King Saud University
Department of Mathematics

106
First Midterm, October 2016

NAME:

Group Number/Instructor name:

ID:

- Duration of the exam: 90 minutes
- Simple calculators are allowed

Question	Grade
I	
II	
III	
IV	
V	
Total	

I) [4 *marks*]

A) Find the function $f(x)$, knowing that $\int f(x)dx = x + \cos x + c$, where c is a constant.

B) **Without evaluating the integrals**, show that

$$\int_0^2 2xdx \leq \int_0^2 (x^2 + 1)dx.$$

C) If $f(x) = x^2 \int_1^x \cos^3 t dt$, find $f'(x)$.

II) [4 *marks*] Let $f(x) = x + 4$ and let R be the region under the graph of f, from 0 to 2. Let P be the **regular** partition of $[0, 2]$ into n subintervals.

A) Find, in terms of n, the Riemann sum R_P, if $w_k = \dfrac{2k}{n}$, for $k = 1, 2, \cdots, n$.

B) Use Riemann sum in (A) to compute the area of the region R.

III) [2 *marks*] Using the trapezoidal rule with $n = 4$, approximate $\int_1^5 \frac{1}{x^2} dx$.

IV) [2 *marks*] Find the value of c that satisfies the Integral Mean Value theorem for the function $f(x) = x^2 - 1$ on $[1, 3]$.

V) [8 *marks*] Compute the following integrals:

(a) $\int x(1+x)dx$

(b) $\int \frac{x^2}{(x+1)^4}dx$

(c) $\int \sec^2 x(1+2\tan x)^4 dx.$

(d) $\int_0^4 \frac{x}{\sqrt{x^2+9}} dx$

(e) $\int_0^3 f(x)dx$, where $f(x) = \begin{cases} 2, & \text{if } 0 \leq x < 1 \\ \frac{1}{x^2}, & \text{if } 1 \leq x \leq 3 \end{cases}$.