

First Mid Term Examination, Semester I, 2024  
Dept. Math. College of Science  
King Saud University  
Math: 107 Full Marks: 25 Time: 90 Minutes

Q 1. Consider the following system of linear equations

$$\begin{aligned}x + y - z &= 2 \\x + y + z &= 3 \\x + y + (\lambda^2 - 5)z &= \lambda\end{aligned}$$

Determine for which values of  $\lambda$  this system has

(i) no solution (ii) infinitely many solutions (iii) unique solution.

Q 2. For the following system of linear equations

$$\begin{aligned}x + y + z &= 4 \\x - y + 2z &= 1 \\y + z &= 3\end{aligned}$$

Find the inverse of the coefficient matrix by using elementary row operations, then find the solution of the given system.

Q 3. Prove the identity

$$\begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{vmatrix} = a(b-a)(c-b)(d-c)$$

Q 4. Let

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 3 & 2 \\ -2 & 1 & -4 \end{bmatrix}$$

Find  $\text{adj}(A)$  (the adjoint of  $A$ ). Use determinant to check whether the matrix  $A$  is invertible or not.

Q 5. Use Cramer's Rule to solve the following linear system

$$\begin{aligned}x - y + z &= 1 \\2x + 3z &= 0 \\x + y - z &= 1.\end{aligned}$$