King Saud University
College of Sciences
Department of Mathematics
Second Semester (1434/1435)

M-106
First Midterm-Exam

The Exam paper contains 5 pages
(5 Multiple choice questions and 5 Full questions)

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 25 Time: 90 minutes

Multiple Choice

Q. No:	1	2	3	4	5
$\{a, b, c, d\}$	c	a	d	a	b

Q. No: 1 The number z that satisfies the conclusion of the mean value theorem of $f(x) = ax$, $a \neq 0$ on $[0, 2]$ is equal to:

$(a)\frac{3}{4}$ $(b)\frac{1}{2}$ $(c)1$ $(d)\frac{3}{2}$

Q. No: 2 The sum $\sum_{k=1}^{n} (4k^3 - 2k)$ is equal to:

(a) $n(n+1)(n^2+n-1)$ (b) $\frac{n}{4}(n+1)(n^2+n+1)$

(c) $\frac{n}{2}(n+1)(n^2+n+1)$ (d) $\frac{n}{4}(n+1)(n^2+n-1)$

Q. No: 3 If $f(x) = x\int_0^{x^2} \cos(\sqrt{t})dt$, then $f'(\frac{\pi}{2})$ is equal to:

(a) 0 $(b)\frac{\pi^2}{2}$ $(c)\frac{\pi^2}{4}$ $(d)\int_0^{\frac{\pi^2}{4}} \cos(\sqrt{t})dt$

Q. No: 4 The integral $\int \frac{\cosh(x)}{\sinh^2 x} dx$ is equal to:

$(a) -\frac{1}{\sinh x} + c$ (b) $\frac{1}{\sinh x} + c$

$(c) \ln(\sinh x) + c$ $(d) \ln(\cosh x) + c$

Q. No: 5 The integral $\int \frac{3^x}{3^x + 2} dx$ is equal to:

(a) $\ln(3^x + 2) + c$ (b) $\frac{\ln(3^x + 2)}{\ln 3} + c$

(c) $(\ln 3)\ln(3^x + 2) + c$ (d) $\frac{\ln(3^x)}{\ln 3} + c$

Full Questions

Question No: 6 [**4** marks] Evaluate the integral $\int \frac{1}{\sqrt{7-e^{2x}}}\,dx$, where $e^x < 1$

Let $u = e^x$, then $du = e^x dx$, [1]

And we will have

$$\begin{aligned}\int \frac{1}{\sqrt{7-e^{2x}}}\,dx &= \int \frac{du}{u\sqrt{7-u^2}}, [1] \\ &= -\frac{1}{\sqrt{7}}\operatorname{sech}^{-1}(\frac{|u|}{\sqrt{7}}) + c, [1] \\ &= -\frac{1}{\sqrt{7}}\operatorname{sech}^{-1}(\frac{e^x}{\sqrt{7}}) + c, [1]\end{aligned}$$

Question No: 7 [**4** marks] If $f(x) = \left(\cosh^{-1}(x) + x^2\right)^{x^2}$, then find $f'(x)$

We have

$$\ln f(x) = x^2 \ln\left(\cosh^{-1}(x) + x^2\right),\ [1]$$

Then

$$\begin{aligned}\frac{f'(x)}{f(x)} &= 2x\ln\left(\cosh^{-1}(x)+x^2\right) + \frac{x^2}{\cosh^{-1}(x)+x^2}\frac{d(\cosh^{-1}(x)+x^2)}{dx}, [1] \\ &= 2x\ln\left(\cosh^{-1}(x)+x^2\right) + \frac{x^2}{\cosh^{-1}(x)+x^2}(\frac{1}{\sqrt{x^2-1}}+2x), [1]\end{aligned}$$

and we will have

$$\begin{aligned}f'(x) &= f(x)(2x\ln\left(\cosh^{-1}(x)+x^2\right) + \frac{x^2}{\cosh^{-1}(x)+x^2}(\frac{1}{\sqrt{x^2-1}}+2x)) \\ &= \left(\cosh^{-1}(x)+x^2\right)^{x^2}(2x\ln\left(\cosh^{-1}(x)+x^2\right) + \frac{x^2}{\cosh^{-1}(x)+x^2}(\frac{1}{\sqrt{x^2-1}}+2x)).[1]\end{aligned}$$

Question No: 8 [**4.5** marks] Evaluate the integral $\int \frac{dx}{\sqrt{x}(1-x)}$, where $0 < x < 1$

let $u = \sqrt{x}$, then $du = \frac{dx}{2\sqrt{x}}$,[1]

And then

$$\begin{aligned}\int \frac{dx}{\sqrt{x}(1-x)} &= 2\int \frac{du}{(1-u^2)}, [1] \\ &= 2\tanh^{-1}(u) + c, [1] \\ &= 2\tanh^{-1}(\sqrt{x}) + c.[1]\end{aligned}$$

Question No: 9 [**4.5** marks] Evaluate the integral $\int \frac{1}{e^x + e^{-x}} dx$

We have

$$\int \frac{1}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x} + 1} dx, [1]$$

Let $u = e^x$, then we have $du = e^x dx$, [1]

And then

$$\begin{aligned} \int \frac{e^x}{e^{2x} + 1} dx &= \int \frac{du}{u^2 + 1}, [1] \\ &= \tan^{-1}(u) + c, [1] \\ &= \tan^{-1}(e^x) + c. [0.5] \end{aligned}$$

Question No: 10 [**3** marks] Approximate the integral $\int_1^2 \frac{1}{\sqrt{3 + x^2}} dx$ using the **Simpson's rule** for a regular partition with $n = 4$

we have $[a, b] = [1, 2]$, then $\Delta x = \frac{2 - 1}{4} = \frac{1}{4} = 0.25$, and $x_0 = 1, x_1 = 1.25, x_2 = 1.5, x_3 = 1.75, x_4 = 2$. [1]

Then we can get

$$\begin{aligned} \int_1^2 \frac{1}{\sqrt{3 + x^2}} dx &\simeq \frac{2 - 1}{3 \times 4} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)), \ [1] \\ &\simeq \frac{2 - 1}{3 \times 4} (\frac{1}{\sqrt{3 + 1}} + 4 \frac{1}{\sqrt{3 + 1.25^2}} + 2 \frac{1}{\sqrt{3 + 1.5^2}} + 4 \frac{1}{\sqrt{3 + 1.75^2}} + \frac{1}{\sqrt{3 + 2^2}}), \ [0.5] \\ &\simeq 0.43734. [0.5] \end{aligned}$$