King Saud University

Math department

April 2026

Midterm1-106

Question1(2+3+3)

a) If $F(x) = \int_{x^4}^{2x^6} \sqrt{1+e^t}\,dt$, find $F'(x)$.

b) Compute the integral $\int \frac{(10x+15\cosh x)dx}{(x^2+3\sinh x)^6}$.

c) Use Simpson's rule with n=4 to approximate $\int_1^5 \frac{1}{\sqrt{1+x^3}}dx$.

Question2 (3+2+3)

a) Find the number c in the mean value theorem for $f(x) = 3\sqrt{x+1}$ on [-1,8].

b) If $y = (2+x^4)^{x^2+3x}$, find $\frac{dy}{dx}$.

c) Evaluate the integral $\int \frac{(1+2x)e^{2x}}{10+x^2e^{4x}}dx$.

Question3(3+3+3)

a) Compute the integral $\int \frac{\ln x+1}{\sqrt{9-(x\ln x)^2}}dx$.

b) Evaluate $\int \frac{dx}{x\sqrt{7-x^8}}$.

c) Find $\int \frac{\sin x\,dx}{\sec x\sqrt{16(\sin x)^4-1}}$.

Grading scheme Midterm 1 106

1) a) $F'(x) = 12x^5\sqrt{1+e^{2x^6}} - 4x^3\sqrt{1+e^{x^4}}$

(1) + (1)

b) $\int \frac{10x + 15\cosh x}{(x^2+3\sinh x)^6}\,dx = 5\int \frac{du}{u^6}$

$= -\frac{1}{(x^2+3\sinh x)^5} + C$

(2) + (1)

c)

$S_4 = \frac{1}{3}\left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{9}} + \frac{2}{\sqrt{28}} + \frac{4}{\sqrt{65}} + \frac{1}{\sqrt{126}}\right)$

$\simeq 1.0012$

(2) + (1)

2) a) $\int_{-1}^{8} 3\sqrt{x+1}\,dx = 54$ (2)

so $6 = 3\sqrt{1+c}$ and $c = 3$ (1)

b) $\ln y = (x^2+3x)\ln(x^4+2)$

(1) $\frac{y'}{y} = (2x+3)\ln(x^4+2) + \frac{4x^3(x^2+3x)}{x^4+2}$

(1) $y' = (\quad\quad)y$

2)

c) $\int \frac{[illegible]x)e^{2x}}{10+x^2e^{4x}}\,dx = \int \frac{du}{10+u^2}$

$= \frac{1}{\sqrt{10}} \tan^{-1}\left(\frac{x e^{2x}}{\sqrt{10}}\right) + C$

(2) + (1)

Q3 a) $\int \frac{\ln x + 1}{\sqrt{9-(x\ln x)^2}}\,dx = \int \frac{du}{\sqrt{9-u^2}}$

(2) + (1) $= \sin^{-1}\left(\frac{x\ln x}{3}\right) + C$

b) $\int \frac{dx}{x\sqrt{7-x^8}} = \frac{1}{4}\int \frac{du}{u\sqrt{7-u^2}}$

(2) + (1) $= -\frac{1}{4\sqrt{7}} \operatorname{sech}^{-1}\left(\frac{x^4}{\sqrt{7}}\right) + C$

c.) $\int \frac{\sin x}{\sec x\sqrt{16\sin^4 x - 1}}\,dx = \frac{1}{8}\int \frac{du}{\sqrt{u^2-1}}$

(2) + (1) $= \frac{1}{8}\cosh^{-1}(4\sin^2 x)$

$+ C$