

MID TERM II EXAM. SEMESTER II, 1445

DEPT. MATH., COLLEGE OF SCIENCE
KING SAUD UNIVERSITY

MATH: 107 FULL MARK: 25 TIME: 90 MINUTES

Q1. [Marks: 4+2+3=9]

(a) Let a constant force $\mathbf{F} = \langle 2, 2, 0 \rangle$ is applied on a particle displacing it from point $(0, 0, 0)$ to $(0, 3, 0)$. Then, find: (i) work done by the force $\mathbf{F}$, and (ii) angle θ between the force $\mathbf{F}$ and the displacement $\mathbf{d}$.

(b) Let $\mathbf{u} = \langle 3, -1, -4 \rangle$, $\mathbf{v} = \langle 2, 5, -2 \rangle$, and $\mathbf{w} = \langle -1, 0, 6 \rangle$.

Compute $\text{comp}_{\mathbf{u}}(\mathbf{v} \times \mathbf{w})$.

(c) Find the area of the triangle $\triangle ABC$, where $A(2, -1, 1)$, $B(-3, 2, 0)$, and $C(4, -5, 3)$.

Q2. [Marks: 2+3+3=8]

(a) Find an equation of the plane through $P(2, 5, -6)$ and parallel to the plane $3x - y + 2z = 10$.

(b) Let l_1 be the line passing through $A(1, 3, 0)$ and $B(0, 4, 5)$, and l_2 be the line passing through $C(-2, -1, 2)$ and $D(5, 1, 0)$. Determine whether l_1 and l_2 are skew lines, that is, neither parallel nor intersecting.

(c) Identify the surface $y = 6x^2 + z^2$. Give its traces, and sketch it.

Q3. [Marks: 2+3+3=8]

(a) Find the domain of the vector-valued function $\mathbf{r}(t) = \ln(1 - t)\mathbf{i} + \sin t\mathbf{j} + t^3\mathbf{k}$.

(b) If $\mathbf{r}(t) = (1 + t)\mathbf{i} + 2t\mathbf{j} + (2 + 3t)\mathbf{k}$ is the position vector of a moving point P , find its velocity, acceleration, and speed at $t = 2$.

(c) Find $\mathbf{r}(t)$ subject to the given conditions: $\mathbf{r}'(t) = 2\mathbf{i} - 4t^3\mathbf{j} + 6\sqrt{t}\mathbf{k}$, $\mathbf{r}(0) = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$.

MT 2 solution

①

Q1 (a) Work done = $\underline{F} \cdot \underline{d}$
(i) $= \langle 2, 2, 0 \rangle \cdot \langle 0, 3, 0 \rangle$
 $= 6 \text{ units} \quad (\text{Mark 2})$

(ii) $\theta = \cos^{-1} \left(\frac{6}{\sqrt{8} \sqrt{9}} \right) = \cos^{-1} \left(\frac{6}{(2\sqrt{2})(3)} \right) = \cos^{-1} \left(\frac{1}{\sqrt{2}} \right)$

$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ$
(Mark 2)

(b) $\text{comp}_{\underline{u}}(\underline{v} \times \underline{w})$
 $= \underline{v} \times \underline{w} \cdot \frac{\underline{u}}{\|\underline{u}\|}$
 $= \langle 30, -10, 5 \rangle \cdot \frac{\langle -3, -1, -4 \rangle}{\sqrt{26}}$
 $= \frac{80}{\sqrt{26}} \quad (\text{Mark 1+1})$

(c) $\Delta ABC = \frac{1}{2} \|\vec{AB} \times \vec{AC}\|$
 $= \frac{1}{2} (2\sqrt{66}) = \sqrt{66}$
(Mark 3)

(2)

Q₂ (a) Equation of the plane is

$$3x - y + 2z = -11 \quad (\text{Mark 2})$$

(b) Here $\underline{a} = \langle -1, 1, 5 \rangle$, $\underline{b} = \langle 7, 2, -2 \rangle$

$$l_1: x = 1 - t, y = 3 + t, z = 5t, \quad t \in \mathbb{R}$$

$$l_2: x = -2 + 7u, y = -1 + 2u, z = 2 - 2u, \quad u \in \mathbb{R}$$

Not Parallel because

$$-\frac{1}{7} \neq \frac{1}{2} \neq \frac{-5}{2} \quad (\text{Mark 1})$$

Not intersecting because as $t = \frac{-22}{9}$
and $u = \frac{7}{9}$ we get

$$z = -\frac{110}{9} \text{ and } z = \frac{4}{9}$$

which means l_1 and l_2 non-intersecting

(Mark 2)

(c) Paraboloid having y-axis as its axis. Traces: xy-trace $y = 6x^2$ a parabola; yz-trace: $y = z^2$ a parabola and xz-trace $6x^2 + z^2 = 0$, origin.

(3)

Q₃ (a) Domain of $\underline{r}$ is

$$D = \{t; t < 1\}$$

since the domain of $\ln(1-t)$ is

$$\{t; t < 1\}.$$

(Mark 2)

(b) For all t ,

$$\underline{v}(t) = \underline{i} + 2\underline{j} + 3\underline{k}$$

$$\underline{a}(t) = 0, \text{ and}$$

$$\|\underline{v}(t)\| = \sqrt{14} \approx 3.74$$

(Mark 1+1+1)

$$(c) \quad \underline{r}'(t) = 2\underline{i} - 4t^3\underline{j} + 6\sqrt{t}\underline{k} \Rightarrow$$

$$\underline{r}(t) = 2t\underline{i} - t^4\underline{j} + 4t^{3/2}\underline{k} + \underline{c}$$

$$\text{Thus, } \underline{r}(t) = (2t+1)\underline{i} + (5-t^4)\underline{j} +$$

$$(4t^{3/2}+3)\underline{k}$$

(Mark 2+1)