King Saud University
College of Sciences
Department of Mathematics
First MidTerm, First Term 1435/1436
M-106

Exercice 1 [5 Marks]:

a. Find the average value of $f(x) = \sin(x) + \cos(x)$ over $[0, \pi]$.

$$\begin{aligned} f_{av} &= \frac{1}{\pi}\int_0^{\pi}(\sin x + \cos x)dx \ [0.5] \\ &= \frac{2}{\pi} \qquad [0.5] \end{aligned}$$

b. Find c if $\sum_{k=1}^{n}(k + \frac{c}{2}) = \frac{n^2}{2}$

we have

$$\begin{aligned} \frac{n(n+1)}{2} + \frac{cn}{2} &= \frac{n^2}{2} \ [0.5] \\ c &= -1 \ \ [0.5] \end{aligned}$$

c. Find the derivative of the integral $\int_x^{x^2} \sqrt{t^2+3}\, dt$

$$\frac{d}{dx}\int_x^{x^2} \sqrt{t^2+3}\, dt = \sqrt{x^4+3}2x - \sqrt{x^2+3}[0.5+0.5]$$

d. Find $f(x)$ if $\int_x^5 f(t)dt = x^2$

$$\begin{aligned} \frac{d}{dx}\int_x^5 f(t)dt &= \frac{d}{dx}x^2 \ [0.5] \\ f(x) &= -2x \quad [0.5] \end{aligned}$$

e. Find x if $\log_5(\frac{5x}{x+1}) = 2$.

$$\begin{aligned} \frac{5x}{x+1} &= 5^2 \qquad [0.5] \\ x &= -\frac{5}{4} \quad [0.5] \end{aligned}$$

Exercice 2 [4 Marks]: Approximate the integral $\int_1^3 \frac{\sqrt{x}}{2+\sqrt[3]{x}}dx$ by using trapeziodal rule and with $n = 4$.

As $n = 2$, then we will have

$$x_0 = 1, x_1 = \frac{3}{2}, x_2 = 2, x_3 = \frac{5}{2}, x_4 = 3.[1]$$

Then

$$\begin{aligned} \int_1^3 \frac{\sqrt{x}}{2+\sqrt[3]{x}}dx &\simeq \frac{2}{2(4)}\left(f(1) + 2f(1.5) + 2f(2) + 2f(\frac{5}{2}) + f(3)\right) \ [2] \\ &\simeq \frac{2}{2(4)}(\frac{\sqrt{1}}{2+\sqrt[3]{1}} + 2\frac{\sqrt{1.5}}{2+\sqrt[3]{1.5}} + 2\frac{\sqrt{2}}{2+\sqrt[3]{2}} + 2\frac{\sqrt{\frac{5}{2}}}{2+\sqrt[3]{\frac{5}{2}}} + \frac{\sqrt{3}}{2+\sqrt[3]{3}}) \ [0.5] \\ &\simeq 0.856\,25 \ [0.5] \end{aligned}$$

Exercice 3 [4 Marks]: Evaluate the integral $\int \frac{(\ln(x^2)+1)^2}{x}dx$.

Let

$$u = \ln(x^2) + 1 \text{ then } du = 2\frac{dx}{x}[1]$$

and we have

$$\begin{aligned} \int \frac{(\ln(x^2)+1)^2}{x}dx &= \frac{1}{2}\int u^2 du \ [0.5] \\ &= \frac{1}{2}\frac{u^3}{3} + C \ [2] \\ &= \frac{1}{2}\frac{(\ln x^2+1)^3}{3} + C \ [0.5] \end{aligned}$$

Exercice 4 [4 Marks]: Evaluate the integral $\int \frac{x}{(x+2)^3}dx$

Let

$$u = x+2 \text{ then } du = dx \text{ [1]}$$

and we have

$$\begin{aligned}\int \frac{x}{(x+2)^3}dx &= \int \frac{u-2}{u^3}du = \int \frac{1}{u^2}du - 2\int \frac{1}{u^3}du \text{ [0.5]} \\ &= -\frac{1}{u} + \frac{1}{u^2} + C \text{ [2]} \\ &= -\frac{1}{x+2} + \frac{1}{(x+2)^2} + C \text{ [0.5]}\end{aligned}$$

Exercice 5 [4 Marks]: Find y' for $y = x^{\tan x}$.

$$\ln y = (\tan x)(\ln x) \text{ [1]}$$

then

$$\frac{y'}{y} = \tan(\frac{1}{x}) + (\sec^2 x)(\ln x) \text{ [2]}$$

and we will have

$$y' = [\tan(\frac{1}{x}) + (\sec^2 x)(\ln x)]x^{\tan x} \text{ [1]}$$

Exercice 6 [4 Marks]: Evaluate the integral $\int \frac{7^x(e^x+2)^2 + e^x}{(e^x+2)^2}dx$

We have

$$\begin{aligned}\int \frac{7^x(e^x+2)^2 + e^x}{(e^x+2)^2}dx &= \int 7^x dx + \int \frac{e^x}{(e^x+2)^2}dx \text{ [2]} \\ &= \frac{7^x}{\ln 7} - \frac{1}{e^x+2} + C \text{ [2]}\end{aligned}$$