King Saud University
College of Sciences
Department of Mathematics
Summer Semester (1433/1434)

M-106
First Midterm-Exam

The Exam paper contains 5 pages
(**5 Multiple choice questions and 5 Full questions**)

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 25 Time: 90 minutes

Marks:

Multiple Choice (1-5)	
Question # 6	
Question # 7	
Question # 8	
Question # 9	
Question # 10	
Total	

Multiple Choice

Q.No:	1	2	3	4	5
$\{a,b,c,d\}$	b	b	a	d	c

Q. No: 1 The average value of $f(x) = \dfrac{2}{\sqrt{x}}$ over $[1, 2]$ is equal to:

$(a)\quad \sqrt{2} - 1 \quad (b)\quad 4\sqrt{2} - 4 \quad (c)\quad 2\sqrt{2} + 1 \quad (d)\quad \sqrt{2}$

Q. No: 2 The approximate value of $\displaystyle\int_0^1 \frac{1}{x^2+1}\,dx$ using trapezoidal rule with $n = 2$ is:

(a) 0.7854 (b) 0.775 (c) 0.3875 (d) 0.8754

Q. No: 3 The derivative of $\tan^{-1}(\sqrt{x+1}) - \pi^\pi$ is equal to:

$(a)\quad \dfrac{1}{2(x+2)\sqrt{1+x}} \quad (b)\quad \dfrac{1}{2(x+2)} - \pi \log_\pi \pi^\pi \quad (c)\quad \dfrac{1}{x+2} \quad (d)\quad \dfrac{2}{(x+2)\sqrt{1+x}} - \pi$

Q. No: 4 $\displaystyle\frac{d}{dx}\left(\int_{x^2}^{0} \sqrt{t^2+3}\,dt\right)$ is equal to:

$(a)\quad \sqrt{x^2+3} \quad (b)\sqrt{x^4+3} \quad (c)2x\sqrt{x^4+3} \quad (d) - 2x\sqrt{x^4+3}$

Q. No: 5 If $\log_2(\dfrac{x}{x-1}) = \dfrac{1}{\ln(2)}$, then x is equal to::

$(a)\quad \dfrac{e^2}{e-1} \quad (b)\quad \dfrac{e}{e+1} \quad (c)\quad \dfrac{e}{e-1} \quad (d)\quad \dfrac{e}{2}$

Full Questions

Question No: 6 Evaluate the integral $\int \frac{(1+3^{-x})^{2013}}{3^x}\,dx$

Let $u = 1 + 3^{-x}$. Then $du = -\ln(3)\cdot 3^{-x}dx = -\frac{\ln(3)}{3^x}dx$ and hence [1]

$$\begin{aligned}
\int \frac{(1+3^{-x})^{2013}}{3^x}\,dx &= -\frac{1}{\ln(3)}\int (1+3^{-x})^{2013}\left(\frac{-\ln(3)}{3^x}\right)dx, && [1] \\
&= -\frac{1}{\ln(3)}\int u^{2013}\,du, && [1] \\
&= -\frac{1}{\ln(3)}\left(\frac{u^{2014}}{2014}\right) + c, && [1] \\
&= -\left(\frac{(1+3^{-x})^{2014}}{2014\cdot\ln(3)}\right) + c, && [1]
\end{aligned}$$

Question No: 7 Find y' for $y = \log(x)^{\ln(x)}$, $x > 0$.

$$y = \log(x)^{\ln(x)} = e^{\ln(x)\ln(\log(x))}, \qquad [1]$$

Then

$$\begin{aligned}
y'(x) &= \log(x)^{\ln(x)}\left[\tfrac{1}{x}\ln(\log(x)) + \ln(x)\tfrac{1}{\log(x)}\tfrac{1}{\ln(10)}\tfrac{1}{x}\right], && [2] \\
&= \tfrac{1}{x}\log(x)^{\ln(x)}\left[\ln(\log(x)) + \tfrac{1}{\ln(10)^2}\right] && [2]
\end{aligned}$$

Question No: 8 Evaluate $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$.

Let $u = e^x$. Then $du = e^x dx$, [1].

$$\begin{aligned} \int \frac{e^x}{\sqrt{1-e^{2x}}}dx &= \int \frac{e^x dx}{\sqrt{1-(e^x)^2}}, && [1] \\ &= \int \frac{du}{\sqrt{1-u^2}}, && [1] \\ &= \sin^{-1}(u) + c && [1] \\ &= \sin^{-1}(e^x) + c, && [1] \end{aligned}$$

Question No: 9 Evaluate $\int \frac{\text{sech}^2(x)}{1+\tanh^2(x)}\,dx$.

Let $u = \tanh(x)$. Then $du = \text{sech}^2(x)dx$, [1]

Thus we have

$$\begin{aligned} \int \frac{\text{sech}^2(x)}{1+\tanh^2(x)}\,dx &= \int \frac{du}{1+u^2}, && [2] \\ &= \tan^{-1}(u) + c, && [1] \\ &= \tan^{-1}(\tanh(x)) + c, && [1] \end{aligned}$$

Question No: 10 Let $f(x) = 3x + 1$,

a) Approximate the area under the graph of f from 0 to 3 by subdividing the interval $[0, 3]$ into n equal parts, using a circumscribed rectangular polygon (the right-hand endpoint).

b) Deduce the area under the graph of f corresponding to the interval $[0, 3]$.

- a) $\Delta x = \dfrac{3-0}{n} = \dfrac{3}{n}$, $w_k = x_k = k\Delta x = \dfrac{3k}{n}$. [1]

  Then we have

$$\begin{aligned} R_n &= \sum_{k=1}^{n} f(w_k)\Delta x = \sum_{k=1}^{n} (\frac{9k}{n} + 1)\frac{3}{n} && [1] \\ &= \frac{3}{n^2}\sum_{k=1}^{n}(9k + n) = \frac{3}{n^2}(9\frac{n(n+1)}{2} + n^2) = \frac{27n(n+1)}{2n^2} + 3 && [1] \end{aligned}$$

- b)

$$\int_0^3 f(x)dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[\frac{27n(n+1)}{2n^2} + 3\right] = \frac{27}{2} + 3 = \frac{33}{2}, \quad [2]$$