King Saud University
Faculty of Sciences
Department of Mathematics

Final Examination	Math 106	Semester I - 1445
	Time: 3H	

Question 1 : (2+3+2)

1. Evaluate $\int \frac{\sinh(\ln x)}{x} dx$.
2. Find the number c in the mean value theorem for the function $f(x) = \frac{1}{\sqrt{x+1}}$ on $[3, 8]$.
3. Compute $\int \frac{\operatorname{sech}^2 x}{4 + \tanh^2 x} dx$.

Question 2 : (3+3+3)

1. Find the indefinite integral $\int \frac{dx}{x\sqrt{1-x^6}}$.
2. Compute $\lim_{x \longrightarrow +\infty} (1 + e^x)^{e^{-x}}$.
3. Evaluate $\int e^{2x} \sin x dx$.

Question 3 : (3+3+3)

1. Compute $\int \tan^2 x \sec^6 x dx$.
2. Find the integral $\int \frac{dx}{(x^2+4)^2}$.

3. Evaluate $\int \frac{2x-1}{x^2+2x+2}dx$.

Question 4 : (3+3+2)

1. Sketch the region bounded by $y = x^2 + 1$, $y = 3x + 1$ and find its area.

2. Find the volume of the solid obtained by revolving the region bounded by the curves $x = y^2 + 1$, $x = 10$ about the $y-$axis.

3. Find the arc length of the curve given by $y = 2x^{\frac{3}{2}} + 1$, $0 \leq x \leq 1$.

Question 5 : (3+4)

1. Find the area of the surface obtained by revolving the curve $y = \frac{1}{2}x^2$, $0 \leq x \leq 1$ about the $y-$axis.

2. Sketch the region inside $r = 3 + 3\sin\theta$ and outside $r = 3$ and find its area.

Solution of the final examination Math 106, Semester I 1445

Question 1 :

1. $\displaystyle\int \frac{\sinh(\ln x)}{x}dx \overset{u=\ln x}{=} \int \sinh u du = \cosh(\ln x)+c.$ **1+1**

2. $\displaystyle\frac{1}{5}\int_3^8 \frac{dx}{\sqrt{x+1}} = \frac{2}{5}$. Then $\sqrt{c+1}=\frac{5}{2} \iff c=\frac{21}{4}$. **1.5+1.5**

3.

$$\begin{aligned}\int \frac{\operatorname{sech}^2 x}{4+\tanh^2 x}dx &\overset{u=\tanh x}{=} \int \frac{du}{4+u^2} \\ &= \frac{1}{2}\tan^{-1}\left(\frac{\tanh x}{2}\right)+c. \qquad \textbf{1+1}\end{aligned}$$

Question 2 :

1. $\displaystyle\int \frac{dx}{x\sqrt{1-x^6}} \overset{t=x^3}{=} \frac{1}{3}\int \frac{dt}{t\sqrt{1-t^2}} = -\frac{1}{3}\operatorname{sech}^{-1}(x^3)+c$ **2+1**

2.

$$\begin{aligned}\lim_{x\longrightarrow+\infty}(1+e^x)^{e^{-x}} &= \lim_{x\longrightarrow+\infty} e^{\frac{\ln(1+e^x)}{e^x}} \\ &= \lim_{x\longrightarrow+\infty} e^{\frac{1}{1+e^x}} = 1. \qquad \textbf{1.5+1.5}\end{aligned}$$

3. By parts $\displaystyle\int e^{2x}\sin x dx = e^{2x}\left(-\frac{1}{5}\cos x+\frac{2}{5}\sin x\right)+c$ **1.5+1.5**

Question 3 :

1.

$$\int \tan^2 x \sec^6 x dx \overset{u=\tan x}{=} \int u^2(1+u^2)^2 du \quad \mathbf{1.5+1.5}$$
$$= \frac{1}{7}\tan^7(x) + \frac{2}{5}\tan^5(x) + \frac{1}{3}\tan^3(x) + c.$$

2.

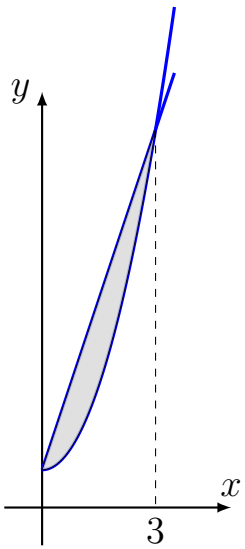
$$\int \frac{dx}{(x^2+4)^2} dx \overset{x=2\tan t}{=} \frac{1}{8}\int \cos^2 t dt$$
$$= \frac{1}{16}(t + \sin t \cos t) + c \quad \mathbf{1.5+1.5}$$
$$= \frac{1}{16}(\tan^{-1}(\frac{x}{2}) + \frac{2x}{4+x^2}) + c.$$

3.

$$\int \frac{2x-1}{x^2+2x+2} dx = \int \frac{2(x+1)-3}{(x+1)^2+1} dx \quad \mathbf{1.5+1.5}$$
$$= \ln(x^2+2x+2) - 3\tan^{-1}(x+1) + c. + c.$$

Question 4 :

1.



0.5+1.5

$$\int_0^3 (3x+1) - (x^2+1) dx = \frac{9}{2}.$$

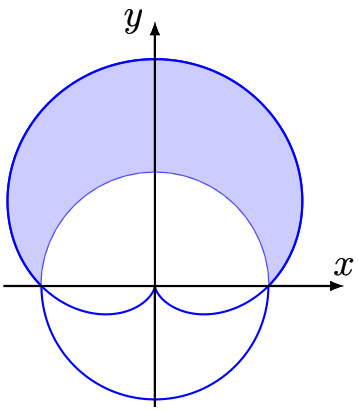
2. $V = \pi \int_{-3}^{3} 10^2 - (y^2+1)^2 dy = \frac{2304\pi}{5}$. **2+1**

3. $L = \int_0^1 \sqrt{1+9x}dx = \frac{2}{27}(10^{\frac{3}{2}} - 1)$. **1.5+0.5**

Question 5 :

1.

$$\begin{aligned} SA &= 2\pi \int_0^1 x\sqrt{1+x^2}dx = \frac{2\pi}{3}\left((1+x^2)^{\frac{3}{2}}\right]_0^1 \\ &= \frac{2\pi}{3}(2^{\frac{3}{2}} - 1). \quad \mathbf{1.5+1.5} \end{aligned}$$

2. 

(1)

$$\begin{aligned} A &= \frac{1}{2}\int_0^{\pi} (3\sin(\theta)+3)^2 - 9d\theta = \frac{9}{2}\int_0^{\pi} (\sin^2\theta + 2\sin\theta)d\theta \\ &= \frac{9}{2}(\frac{\pi}{2}+4). \quad \mathbf{1.5+1.5} \end{aligned}$$