King Saud University
College of Sciences
Department of Mathematics
First Semester (1434/1435)

M-106
First Midterm-Exam

The Exam paper contains 5 pages

(5 Multiple choice questions and 5 Full questions)

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 25 Time: 90 minutes

Multiple Choice

Q.No:	1	2	3	4	5
$\{a, b, c, d\}$	a	a	a	a	c

Q. No: 1 The avarage value of $T(t) = 50 + 14\sin(\frac{\pi t}{12})$ over the interval $[0, 12]$ is equal to:

$(a) 50 + \frac{28}{\pi}$ $\quad (b) 600 + \frac{168}{\pi}$ $\quad (c) 50 - \frac{28}{\pi}$ $\quad (d) 600 + \frac{168}{\pi}$

Q. No: 2 The approximate value of $\int_0^4 2^x dx$ using simpson's rule with $n = 2$ is:

(a) 22 $\quad (b)$ 21.64 $\quad (c)$ 25 $\quad (d)$ 33

Q. No: 3 The derivatives of $f(x) = \ln(\cosh^{-1}(4x))$ is equal to:

(a) $\frac{1}{\cosh^{-1}(4x)} \frac{4}{\sqrt{(4x)^2 - 1}}$ $\quad (b)$ $\frac{1}{\cosh(x)} \frac{4}{\sqrt{(4x)^2 - 1}}$

(c) $\frac{4}{\sqrt{(\cosh^{-1}(4x))^2 - 1}}$ $\quad (d)$ $\frac{4}{\sqrt{(\cosh(4x))^2 - 1}}$

Q. No: 4 $\frac{d}{dx}\left(\int_{\cos x}^{3x} \cos(t^3)\, dt\right)$ is equal to:

$(a) 3\cos(27x^3) + \sin(x)\cos(\cos^3(x))$ $\quad (b)$ $\cos(27x^3) - \cos(\cos^3(x))$

$(c) \cos(27x^3) + \cos(\cos^3(x))$ $\quad (d) 3\cos(27x^3) - \sin(x)\cos(\cos^3(x))$

Q. No: 5 If $f(x) = \log(2x + 5)$, then $f'(-2)$ is equal to:

(a) 0 $\quad (b)$ $\frac{1}{\ln 10}$ $\quad (c)$ $\frac{2}{\ln 10}$ $\quad (d)$ ∞

Full Questions

Question No: 6 [4 marks] Evaluate the integral $\int \frac{e^x}{4 - e^{2x}} dx$,, where $e^x < 2$

$$= \int \frac{e^x}{(2)^2 - (e^x)^2} dx$$

Put $u = e^x$

$du = e^x dx$

$$= \int \frac{du}{4 - u^2} = \frac{1}{2} \tanh^{-1}(u/2) + c$$

$$= \frac{1}{2} \tanh^{-1}(e^x/2) + c$$

Question No: 7 [4 marks] If $f(x) = \left(x + \frac{1}{x}\right)^{x^2}$, then find $f'(x)$.

$$\ln|f(x)| = x^2 \ln\left(x + \frac{1}{x}\right)$$

$$\frac{1}{f(x)} f'(x) = (2x) \ln\left(x + \frac{1}{x}\right) + x^2 \left[\frac{1}{x + \frac{1}{x}} \left(1 - 1/x^2\right)\right]$$

Question No: 8 [4 marks] Evaluate the integral $\int \frac{2}{x\sqrt{1-\frac{\ln^2(x)}{4}}}dx$

$$= \int \frac{2}{x\sqrt{1-\left(\frac{\ln x}{2}\right)^2}}\, dx$$

Put $u = \frac{\ln x}{2}$

$du = \frac{1}{2x}\, dx$

$2\,du = \frac{1}{x}\, dx$

$$= \int \frac{4}{\sqrt{1-u^2}}\, du = 4 \sin^{-1}(u) + c$$

$$= 4 \sin^{-1}\left(\frac{\ln x}{2}\right) + c$$

Question No: 9 [4 marks] Evaluate the integral $\int \frac{e^x - e^{-x}}{e^{2x} + 2 + e^{-2x}}dx$.

$$= \int \frac{e^x - e^{-x}}{(e^x + e^{-x})^2}\, dx$$

$$= \int (e^x + e^{-x})^{-2}(e^x - e^{-x})\, dx$$

$$= \frac{(e^x + e^{-x})^{-1}}{-1} + c$$

Question No: 10 [4 marks] Let $f(x) = \frac{x^2}{2}$, and let P be the partition of $[0,6]$ into the five subintervals determided by:

$$x_0 = 0,\ x_1 = 1.5,\ x_2 = 2.5,\ x_3 = 4.5,\ x_4 = 5,\ x_5 = 6$$

a) Find the norm of the partition.

b) Find Riemann sum R_p, if for $k = 1,2,3,4,5$, $w_k \in [x_{k-1}, x_k]$ are given by:

$$w_1 = 1,\ w_2 = 2,\ w_3 = 3.5,\ w_4 = 5,\ w_5 = 5.5$$

$\Delta x_1 = 1.5,\ \Delta x_2 = 1,\ \Delta x_3 = 2,\ \Delta x_4 = 0.5,\ \Delta x_5 = 1$

(a) Norm $= \|P\| = 2$

$R_R = f(1)(1.5) + f(2)(1) + f(3.5)(2) + f(5)(0.5) + f(5.5)(1)$

$= \frac{1}{2}\left(\frac{3}{2}\right) + (2)(1) + \left(\frac{49}{4}\right)(2) + \frac{25}{2}\,\frac{1}{2} + \frac{121}{4}$

$= \frac{3}{4} + 2 + \frac{49}{2} + \frac{25}{4} + \frac{121}{4}$

$= \frac{3+4+98+25+121}{4} = \frac{251}{4} = 62.75$