

Grading scheme

Ex 1

$$a) \sum_1^{10} b^2 - a \sum_1^{10} b = 0 \quad (0.5)$$

$$\frac{10(10+1)(21)}{6} - a \frac{10(10+1)}{2} = 0 \quad (2.5)$$

$$a = \frac{21}{3} = 7 \quad (0.5)$$

$$b) F(x) = \cos x^2 \cdot \int_0^{x^2} \cos^4 t \, dt$$

$$(1.5) F'(x) = -2x \sin x^2 \int_0^{x^2} \cos^4 t \, dt + \cos x^2 \cdot (\cos^4 x^2) \cdot 2x$$

$$(0.5) F'(0) = 0$$

$$c) P = \left\{ -1, -\frac{1}{2}, 0, \frac{1}{2}, 1 \right\} \quad (0.5)$$

$$S_4 = \frac{2}{3 \cdot 4} (f(-1) + 4f(-\frac{1}{2}) + 2f(0) + 4f(\frac{1}{2}) + f(1))$$

$$= \frac{1}{6} (\sqrt{2} + 4\sqrt{\frac{17}{16}} + 2 + 4\sqrt{\frac{17}{16}} + \sqrt{2}) \quad (2)$$

$$\approx 2.17911 \quad (0.5)$$

$$2) \quad a) \quad u = x^{5/4} - 4 \quad (1)$$

$$du = \frac{5}{4} x^{1/4} dx$$

$$\int x^{1/4} (x^{5/4} - 4)^3 dx = \frac{4}{5} \int u^3 du$$

$$= \frac{1}{5} u^4 + C$$

$$= \frac{1}{5} (x^{5/4} - 4)^4 + C \quad (2)$$

$$b) \quad u = 1 + \ln x \quad du = \frac{dx}{x} \quad (1)$$

$$\int_1^e \frac{\sqrt[3]{1 + \ln x}}{x} dx = \int_1^2 u^{1/3} du = \frac{3}{4} \left[u^{4/3} \right]_1^2 \quad (1.5)$$

$$= \frac{3}{4} (\sqrt[3]{16} - 1) \quad (2)$$

$$c) \quad y = x^{x^2} \quad \ln y = x^2 \ln x \quad (0.5)$$

$$\frac{y'}{y} = 2x \ln x + x$$

$$y' = (2x \ln x + x) x^{x^2} \quad (1.5)$$

$$= (2 \ln x + 1) x^{x^2 + 1}$$

$$3) \quad a) \quad u = \cos x \quad du = -\sin x \, dx \quad (1)$$

$$\int \frac{dx}{\csc x \sqrt{9 - \cos^2 x}} = - \int \frac{du}{\sqrt{9 - u^2}}$$
$$= -\sin^{-1}\left(\frac{u}{3}\right) + C$$
$$= -\sin^{-1}\left(\frac{\cos x}{3}\right) + C \quad (2)$$

$$b) \quad u = e^{5x} \quad du = 5e^{5x} \, dx \quad (1)$$

$$\int \frac{dx}{\sqrt{e^{10x} - 25}} = \int \frac{du}{5u \sqrt{u^2 - 5^2}}$$
$$= \frac{1}{25} \sec^{-1}\left(\frac{u}{5}\right) + C$$
$$= \frac{1}{25} \sec^{-1}\left(\frac{e^{5x}}{5}\right) + C \quad (2)$$

$$c) \quad u = \sqrt{x} \quad du = \frac{dx}{2\sqrt{x}} \quad (1)$$

$$\int \frac{dx}{\sqrt{x}(x+16)} = 2 \int \frac{du}{u^2 + 16}$$
$$= \frac{1}{2} \tan^{-1}\left(\frac{\sqrt{x}}{4}\right) + C \quad (2)$$