > 5,\ n_2q_2 > 5)$

$$* \text{ Point estimate for } P_1 - P_2 = \hat{p}_1 - \hat{p}_2 = \frac{x_1}{n_1} - \frac{x_2}{n_2}$$

$$* \text{ Interval estimate for } P_1 - P_2 \text{ is: } (\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

Question 9:

A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smoker in school "A" and let p_2 be the proportion of smoker in school "B".

1. Find a point estimate for $P_1 - P_2$.

$$n_1 = 100\ , x_1 = 15 \ \rightarrow \ \hat{p}_1 = \frac{15}{100} = \boxed{0.15} \ \Longrightarrow \hat{q}_1 = 1 - 0.15 = \boxed{0.85}$$

$$n_2 = 200\ , x_2 = 20 \ \rightarrow \ \hat{p}_2 = \frac{20}{200} = \boxed{0.10} \ \Longrightarrow \hat{q}_2 = 1 - 0.10 = \boxed{0.90}$$

$$\boxed{\hat{p}_1 - \hat{p}_2 = 0.15 - 0.1 = 0.05}$$

2. Find 95% confidence interval for $P_1 - P_2$.

$$95\% \rightarrow \alpha = 0.05 \ \rightarrow \ Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$(\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.05) \quad \pm \left(1.96 \times \sqrt{\frac{(0.15)(0.85)}{100} + \frac{(0.1)(0.9)}{200}}\right)$$

$$= 0.05 \pm \left(1.96 \times \sqrt{0.001725}\right)$$

The 95% confidence interval is: $(-0.031,\ 0.131)$

Question 10:

a first sample of 100 store customers, 43 used a MasterCard. In a second sample of 100 store customers, 58 used a Visa card. To find the 95% confidence interval for difference in the proportion $(P_1 - P_2)$ of people who use each type of credit card?

1. The value of α is:

A	0.95	B	0.50	C	0.05	D	0.025

2. The upper limit of 95% confidence interval for the proportion difference is:

$$n_1 = 100\ ,\ x_1 = 43 \rightarrow \hat{p}_1 = \frac{43}{100} = \boxed{0.43} \Longrightarrow \hat{q}_1 = 1 - 0.43 = \boxed{0.57}$$

$$n_2 = 100\ ,\ x_2 = 58 \rightarrow \hat{p}_2 = \frac{58}{100} = \boxed{0.58} \Longrightarrow \hat{q}_2 = 1 - 0.58 = \boxed{0.42}$$

$$(\hat{p}_1 - \hat{p}_2) + \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.43 - 0.58) + \left(1.96 \times \sqrt{\frac{(0.43)(0.57)}{100} + \frac{(0.58)(0.42)}{100}}\right) = -0.013$$

3. The lower limit of 95% confidence interval for the proportion difference is:

$$(\hat{p}_1 - \hat{p}_2) - \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.43 - 0.58) - \left(1.96 \times \sqrt{\frac{(0.43)(0.57)}{100} + \frac{(0.58)(0.42)}{100}}\right) = -0.287$$

Question from previous midterms and finals:

- In procedure of construction $(1-\alpha)100\%$ confidence interval for the population mean (μ) of a normal population with a known standard deviation (σ) based on a random sample of size n.

1. The width of $(1-\alpha)100\%$ confidence interval for (μ) is:

A	$2\,Z_{1-\alpha}\frac{\sigma^2}{n}$	B	$2\,Z_{1-\alpha}\frac{\sigma}{\sqrt{n}}$	C	$2\,Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$	D	$2\,Z_{1-\alpha}\frac{\sigma^2}{\sqrt{n}}$

2. For $n = 70$ and $\sigma = 4$ the width of a 95% confidence interval for (μ) is:

The width $= 2\,Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
$= 2\,(1.96)\frac{4}{\sqrt{70}}$
$= 1.874118$

$95\% \longrightarrow \alpha = 0.05$
$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}}$
$= Z_{0.975} = 1.96$

A	3.1458	B	1.5153	C	6.1601	D	1.8741

3. The most typical form of a calculated confidence interval is:

A	Point estimate $\pm$ standard error
B	Population parameter $\pm$ margin of error
C	Population parameter $\pm$ standard error
D	Point estimate $\pm$ margin of error

4. For $\overline{X} = 60$ and a 95% confidence interval for μ is $(57\,,\ k)$, then the value of the upper confidence limit k is:

(Point estimate − magin of error , Point estimate + magin of error)
(57 , k)
(60 − magin of error , 60 + magin of error)

60 − magin of error = 57
magin of error = 3

k = 60 + magin of error
k = 60 + 3 = 63

A	64.5	B	66	C	61.5	D	63

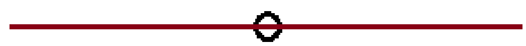
5. The following C.I. is obtained for a population proportion $(0.505, 0.545)$, then the margin of error equals (let $\hat{p} = 0.525$)

$$(\text{Point estimate} - \text{magin of error} \quad , \quad \text{Point estimate} + \text{magin of error})$$
$$(0.505 \, , \quad 0.545)$$
$$(0.525 - \text{magin of error} \quad , \quad 0.525 + \text{magin of error})$$

$$0.525 - \text{magin of error} = 0.505$$
$$\text{magin of error} = 0.02$$
or
$$0.525 + \text{magin of error} = 0.545$$
$$\text{magin of error} = 0.02$$

A	0.01	B	0.04	C	0.03	D	0.02

6. When comparing the width of the 95% confidence interval (C.I.) for μ with that of 90% C.I., we found that:

	99% confidence interval
	95% confidence interval
	90% confidence interval

	95% C.I. is shorter	B	95% C.I. is wider	C	They have the same width	D	We can't decide

7. When the sample size n increase, the width of the C.I. will:

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$ ← كل مازاد حجم العينة تصغر الفتره

A	Decrease	B	Increase	C	Not be change	D	We can't decide

8. Confidence intervals are useful when trying to estimate .... parameter:

A	Sample	B	Statistics	C	Population	D	None of these

Chapter 7 Hypotheses Testing

Hypothesis Testing

Answering research questions about the unknown parameters of the population (confirming or denying (نقبل أو نرفض) some conjectures or statements about the unknown parameters).

- A hypothesis is a statement about one or more populations.
- A research hypothesis is the conjecture or supposition that motivates the research.
- A statistical hypothesis is a conjecture (or a statement) concerning the population which can be evaluated by appropriate statistical technique.
- For example, if θ is an unknown parameter of the population, we might be interested in testing the conjecture sating (رضى) that $\theta \geq \theta_0$ against $\theta < \theta_0$.
- We usually test the null hypothesis (H_0) against the alternative hypothesis (the research hypothesis) (H_1 or H_A) by choosing one of the following situations:

  i. $H_0: \theta = \theta_0 \quad vs \quad H_A: \theta \neq \theta_0$
  ii. $H_0: \theta \geq \theta_0 \quad vs \quad H_A: \theta < \theta_0$
  iii. $H_0: \theta \leq \theta_0 \quad vs \quad H_A: \theta > \theta_0$

- Equality sign must appear in the null hypothesis.
- H_0 and H_A are complement of each other.
- The null hypothesis (H_0) is also called "the hypothesis of no difference".
- The alternative hypothesis (H_A) is also called the research hypothesis.

Determining a test statistic (T.S.):
We choose the appropriate test statistic based on the point estimator of the parameter. The test statistic has the following form:

$$\text{Test statistic} = \frac{\text{Estimate} - \text{hypothesized parameter}}{\text{Standard error of the estimate}}$$

Hypotheses Testing

Test statistic (TS)			
Single mean	$Z=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$ σ known	Single proportion	$Z=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$
	$T=\frac{\bar{X}-\mu_0}{S/\sqrt{n}}$ σ unknown		
Two means	$Z=\frac{(\bar{X}_1-\bar{X}_2)-d}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$ σ_1 and σ_2 known	Two proportions	$Z=\frac{(\hat{p}_1-\hat{p}_2)-d}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$
	$T=\frac{(\bar{X}_1-\bar{X}_2)-d}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ σ_1 and σ_2 unknown		

The pooled variance $S_p^2=\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}$ The pooled proportion $\bar{p}=\frac{x_1+x_2}{n_1+n_2}$

The test statistic has the following form:

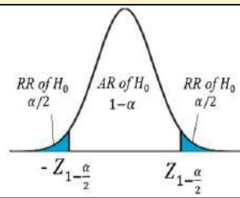
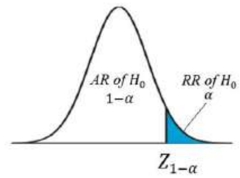
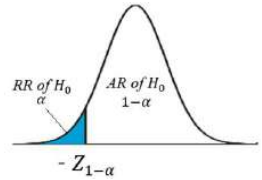
$$\text{Test statistic} = \frac{\text{Estimate} - \text{hypothesized parameter}}{\text{Standard error of the estimate}}$$

	H_0 is true	H_0 is false
Accepting H_0	Correct decision	Type II error (β)
Rejecting H_0	Type I error (α)	Correct decision

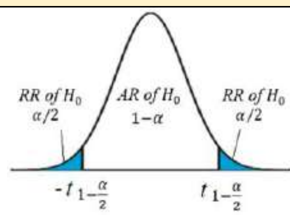
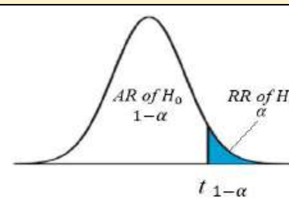
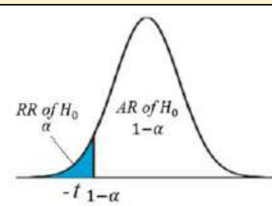
Type I error = Rejecting H_0 when H_0 is true $P(\text{Type I error}) = P(\text{Rejecting } H_0 \mid H_0 \text{ is true}) = \alpha$	Type II error = Accepting H_0 when H_0 is false $P(\text{Type II error}) = P(\text{Accepting } H_0 \mid H_0 \text{ is false}) = \beta$

<u>1-Single Mean</u>

(*if* σ *known*):

Hypothesis *Null* H_0 *Alternative (Research)* H_A	$H_0: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_0: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_0: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
Test Statistics (TS)	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of* H_0			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

(*if* σ *unknown*):

Hypothesis *Null* H_0 *Alternative (Research)* H_A	$H_0: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_0: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_0: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
Test Statistics (TS)	$t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of* H_0			
Decision	We reject H_0 at the significance level α if		
	$t < -t_{1-(\alpha/2)}$ *or* $t > t_{1-(\alpha/2)}$ *Two sides test*	$t > t_{1-\alpha}$ *One side test*	$t < -t_{1-\alpha}$ *One side test*

Question 1:

Suppose that we are interested in estimating the true average time in seconds it takes an adult to open a new type of tamper-resistant aspirin bottle. It is known that the population standard deviation is $\sigma = 5.71$seconds. A random sample of 40 adults gave a mean of 20.6 seconds. Let μ be the population mean, then, to test if the meanμ is 21 seconds at level of significant 0.05 ($H_0: \mu = 21$ vs $H_A: \mu \neq 21$) then:

(1) The value of the test statistic is:

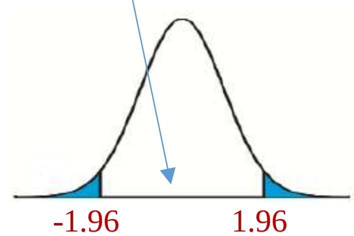
$$\sigma = 5.71 \quad n = 40 \quad \bar{X} = 20.6$$

$$Z = \frac{\bar{X} - \mu_o}{\sigma/\sqrt{n}} = \frac{20.6 - 21}{5.71/\sqrt{40}} = -0.443$$

A	0.443	B	-0.012	C	-0.443	D	0.012

(2) The acceptance area is:

$$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.96$$



A	$(-1.96, 1.96)$	B	$(1.96, \infty)$	C	$(-\infty, 1.96)$	D	$(-\infty, 1.645)$

(3) The decision is:

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

$$P-value = 2 \times P(Z < -0.443) \quad = 2 \times 0.32997 \quad = 0.66 > 0.05$$

or

$$P-value = 2 \times P(Z > |-0.443|) = 2 \times P(Z > 0.443) = 0.66 > 0.05$$

Question 2:

If the hemoglobin level of pregnant women (امرأه حامل) is normally distributed, and if the mean and standard deviation of a sample of 25 pregnant women were $\overline{X} = 13$ (g/dl), $s = 2$ (g/dl). Using $\alpha = 0.05$, to test if the average hemoglobin level for the pregnant women is greater than 10 (g/dl) [H_0: $\mu \leq 10$, H_A: $\mu > 10$].

$$\boxed{s = 2 \ , \ n = 25 \ , \ \bar{X} = 13}$$

1. The test statistic is:

A	$Z = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$	B	$Z = \frac{\bar{X}-10}{S/\sqrt{n}}$	C	$t = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$	D	$t = \frac{\bar{X}-10}{S/\sqrt{n}}$

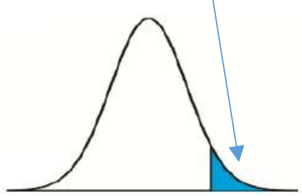
2. The value of the test statistic is:

$$t = \frac{\bar{X} - \mu_o}{S/\sqrt{n}} = \frac{13 - 10}{2/\sqrt{25}} = \boxed{7.5}$$

A	10	B	1.5	C	7.5	D	37.5

3. The rejection of H_0 is:

$$t_{1-\alpha,n-1} = t_{0.95,24} = 1.711$$



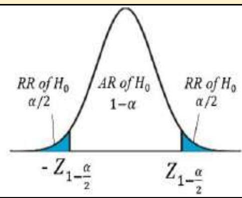
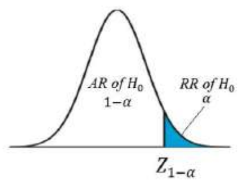
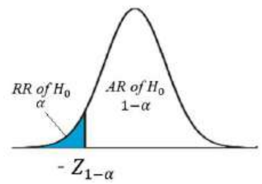
A	$Z < -1.645$	B	$Z > 1.645$	C	$t < -1.711$	D	$t > 1.711$

4. The decision is:

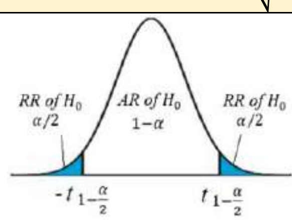
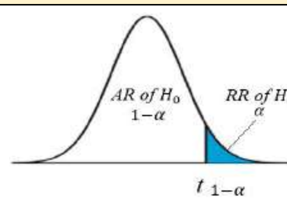
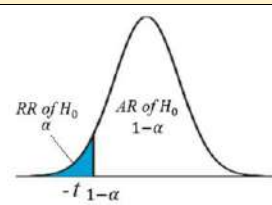
A	Reject H_0
B	Do not reject (Accept) H_0.
C	Accept both H_0 and H_A.
D	Reject both H_0 and H_A.

2-Two Means:

(if σ_1 and σ_2 known):

Hypothesis *Null H_0* *Alternative (Research) H_A*	$H_0: \mu_1 - \mu_2 = d$ $H_A: \mu_1 - \mu_2 \neq d$	$H_0: \mu_1 - \mu_2 \leq d$ $H_A: \mu_1 - \mu_2 > d$	$H_0: \mu_1 - \mu_2 \geq d$ $H_A: \mu_1 - \mu_2 < d$
Test Statistics (TS)	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of H_0*			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

(if σ_1 and σ_2 unknown):

Hypothesis *Null H_0* *Alternative (Research) H_A*	$H_0: \mu_1 - \mu_2 = d$ $H_A: \mu_1 - \mu_2 \neq d$	$H_0: \mu_1 - \mu_2 \leq d$ $H_A: \mu_1 - \mu_2 > d$	$H_0: \mu_1 - \mu_2 \geq d$ $H_A: \mu_1 - \mu_2 < d$
Test Statistics (TS)	$t = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of H_0*			
Decision	We reject H_0 at the significance level α if		
	$t < -t_{1-(\alpha/2)}$ *or* $t > t_{1-(\alpha/2)}$ *Two sides test*	$t > t_{1-\alpha}$ *One side test*	$t < -t_{1-\alpha}$ *One side test*

The pooled variance $S_p^2 = \frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1 + n_2 - 2}$

Question 3:

A standardized chemistry test was given to 50 girls and 75 boys. The girls made an average of 84, while the boys made an average grade of 82. Assume the population standard deviations are 6 and 8 for girls and boys respectively. To test the null hypothesis

$$H_0: \mu_1 - \mu_2 \leq 0 \quad vs \; H_A: \mu_1 - \mu_2 > 0 \quad use \; \alpha = 0.05$$

(1) The standard error of $(\overline{X}_1 - \overline{X}_2)$ is:

$$girls: \quad n_1 = 50, \; \bar{X}_1 = 84, \; \sigma_1 = 6$$
$$boys: \quad n_2 = 75, \; \bar{X}_2 = 82, \; \sigma_2 = 8$$

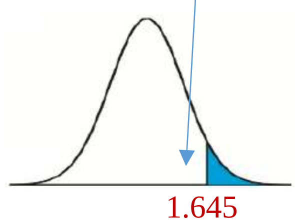
$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{6^2}{50} + \frac{8^2}{75}} = 1.2543$$

(2) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(84-82)}{\sqrt{\frac{6^2}{50} + \frac{8^2}{75}}} = \frac{2}{1.2543} = \boxed{1.5945}$$

(3) The rejection region (RR) of H_0 is:

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$



A	(1.645, ∞)	B	(−∞, −1.645)	C	(1.96, ∞)	D	(−∞, −1.96)

(4) The decision is:

A	Reject H_0
B	Do not reject (Accept) H_0.
C	Accept both H_0 and H_A.
D	Reject both H_0 and H_A.

$$P - value = P(Z > 1.59) = 1 - P(Z < 1.59) = 0.056 > 0.05$$

Question 4:

Cortisol level determinations were made on two samples of women at childbirth. Group 1 subjects underwent emergency cesarean section (عملية قيصرية) following induced labor. Group 2 subjects natural childbirth route following spontaneous labor (الولادة الطبيعية). The random sample sizes, mean cortisol levels, and standard deviations were $(n_1 = 40, \bar{x}_1 = 575, \sigma_1 = 70)$, $(n_2 = 44, \bar{x}_2 = 610, \sigma_2 = 80)$.

If we are interested to test if the mean Cortisol level of group 1 (μ_1) is less than that of group 2 (μ_2) at level 0.05 (or $H_0: \mu_1 \geq \mu_2$ vs $H_1: \mu_1 < \mu_2$), then:

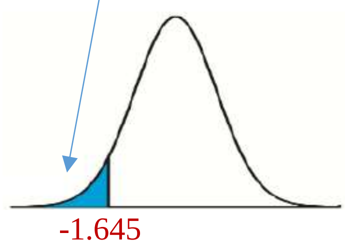
(1) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(575-610)}{\sqrt{\frac{70^2}{40} + \frac{80^2}{44}}} = \boxed{-2.138}$$

A	-1.326	B	-2.138	C	-2.576	D	-1.432

(2) Reject H_0 if :

$Z_{1-\alpha} = Z_{0.95} = 1.645$



A	$Z > 1.645$	B	$T > 1.98$	C	$Z < -1.645$	D	$T < -1.98$

(3) The decision is:

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

$P-value = P(Z < -2.138) = 0.01618 < 0.05$

Question 5:

An experiment was conducted to compare time length (duration time in minutes) of two types of surgeries (A) and (B). 10 surgeries of type (A) and 8 surgeries of type (B) were performed. The data for both samples is shown below.

Surgery type	A	B
Sample size	10	8
Sample mean	14.2	12.8
Sample standard deviation	1.6	2.5

Assume that the two random samples were independently selected from two normal populations with equal variances. If μ_A and μ_B are the population means of the time length of surgeries of type (A) and type (B), then, to test if μ_A is greater than μ_B at level of significant 0.05 ($H_0: \mu_A \leq \mu_B$ vs $H_A: \mu_A > \mu_B$) then:

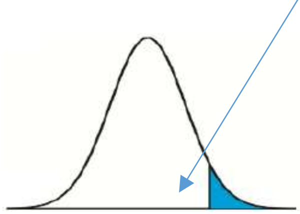
1. The value of the test statistic is:

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{1.6^2(10-1)+2.5^2(8-1)}{10+8-2} = 4.174$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(14.2-12.8)}{\sqrt{4.174}\sqrt{\frac{1}{10}+\frac{1}{8}}} = \boxed{1.44}$$

2. Reject H_0 if:

$$t_{1-\alpha,n_1+n_2-2} = t_{0.95,10+8-2} = t_{0.95,16} = 1.746$$



1.746

A	$Z > 1.645$	B	$Z < -1.645$	C	$T > 1.746$	D	$T < -1.746$

3. The decision is:

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

Question 6:

A researcher was interested in comparing the mean score of female students μ_1 ,with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	Male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\overline{X}_1 = 82.63$	$\overline{X}_2 = 80.04$
Variance	$S_1^2 = 15.05$	$S_2^2 = 20.79$

Test that is there is a difference between the mean score of female students and the mean score of male students.

1. The hypotheses are:

A	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \neq \mu_2$	B	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 < \mu_2$	C	$H_0: \mu_1 < \mu_2$ $H_A: \mu_1 > \mu_2$	D	$H_0: \mu_1 \leq \mu_2$ $H_A: \mu_1 > \mu_2$

2. The value of the test statistic is:

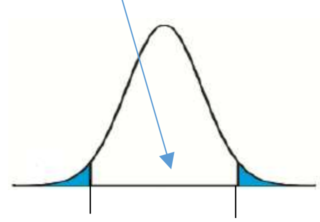
$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{15.05(4)+20.79(6)}{5+7-2} = 18.494$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{82.63-80.04}{\sqrt{18.494}\sqrt{\frac{1}{5}+\frac{1}{7}}} = \boxed{1.029}$$

A	1.3	B	1.029	C	0.46	D	0.93

3. The acceptance region (AR) of H_0 is:

$$t_{1-\frac{\alpha}{2},n_1+n_2-2}$$
$$= t_{1-\frac{0.05}{2},5+7-2}$$
$$= t_{0.975,10}$$
$$= 2.228$$



A	$(2.228, \infty)$	B	$(-\infty, -2.228)$	C	(-2.228, 2.228)	D	(-1.96, 1.96)

Question 7:

A nurse researcher wished to know if graduates of baccalaureate (بكالوريس) nursing program and graduate of associate degree (الزمالة) nursing program differ with respect to mean scores on personality inventory at α = 0.02. A sample of 50 associate degree graduates (sample A) and a sample of 60 baccalaureate graduates (sample B) yielded the following means and standard deviations:

$$\bar{X}_A = 88.12,\ \ S_A = 10.5,\ \ n_A = 50$$
$$\bar{X}_B = 83.25,\ \ S_B = 11.2,\ \ n_B = 60$$

1) The hypothesis is:

A	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \neq \mu_2$	B	$H_0: \mu_1 \leq \mu_2$ $H_A: \mu_1 > \mu_2$	C	$H_0: \mu_1 \geq \mu_2$ $H_A: \mu_1 < \mu_2$	D	None of these

2) The test statistic is:

A	Z	B	t	C	F	D	None of these

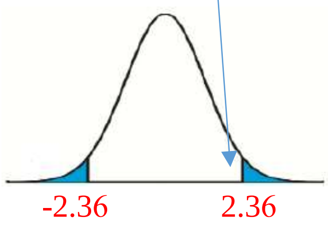
3) The computed value of the test statistic is:

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{10.5^2(50-1)+11.2^2(60-1)}{50+60-2} = 118.55$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{88.12-83.25}{\sqrt{118.55}\sqrt{\frac{1}{50}+\frac{1}{60}}} = \boxed{2.34}$$

4) The critical region (rejection area) is:

$$t_{1-\frac{\alpha}{2}, n_1+n_2-2}$$
$$= t_{1-\frac{0.02}{2},\ 50+60-2}$$
$$= t_{0.99,108} = 2.36$$

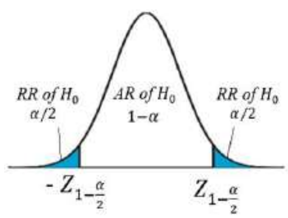
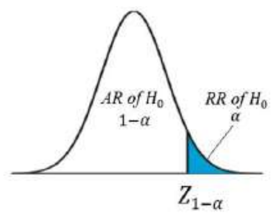
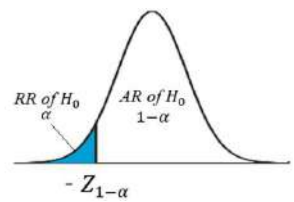


A	2.60 or − 2.60	B	2.06 or − 2.06	C	2.36 or − 2.36	D	2.58

5) Your decision is:

A	Reject H_0	B	Accept H_0	C	Accept H_A	D	No decision

3- Single proportion:

Hypothesis *Null H_0* *Alternative (Research) H_A*	$H_0: p = p_o$ $H_A: p \neq p_o$	$H_0: p \leq p_o$ $H_A: p > p_o$	$H_0: p \geq p_o$ $H_A: p < p_o$
Test Statistics (TS)	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim N(0,1)$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of H_0*			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

Question 8:

Toothpaste (معجون الأسنان) company claims that more than 75% of the dentists recommend their product to the patients. Suppose that 161 out of 200 dental patients reported receiving a recommendation for this toothpaste from their dentist. Do you suspect that the proportion is actually morethan75%. If we use 0.05 level of significance to test $H_0: P \leq 0.75$, $H_A: P > 0.75$, then:

(1) The sample proportion $\hat{p}$ is:

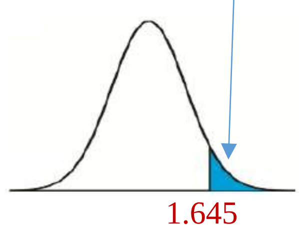
$$n = 200, \ \hat{p} = \frac{161}{200} = 0.8050$$

(2) The value of the test statistic is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.805 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{200}}} = \boxed{1.7963}$$

(3) The decision is:

$$\alpha = 0.05$$
$$Z_{1-\alpha} = Z_{0.95} = 1.645$$



A	Reject H_0 (agree with the claim)
B	Do not reject (Accept) H_0
C	Accept both H_0 and H_A
D	Reject both H_0 and H_A

$$P - value = P(Z > 1.7963) = 1 - P(Z < 1.7963) = 1 - 0.96407 = 0.03593 < 0.05$$

Question 9:

A researcher was interested in studying the obesity (السمنة) disease in a certain population. A random sample of 400 people was taken from this population. It was found that 152 people in this sample have the obesity disease. If p is the population proportion of people who are obese. Then, to test if p is greater than 0.34 at level 0.05 ($H_0: p \leq 0.34$ vs $H_A: p > 0.34$) then:

(1) The value of the test statistic is:

$$n = 400, \quad \hat{p} = \frac{152}{400} = 0.38$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.38 - 0.34}{\sqrt{\frac{0.34 \times 0.66}{400}}} = \boxed{1.69}$$

A	0.023	B	1.96	C	2.50	D	1.69

(2) The P-value is

$$P - value = P(Z > 1.69) = 1 - P(Z < 1.69) = 1 - 0.9545 = 0.0455$$

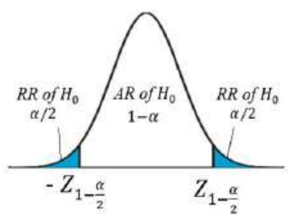
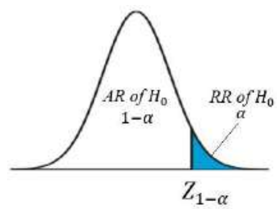
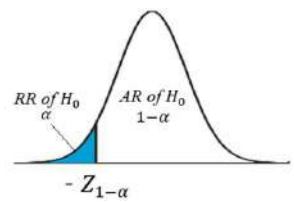
A	0.9545	B	0.0910	C	0.0455	D	1.909

(3) The decision is:

$$P - value = 0.0455 < 0.05$$

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

4-Two proportions:

Hypothesis *Null H_0* *Alternative (Research) H_A*	$H_0: p_1 - p_2 = d$ $H_A: p_1 - p_2 \neq d$	$H_0: p_1 - p_2 \leq d$ $H_A: p_1 - p_2 > d$	$H_0: p_1 - p_2 \geq d$ $H_A: p_1 - p_2 < d$
Test Statistics (TS)	$Z = \frac{(\hat{p}_1 - \hat{p}_2) - d}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{(\hat{p}_1 - \hat{p}_2) - d}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of H_0*			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

The pooled proportion (The proportion for both samples)

Question 10:

In a first sample of 200 men, 130 said they used seat belts and a second sample of 300 women, 150 said they used seat belts. To test the claim that men are more safety-conscious than women ($H_0: p_1 - p_2 \leq 0, H_1: p_1 - p_2 > 0$), at 0.05 level of significant:

(1) The value of the test statistic is:

Men: $n_1 = 200 \quad x_1 = 130$ → $\hat{p}_1 = \frac{x_1}{n_1} = \frac{130}{200} = 0.65$

Women: $n_2 = 300 \quad x_2 = 150$ → $\hat{p}_2 = \frac{x_2}{n_2} = \frac{150}{300} = 0.5$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{130+150}{200+300} = \frac{280}{500} = 0.56$$

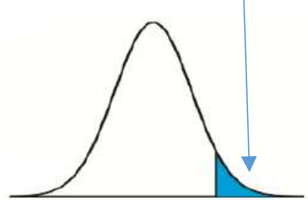
Who used seat belt for both samples (male and female) (280)

Sum of all samples (male and female) (500)

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.65 - 0.5)}{\sqrt{(0.56)(0.44)\left(\frac{1}{200} + \frac{1}{300}\right)}} = \boxed{3.31}$$

(2) The decision is:

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$



1.645

A	Reject H_0
B	Do not reject (Accept) H_0
C	Accept both H_0 and H_A
D	Reject both H_0 and H_A

$$P - value = P(Z > 3.31) = 1 - P(Z < 3.31) = 1 - 0.99953 = 0.00047 < 0.05$$

Question 11:

In a study of diabetes, the following results were obtained from samples of males and females between the ages of 20 and 75. Male sample size is 300 of whom 129 are diabetes patients, and female sample size is 200 of whom 50 are diabetes patients. If P_M, P_F are the diabetes proportions in both populations and $\hat{p}_M, \hat{p}_F$ are the sample proportions, then:
A researcher claims that the Proportion of diabetes patients is found to be more in males than in female (H_0: $P_M - P_F \leq 0$ vs H_A: $P_M - P_F > 0$) . Do you agree with his claim, take $\alpha = 0.10$

$$n_m = 300 \quad , \quad x_m = 129 \quad \Longrightarrow \hat{p}_1 = \frac{129}{300} = 0.43$$

$$n_f = 200 \quad , \quad x_f = 50 \quad \Longrightarrow \hat{p}_2 = \frac{50}{200} = 0.25$$

(1) The pooled proportion is:

$$\bar{p} = \frac{x_m + x_f}{n_m + n_f} = \frac{129 + 50}{300 + 200} = 0.358$$

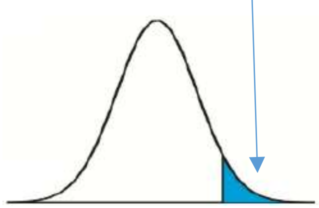
A	0.43	B	0.18	C	0.358	D	0.68

(2) The value of the test statistic is:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(0.43-0.25)}{\sqrt{(0.358)(1-0.358)\left(\frac{1}{300}+\frac{1}{200}\right)}} = \boxed{4.11}$$

(3) The decision is:

$$Z_{1-\alpha} = Z_{1-0.10} = Z_{0.90} = 1.285$$

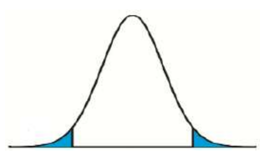
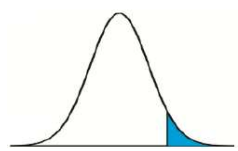
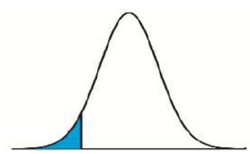


A	Agree with the claim (Reject H_o)
B	do not agree with the claim
C	Can't say

$$P - value = P(Z > 4.11) = 1 - P(Z < 4.11) = 1 - 1 = 0 < 0.05$$

- ***P – value:***

P–value is the smallest value of **α** for which we can reject the null hypothesis H_0.

Hypothesis	$H_0: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_0: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_0: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
Reject region (RR) of H_0			
P-value	$2 \times P(Z > \lvert TS \rvert)$	$P(Z > TS)$	$P(Z < TS)$

$2 \times P(Z > TS)$ *If* $TS > 0$	$2 \times P(Z < TS)$ *If* $TS < 0$

If P-value < α $\Longrightarrow$ Reject H_0

	population normal or not normal n large ($n \geq 30$)		population normal n small ($n < 30$)	
	σ known	σ unknown	σ known	σ unknown
Testing	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$Z = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$T = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$

- **Two Samples Test for Paired Observation**

<u>Question 1:</u>

The following contains the calcium levels of nine test subjects at zero hours and three hours after taking a multi-vitamin containing calcium.

Pair	0 hour (X_i)	3 hours (Y_i)	Difference $D_i = X_i - Y_i$
1	17.0	17.0	0.0
2	13.2	12.9	0.3
3	35.3	35.4	-0.1
4	13.6	13.2	0.4
5	32.7	32.5	0.2
6	18.4	18.1	0.3
7	22.5	22.5	0.0
8	26.8	26.7	0.1
9	15.1	15.0	0.1

The sample mean and sample standard deviation of the differences D are 0.144 and 0.167, respectively. To test whether the data provide sufficient evidence to indicate a difference in mean calcium levels ($H_0: \mu_1 = \mu_2$ against$H_1: \mu_1 \neq \mu_2$) with $\alpha = 0.10$ we have: $\boxed{\bar{D} = 0.144 \ , \ S_d = 0.167 \ , \ n = 9}$

$$\boxed{\begin{matrix} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{matrix}} \Longrightarrow \boxed{\begin{matrix} H_0: \mu_1 - \mu_2 = 0 \\ H_1: \mu_1 - \mu_2 \neq 0 \end{matrix}} \Longrightarrow \boxed{\begin{matrix} H_0: \mu_D = 0 \\ H_1: \mu_D \neq 0 \end{matrix}}$$

[1]. The reliability coefficient (the tabulated value) is:

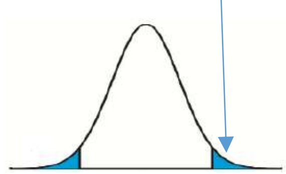
$$t_{1-\frac{\alpha}{2},n-1} = t_{1-\frac{0.1}{2},9-1} = t_{0.95,8} = \boxed{1.860}$$

[2]. The value of the test statistic is:

$$T = \frac{\bar{D} - \mu_D}{S_d/\sqrt{n}} = \frac{0.144 - 0}{0.167/\sqrt{9}} = \boxed{2.5868}$$

[3]. The decision is:

We reject H_0



-1.86 1.86

Question 2:

Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. Reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

Pair	DD method (X_i)	TW method (Y_i)	Difference $D_i = X_i - Y_i$
1	1509	1498	11
2	1418	1254	164
3	1561	1336	225
4	1556	1565	-9
5	2169	2000	169
6	1760	1318	442
7	1098	1410	-312
8	1198	1129	69
9	1479	1342	137
10	1281	1124	157
11	1414	1468	-54
12	1954	1604	350
13	2174	1722	452
14	2058	1518	540

1. The sample mean of the differences $\overline{D}$ is:

$$\overline{D} = \frac{11+164+225-9+169+442-312+\cdots+540}{14} = 167.21$$

A	167.21	B	0.71	C	0.61	D	0.31

2. The sample standard deviation of the differences S_D is:

$$S_D = \sqrt{\frac{\sum(D_i - \overline{D})^2}{n-1}} = 228.21$$

A	3.15	B	−0.71	C	71.53	D	228.21

3. The reliability coefficient to construct 90% confidence interval for the true average difference between intake values measured by the two methods μ_D is:

$$\text{The reliability coefficient} = t_{1-\frac{\alpha}{2},n-1} = t_{0.95,13} = 1.771$$

A	1.96	B	1.771	C	2.58	D	1.372

4. The 90% lower limit for μ_D is:

$$= \bar{D} - \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 167.21 - \left(1.771 \times \frac{228.12}{\sqrt{14}}\right) = 59.19$$

A	24.92	B	22.55	C	59.19	D	44.96

5. The 90% upper limit for μ_D is:

$$= \bar{D} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 167.21 + \left(1.771 \times \frac{228.12}{\sqrt{14}}\right) = 275.23$$

A	224.92	B	322.55	C	275.23	D	24.96

To test $H_0: \mu_D = 0$ versus $H_A: \mu_D \neq 0$, $\alpha = 0.10$ as a level of significance we have:

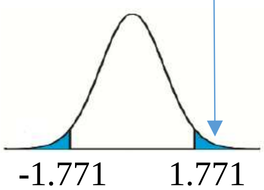
6. The value of the test statistic is:

$$T = \frac{\bar{D}-\mu_D}{S_d/\sqrt{n}} = \frac{167.21-0}{228.12/\sqrt{14}} = \boxed{2.74}$$

A	2.74	B	−0.7135	C	−7.153	D	−0.3157

7. The decision is:

Reject H_0



A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

Question 3:

In a study of a surgical procedure used to decrease the amount of food that person can eat. A sample of 10 persons measures their weights before and after one year of the surgery, we obtain the following data:

Before surgery (X)	148	154	107	119	102	137	122	140	140	117
After surgery (Y)	78	133	80	70	70	63	81	60	85	120
$D_i = X_i - Y_i$	70	21	27	49	32	74	41	80	55	-3

We assume that the data comes from normal distribution.
For 90% confidence interval for μ_D, where μ_D is the difference in the average weight before and after surgery.

1. The sample mean of the differences $\overline{D}$ is:

$$\overline{D} = \frac{70+21+27+49+32+74+41+80+55-3}{10} = 44.6$$

2. The sample standard deviation of the differences S_D is:

$$S_D = \sqrt{\frac{\sum(D_i - \overline{D})^2}{n-1}} = 26.2$$

3. The 90% upper limit of the confidence interval for μ_D is:

$$t_{1-\frac{\alpha}{2},n-1} = t_{0.95,9} = 1.833$$

$$= \overline{D} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

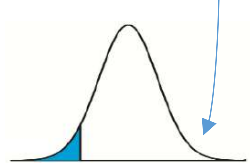
$$= 44.6 + \left(1.833 \times \frac{26.2}{\sqrt{10}}\right) = 59.79$$

4. To test $H_0: \mu_D \geq 43$ versus $H_A: \mu_D < 43$, with $\alpha = 0.10$ as a level of significance, the value of the test statistic is:

$$T = \frac{\overline{D} - \mu_D}{S_d/\sqrt{n}} = \frac{44.6-43}{26.2/\sqrt{10}} = \boxed{0.19}$$

5. The decision is:

$$t_{1-\alpha,n-1} = t_{0.90,9} = 1.383$$



-1.383

A	Reject H_0	B	Do not reject H_0	C	No decision	D	None of these

Questions 4:

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water.

The Data is given below:

	zinc concentration in Bottom water	zinc concentration in Surface water	Difference
1	0.43	0.415	0.015
2	0.266	0.238	0.028
3	0.567	0.39	0.177
4	0.531	0.41	0.121
5	0.707	0.605	0.102
6	0.716	0.609	0.107
7	0.651	0.632	0.019
8	0.589	0.523	0.066
9	0.469	0.411	0.058
10	0.723	0.612	0.111

Note that the mean and the standard deviation of the difference are given respectively by $\overline{D}$ = 0.0804 and S_D = 0.0523 We want to determine the 95 % confidence interval for $\mu_1 - \mu_2$, where μ_1 and μ_2 represent the true mean zinc concentration in Bottom water and surface water respectively. Assume the distribution of the differences to be approximately normal.

1. The 95% lower limit for $\mu_1 - \mu_2$ equals to:

A	0.02628	B	0.13452	C	0.04299	D	0.11781

2. The 95% upper limit for $\mu_1 - \mu_2$ equals to:

A	0.02628	B	0.13452	C	0.04299	D	0.11781

Questions 5:

The following data shows the measurements of the cumulative diabetes level for a sample of 10 elderly individuals in the first stage of infection before and after following a specific diet program.

Individuals	1	2	3	4	5	6	7	8	9	10
Diabetes level before diet (X)	7.5	7.4	7.5	7.2	7.3	6.8	7.5	7.7	7.5	7.3
Diabetes level after diet (Y)	5.7	4.8	6.8	5.9	5.8	5.4	6.2	6	6.1	6
D=X-Y	1.8	2.6	0.7	1.3	1.5	1.4	1.3	1.7	1.4	1.3

Let $\mu_D = \mu_X - \mu_Y$, and assume that the population are normal. The researcher claims that following a diet program reduce the level of diabetes
(هل البرنامج الغذائي يقلل مستوى السكر).

1. The value of the mean and standard deviation respectively $(\bar{D}, S_D)$ are:

A	1.4 , 0.4604	B	1.5 , 0.4853	C	-1.5 , 0.4853	D	1.5 , 0.4604

2. The alternative hypotheses given by:

A	$H_0: \mu_D = 0$	B	$H_A: \mu_D < 0$	C	$H_A: \mu_D > 0$	D	$H_A: \mu_D \neq 0$

3. The value of the test statistic equal to:

A	7.5	B	8.213	C	5.37	D	9.774

4. The $\alpha = 0.05$, the rejection region is to:

A	The right of value (2.262)	B	The left of value (-1.833)
C	The right of value (1.9) or left (-1.9)	D	The right of value (1.833)

5. The lower bound of 90% confidence interval of μ_D equal to:

A	1.219	B	1.153	C	1.288	D	1.781

	Estimation	Testing
Single mean	$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ σ known	$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$ σ known
	$\bar{X} \pm t_{1-\frac{\alpha}{2},(n-1)} \frac{S}{\sqrt{n}}$ σ unknown	$T = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$ σ unknown
Two means	$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ σ_1 and σ_2 known	$Z = \frac{(\bar{X}_1-\bar{X}_2)-d}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$ σ_1 and σ_2 known
	$(\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2},\ (n_1+n_2-2)}\ S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ σ_1 and σ_2 unknown	$T = \frac{(\bar{X}_1-\bar{X}_2)-d}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ σ_1 and σ_2 unknown
Single proportion	$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$	$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$
Two proportions	$(\hat{p}_1 - \hat{p}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$	$Z = \frac{(\hat{p}_1-\hat{p}_2)-d}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

Paired	$\bar{D} \pm t_{1-\frac{\alpha}{2},(n-1)} \frac{S_d}{\sqrt{n}}$ σ unknown	$T = \frac{\bar{D}-\mu_D}{S_d/\sqrt{n}}$ σ unknown

The pooled variance $S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}$ The pooled proportion $\bar{p} = \frac{x_1+x_2}{n_1+n_2}$

	H_0 is true	H_0 is false
Accepting H_0	Correct decision ✓	Type II error (β)
Rejecting H_0	Type I error (α)	Correct decision ✓

Type I error = Rejecting H_0 when H_0 is true $P(\text{Type I error}) = P(\text{Rejecting } H_0 \mid H_0 \text{ is true}) = \alpha$	Type II error = Accepting H_0 when H_0 is false $P(\text{Type II error}) = P(\text{Accepting } H_0 \mid H_0 \text{ is false}) = \beta$

- **Question from previous finals:**

Question 1: In the procedure of testing the statistical hypotheses H_0 against H_A using a significance level α

1. The type I error occur if we:

A	Rejecting H_0 when H_0 is true	B	Rejecting H_0 when H_0 is false	C	Accepting H_0 when H_0 is true	D	Accepting H_0 when H_0 is false

2. The probability of type I error is:

A	β	B	α	C	$1-\beta$	D	$1-\alpha$

3. The level of significance is:

A	The probability of rejecting H_A	B	The probability of rejecting H_0
C	The probability of Type I error	D	The probability of Type II error

4. The level of significance α is:

A	Probability (Accept H_0 when H_0 is false)	B	Accept H_0 when H_0 is false
C	Probability (Reject H_0 when H_0 is true)	D	Reject H_0 when H_0 is true

5. When we use P-value method, we reject H_0 if

A	P- value > α	B	P- value < α	C	P- value < β	D	P- value > β

6. If the P-value = 0.0625 and α = 0.05, the decision is:

A	Reject H_0	B	Accept H_0	C	Reject H_A	D	Accept H_A

7. If the P-value = 0.05609, in which level of significance can reject H_0 is:

A	$\alpha = 0.05$	B	$\alpha = 0.025$	C	$\alpha = 0.1$	D	$\alpha = 0.01$

8. To determine the rejection region for $\mathrm{H_0}$, it depends on:

A	α and $\mathrm{H_A}$	B	$\mathrm{H_0}$	C	α and $\mathrm{H_0}$	D	β

9. Which one is an example of two-tailed test:

A	$\mathrm{H_A}: \mu = 0$	B	$\mathrm{H_A}: \mu \neq 0$	C	$\mathrm{H_A}: \mu < 0$	D	$\mathrm{H_A}: \mu > 0$

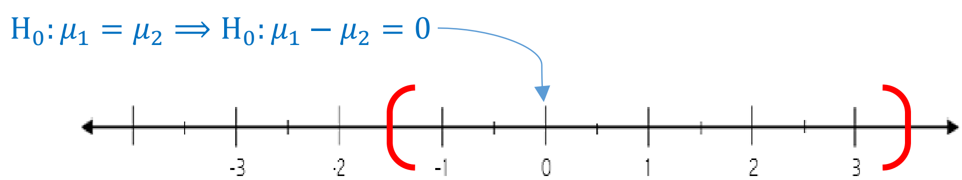
10. If the distribution of the random sample is normal and standard deviation of the population is known, which type of the interval should be considered?

A	z - interval	B	x - interval	C	t - interval	D	c - interval

11. In testing hypothesis for population mean (μ) of non-normal population with unknown standard deviation and large sample size. The test statistic is used

A	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	B	$T = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	C	$Z = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$	D	$T = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$

12. An appropriate 95% CI for μ has been calculated as (-1.5 , 3.5) based on $\mathrm{n_1} = 15$, $\mathrm{n_2} = 17$ observations from two independent population with normal distribution. The hypotheses of interest $\mathrm{H_0}: \mu_1 = \mu_2$ vs $\mathrm{H}_A: \mu_1 \neq \mu_2$. Based on this CI and at $\alpha = 0.05$,



داخل الفتره Accept خارج الفترة Reject

A	We should reject $\mathrm{H_0}$	B	We should not reject $\mathrm{H_0}$

Question 2:

To compare the mean times spent waiting for a heart transplant for two age groups, you randomly select several people in each age group who have had a heart transplant. The result is shown below. Assume both population is are normally distributed with equal variance.

Sample statistics for heart transplant		
Age group	18-34	35-49
Mean	171 days	169 days
Standard deviation	8.5 days	11.5 days
Sample size	20	17

Do this data provide sufficient evident to indicate a difference among the population means at $\alpha = 0.05$

1. The alternative hypothesis is:

A	$H_A: \mu_1 \neq \mu_2$	B	$H_A: \mu_1 \leq \mu_2$	C	$H_A: \mu_1 > \mu_2$	D	$H_A: \mu_1 = \mu_2$

2. The pooled estimator of the common variance S_p^2 is:

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{8.5^2(19)+11.5^2(16)}{35} = 99.67857$$

A	9935.82	B	105.5214	C	10.4429	D	99.6786

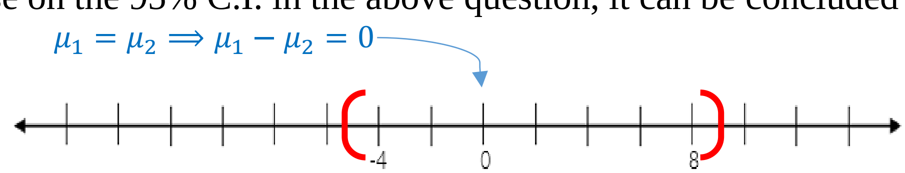
3. The appropriate test statistics is:

A	$Z = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$	B	$Z = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$	C	$T= \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$	D	$T = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$

4. The 95% confidence interval for the different in mean times spent waiting for heart transplant for the two age groups:

A	$(-3.548, 7.565)$	B	$(-0.1306, 4.1306)$	C	$(-4.6862, 8.6862)$	D	$(-4.8519, 8.8519)$

5. Base on the 95% C.I. in the above question, it can be concluded that:



A	$\bar{X}_1 = \bar{X}_2$	B	$\mu_1 \neq \mu_2$	C	$\mu_1 = \mu_2$	D	None of these

Question 3:

A random sample of 30 male babies has the mean birth heights of 75 cm. Assume normal population with standard deviation of 3 cm.

- Consider a 90% confidence interval for the mean height of all babeis.

1. The point estimate of μ is

A	3	B	0.548	C	9	D	75

2. The reliability coefficient (at 90% confident level) is

A	1.699	B	1.645	C	1.311	D	1.285

3. The 90% upper limit of the confidence interval for the population mean μ is

A	75.704	B	74.099	C	75.901	D	74.069

- Can it be concluded from these data that population mean height of all male babies is greater than 74 cm? $\alpha = 0.1$

4. The test statistic equals

A	1.83	B	3.16	C	0.608	D	0.333

5. The p – value equals

A	0.0007	B	0.03362	C	0.50399	D	0.96638

6. The decision is

A	Reject H_0	B	Accept H_0

Question 4:

If the hemoglobin level of pregnant women has normal distribution. A sample of 25 pregnant women was taking and found that the average of their hemoglobin level is 4.7 (g/dl) with sample standard deviation 3 (g/dl). Then

- To construct 95% confidence interval for the population mean μ .

1. The estimated standard error for $\overline{X}$ is

A	0.638	B	0.12	C	0.6	D	0.75

2. The 95% upper limit of the confidence interval for μ is

A	3.462	B	2.131	C	6.171	D	5.938

- Use $\alpha = 0.05$, test if mean of hemoglobin level of pregnant women equal 4

3. The hypotheses are:

A	$H_0: \mu = 4$ $H_A: \mu > 4$	B	$H_0: \mu = 4$ $H_A: \mu \neq 4$	C	$H_0: \mu \geq 4$ $H_A: \mu < 4$	D	$H_0: \mu < 4$ $H_A: \mu > 4$

4. The decision is

A	Reject H_0	B	Accept H_0

Question 5:

We believe that healthy men and women are different with respect to their body temperature. Suppose that the two populations are normal with standard deviation 0.8 for women and 0.9 for men. Two random independent samples are observed. The showed the samples result.

	Women	Men
Sample size	25	28
Sample mean	98.2	98.4

- Let μ_w represents mean of body temperature for women and μ_m represents of body temperature for men.

1. The standard error for $\mu_w - \mu_m$ is

A	0.233514	B	0.06414	C	0.054528	D	0.253259

2. The margin of error of 90% confident interval of $\mu_w - \mu_m$ is

A	0.4577	B	0.3841	C	0.4964	D	0.4166

3. The lower limit of 90% confidence interval of $\mu_w - \mu_m$ is

A	-0.1841	B	0.5841	C	-0.5841	D	0.1841

- Test $H_0: \mu_w - \mu_m \geq 0$ vs $H_A: \mu_w - \mu_m < 0$, at level of significance of 0.01

4. The test statistic is:

A	$Z = -0.86$	B	$T = -0.86$	C	$Z = -0.79$	D	$T = -0.79$

5. The decision is

A	Reject H_0	B	Accept H_0

Question 6:

The average amount of time boys and girls aged 7 to 11 spend playing sports each day is believed to be the same. Two independent samples gave the following results:

	Sample size	Average number of hours playing sport per day	Sample standard deviation
Boys	9	4.5	1
Girls	11	2	0.5

Assume both populations have a normal distribution with equal variances.

- Is there difference in the mean amont of time boys and girls spend playing sports each day? $\alpha = 0.01$

1. The pooled estimate of the common variance (S_p^2) is:

A	0.7222	B	0.8065	C	0.6528	D	0.5833

2. The rejection region of the null hypothesis (H_0) is

A	$T > 2.878$ *or* $T < -2.878$	B	$Z > 2.325$ *or* $Z < -2.325$	C	$Z < -2.325$	D	$T > 2.878$

3. The decision is

A	Reject H_0	B	Accept H_0

- To construct 99% confidence interval for the difference in mean amont of time boys and girls spend play sports esch day $\mu_{Boy} - \mu_{Girl}$, then

4. The 99% confidence interval of $\mu_{Boy} - \mu_{Girl}$ is

A	$(-3.486, -1.511)$	B	$(1.7018, 3.2981)$
C	$(1.5121, 3.4879)$	D	$(0.3253, 2.6747)$

5. The length of the interval is:

A	1.5963	B	2.1065	C	1.9758	D	2.1072

Question 7:

We a study of Alzheimer disease, it is found that in a sample of 80 male, 28 have Alzheimer disease.

1. The ponit eatimate of the population proportion of males who had the Alzeimer disease is

A	0.35	B	0.6	C	0.45	D	28

2. The upper limit of a 95% confidence interval for the population proportion of males who had the Alzheimer disease is

A	0.245	B	0.262	C	0.455	D	0.5455

- If we test the hypothesis that the population proportion of males who had the Alzheimer disease is greater than 25%, then

3. The value of the test statistic is

A	-1.875	B	2.066	C	0.938	D	1.875

4. The critical value is (use $\alpha = 0.05$)

A	1.96	B	-1.645	C	1.645	D	-1.96

5. The conclution is

A	The population proportion of males who had the Alzheimer disease is equal to 25%
B	The population proportion of males who had the Alzheimer disease is less than or equal to 25%
C	The population proportion of males who had the Alzheimer disease is greater than or equal to 25%
D	None of these

Question 8:

A researcher is interested in studying a certain infection proportion in two different cities, City A and City B. The data of the two different proportion for is summarized in the table:

	City A	City B
Infection people number	78	70
Sample size	120	100

Let P_A be the proportion for city A and P_B be the proportion for city B, then:

1. The standard error for the different between the two sample proportions is

A	0.0040	B	0.06321	C	0.01241	D	0.11142

2. The 90% confidence interval for $P_A - P_B$ is

A	$(-0.0566, 0.0434)$	B	$(-0.0673, 0.0308)$	C	$(-0.15398, 0.05398)$	D	$(-0.0551, -0.0449)$

3. Are P_A and P_B equal ?

A	No	B	Yes

- To test $H_0: P_A = P_B \ vs \ H_A: P_A \neq P_B$ assume equal proportion at $\alpha = 0.05$, then

4. The pooled estimator of the common proportion $(\bar{P})$ is

A	0.6727	B	0.8202	C	0.1908	D	0.0364

5. The test statistic is

A	1.972	B	0.787	C	-1.972	D	-0.787

6. The acceptance region of H_0 is

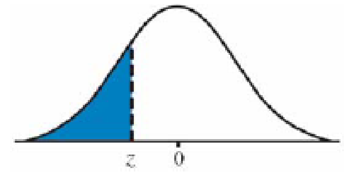
A	$(-1.96, 1.96)$	B	$(\infty, 1.96)$	C	$(1.96, \infty)$	D	$(-1.96, \infty)$

7. The decision is

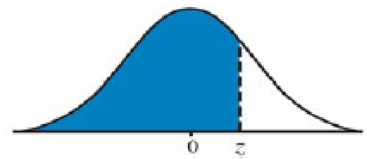
A	Accept H_0	B	Reject H_0

Standard Normal Table

Areas Under the Standard Normal Curve



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

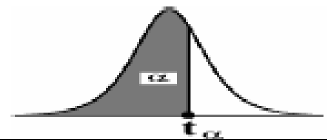


Standard Normal Table (continued)

Areas Under the Standard Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	**0.00**
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	**0.10**
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	**0.20**
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	**0.30**
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	**0.40**
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	**0.50**
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	**0.60**
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	**0.70**
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	**0.80**
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	**0.90**
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	**1.00**
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	**1.10**
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	**1.20**
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	**1.30**
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	**1.40**
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	**1.50**
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	**1.60**
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	**1.70**
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	**1.80**
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	**1.90**
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	**2.00**
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	**2.10**
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	**2.20**
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	**2.30**
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	**2.40**
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	**2.50**
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	**2.60**
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	**2.70**
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	**2.80**
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	**2.90**
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.9990	**3.00**
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	**3.10**
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	**3.20**
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	**3.30**
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	**3.40**
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	**3.50**

Critical Values of the t-distribution (t_α)



ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387