CHAPTER 13

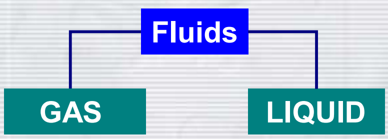
(Static Fluids) الموائع الساكنة

The Mechanics of Nonviscous Fluids

هناك ثلاث مراحل (مشتركة) للمادة:

1. الصلبة Solid : يحافظ على الشكل والحجم (تقريبا) ، حتى تحت قوى كبيرة.
2. السائل Liquid : لا شكل ثابت. يأخذ شكل الإناء الحاوى لها أي يكون لها حجم معين وتقاوم أي ضغط يقع عليها.
3. الغاز Gas : ليس لها شكل محدد كما أنها لا تأخذ شكل الإناء لها فحسب بل تشغله تماما وتتميز بأنها قابلة للانضغاط

تعرف الموائع Fluids بأنها المواد التي تتميز بقدرتها علي الانسياب ولاتتخذ شكلا محددا



Fluids present

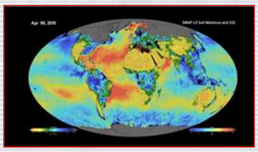
- Weather & climate
- Vehicles: automobiles, trains, ships, and planes, etc.
- Environment
- Physiology and medicine
- Sports & recreation
- Many other examples!

Weather & climate

Tornadoes

Hurricanes

Global Climate



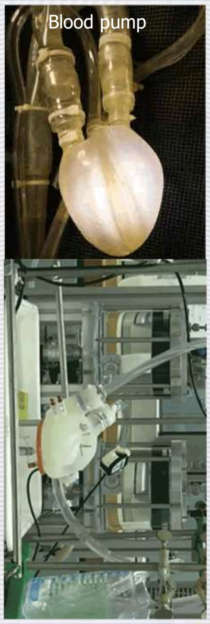
Thunderstorm

Vehicles

Environment

Air pollution

Physiology and Medicine



Sports & Recreation

Density and Pressure

دراسة حركة وتحريك أي جسم تتطلب معرفة كتلته،

دراسة ميكانيك السوائل والغازات ليست سهلة لأن تحديد كتلة الجسم عند دراسة حركة الرياح في الجو والأمواج في المحيطات والماء في الأنهار صعب.

ولذا نختار حجما معينا من السائل المدروس ونحدد كتلته ونطبق قوانين التحريك عليه ونعمم النتائج على السائل المدروس كله.

في السوائل ، يكون الإهتمام بخصائص يمكن أن تختلف من نقطة إلى أخرى.

وبالتالي ، من المفيد الحديث عن الكثافة والضغط بدلا من الكتلة والقوة.

$$\rho = \frac{m}{V} \quad \text{(uniform density)}$$

Density is a scalar, the SI unit is $kg/m^3 = 10^{-3}\ g/cm^3$.

وتعتمدعلى عدةعوامل كدرجة الحرارة والضغط وطبيعة المادة المعنية سواء كانت صلبة أم سائلة أم غازية. ولاتتغير كثافةالأجسام الصلبة والسائلة كثيرا بالمقارنة مع الغازات التي تتغير كثافتها بشكل ملحوظ مع درجةالحرارة والضغط

المادة	الكثافة (kg/m³)	المادة	الكثافة (kg/m3)
الألمنيوم	7.1	الماء (4 °C)	7
الزنبق	9.1	الهواء (0 °C)	6.1
النحاس	11	الهواء (25 °C)	14
الحديد	20	دم الإنسان (25 °C)	16
الفضة	35	ماء البحر	20
الرصاص	6.5-7.8	زيت الزيتون (15 °C)	5-5.5
الذهب	5.6	البنزين (15 °C)	2.7
الفولاذ	_	الجليد	0.21

TABLE Some Densities

Material or Object	Density (kg/m³)
Interstellar space	10^{-20}
Best laboratory vacuum	10^{-17}
Air: 20°C and 1 atm pressure	1.21
20°C and 50 atm	60.5
Styrofoam	1×10^{2}
Ice	0.917×10^{3}
Water: 20°C and 1 atm	0.998×10^{3}
20°C and 50 atm	1.000×10^{3}
Seawater: 20°C and 1 atm	1.024×10^{3}
Whole blood	1.060×10^{3}
Iron	7.9×10^{3}
Mercury (the metal)	13.6×10^{3}
Earth: average	5.5×10^{3}
core	9.5×10^{3}
crust	2.8×10^{3}
Sun: average	1.4×10^{3}
core	1.6×10^{5}
White dwarf star (core)	10^{10}
Uranium nucleus	3×10^{17}
Neutron star (core)	10^{18}
Black hole (1 solar mass)	10^{19}

105 PHYS Dr. Abdallah M.Azzeer

Solids (0.0°C)		Liquids (0.0°C)		Gases (0.0°C, 101.3 kPa)	
Substance	ρ(kg/m³)	Substance	ρ(kg/m³)	Substance	ρ(kg/m³)
Aluminum	2.70×10^{3}	Benzene	8.79×10^{2}	Air	1.29×10^{0}
Bone	1.90×10^{3}	Blood	1.05×10^{3}	Carbon dioxide	1.98×10^{0}
Brass	8.44×10^{3}	Ethyl alcohol	8.06×10^{2}	Carbon monoxide	1.25×10^{0}
Concrete	2.40×10^{3}	Gasoline	6.80×10^{2}	Helium	1.80×10^{-1}
Copper	8.92×10^{3}	Glycerin	1.26×10^{3}	Hydrogen	9.00×10^{-2}
Cork	2.40×10^{2}	Mercury	1.36×10^{4}	Methane	7.20×10^{-2}
Earth's crust	3.30×10^{3}	Olive oil	9.20×10^{2}	Nitrogen	1.25×10^{0}
Glass	2.60×10^{3}			Nitrous oxide	1.98×10^{0}
Gold	1.93×10^{4}			Oxygen	1.43×10^{0}
Granite	2.70×10^{3}				
Iron	7.86×10^{3}				
Lead	1.13×10^{4}				
Oak	7.10×10^{2}				
Pine	3.73×10^{2}				
Platinum	2.14×10^{4}				
Polystyrene	1.00×10^{2}				
Tungsten	1.93×10^{4}				
Uranium	1.87×10^{3}				

Densities of Some Common Substances

105 PHYS Dr. Abdallah M.Azzeer 8

ونظرا لأنه يمكن من الناحية التجريبية قياس الكتلة بشكل أكثر دقة من قياس الحجم
فإنه للتخلص من مصدر الخطأ في قياس الحجم لجأ العلماء إلى استخدام مفهوم الكثافة النسبية (Relative
Density) (النوعية) للمادة وتعرف بأنها النسبه بين كثافة المادة إلى كثافة الماء عند الصفر المئوي

$$\rho_{rel} = \frac{\rho_{obj}}{\rho_{water}}$$

الكثافة النسبية ليس لها وحدة

What do you think would happen to the substance in the water dependent on SG?

RD > 1 Sink in the water

RD < 1 Float on the surface

105 PHYS Dr. Abdallah M.Azzeer 9

105 PHYS Dr. Abdallah M.Azzeer 10

مثال: تحديد كثافة مادة غيرمعروفة

من المعروف أن أي سائلين لهما نفس الكثافة يمتزجان ببعضهما دون أن يطفو أحدهما فوق الآخر. وتستخدم هذه النتيجة لتحديد كثافة سوائل مجهولة.

ففي تجربة لقياس كثافة سائل مجهول يخلط بنزين ($\rho=874\ kg/m^3$) وكلوروفورم ($\rho=1527kg/m^3$) بنسبة 78 % و 22% من الحجم الكلي، على التوالي،

يلاحظ أن قطرات من السائل المجهول تصبح معلقة في الخليط. ماكثافته؟

إذا كانت قطرات السائل المجهول معلقة تكون كثافتها مساوية لكثافة خليط البنزين والكلوروفورم .

نفترض أن حجم البنزين V_1 وحجم الكلوروفورم V_2

الحجم الكلي : $V_T=V_1+V_2$

$M_1=\rho_1 V_1$, $M_2=\rho_2 V_2$

$$M_T=\rho_1 V_1+\rho_2 V_2$$

وتصير كثافة الخليط

$$\rho=\frac{M_T}{V_T}=\frac{\rho_1 V_1+\rho_2 V_2}{V_1+V_2}=\rho_1\left(\frac{V_1}{V_T}\right)+\rho_2\left(\frac{V_2}{V_T}\right)$$

وبتعويض $V_1=0.78V_T$ و $V_2=0.22V_T$ و $\rho_1=874\ kg/m^3$ و $\rho_2=1527\ kg/m^3$ نجد:

$$\rho=1017.7\ kg/m^3$$

105 PHYS Dr. Abdallah M.Azzeer 11

Pressure

$$p=\frac{F}{A}\quad \text{(pressure of uniform force on flat area)}$$

- F is the magnitude of the normal force on area A.
- The SI unit of pressure is N/m^2, called the pascal (Pa).
- The tire pressure of cars are in kilopascals.

units :
1 N/m^2 = 1 Pa (Pascal)
1 bar = 10^5 Pa
1 mbar = 10^2 Pa
1 torr = 133.3 Pa

1 atm = 1.013×10^5 Pa
= 76 cm Hg
=1013 mbar
= 760 Torr
= 14.7 lb/m^2 (=PSI)

- The torr (named for Evangelista Torricelli, who invented the mercury barometer) is equal to 1 mm of Hg.

105 PHYS Dr. Abdallah M.Azzeer

TABLE Some Pressures

	Pressure (Pa)
Center of the Sun	2×10^{16}
Center of Earth	4×10^{11}
Highest sustained laboratory pressure	1.5×10^{10}
Deepest ocean trench (bottom)	1.1×10^{8}
Spike heels on a dance floor	1×10^{6}
Automobile tire[a]	2×10^{5}
Atmosphere at sea level	1.0×10^{5}
Normal blood pressure[ab]	1.6×10^{4}
Best laboratory vacuum	10^{-12}

[a] Pressure in excess of atmospheric pressure.
[b] The systolic pressure, corresponding to 120 torr on the physician's pressure gauge.

Example ;

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.

(a) What does the air in the room weigh when the air pressure is 1.0 atm?

(b) What is the magnitude of the atmosphere's force on the floor of the room?

SOLUTION:

$$mg = (\rho V)g \quad \text{(Use } \rho \text{ of air from Table)}$$
$$= (1.21\, kg/m^3)(3.5\, m \times 4.2\, m \times 2.4\, m)(9.8\, m/s^2) = 418\, N \approx 420\, N$$

This is the weight of about 110 cans of soda.

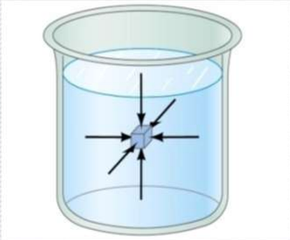
(b) $$F = pA = (1.0\, atm)\left(\frac{1.01 \times 10^5\, N/m^2}{1.0\, atm}\right)(3.5\, m)(4.2\, m) = 1.5 \times 10^6\, N$$

This enormous force is equal to the weight of the column of air that covers the floor and extends all the way to the top of the atmosphere.

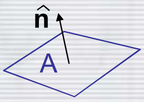
(Need to understand the Kinetic Theory of gases)

Pressure of a Fluids

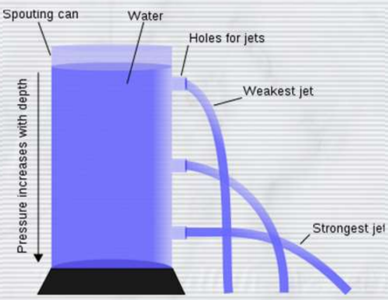
- What parameters do we use to describe fluids?
- Matter can exert the same force in all directions.
- The ratio of the force exerted to the area is the pressure.



$$\vec{F} = pA\hat{n}$$



- It's a scalar, not a vector

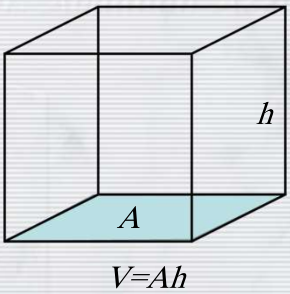


105 PHYS Dr. Abdallah M.Azzeer

- Examine the area at the bottom of fluid
  - It has a cross-sectional area A
  - Extends to a depth h below the surface
- Force act on the region is the weight of fluid

$$P = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h$$

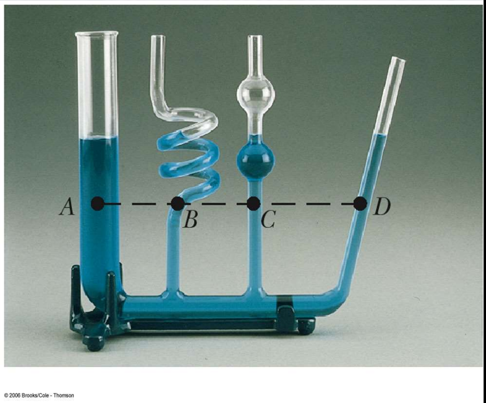
$$P = \rho g h$$



105 PHYS Dr. Abdallah M.Azzeer

Pressure and Depth equation

$$P = P_a + \rho g h$$



- P_a is normal atmospheric pressure
  - 1.013×10^5 Pa = 14.7 lb/in^2 (psi)
- The pressure does not depend upon the shape of the container

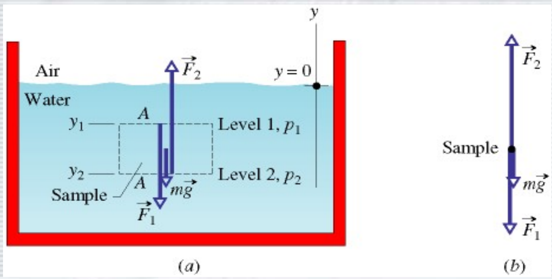
105 PHYS Dr. Abdallah M.Azzeer

Fluids at Rest

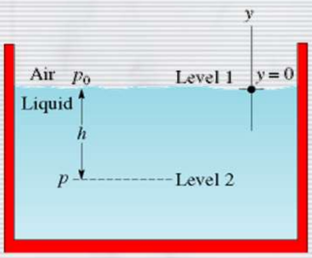
$F_2 = F_1 + mg$

$F_1 = P_1 A$ and $F_2 = P_2 A$

$P_2 A = P_1 A + \rho g A h$



- If we choose $P_1 = P_0$ and $h = y_1 - y_2$ then we get
  $P = P_0 + \rho g h$ (pressure at depth h)
- The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container.
- P is the absolute pressure, P_0 is the atmospheric pressure and $(P - P_0)$ is the gauge pressure.



$$P = P_0 + \rho g h$$

105 PHYS Dr. Abdallah M.Azzeer

Example **Finding absolute and gauge pressures**

Water stands 12.0 m deep in a storage tank whose top is open to the atmosphere. What are the absolute and gauge pressures at the bottom of the tank?

SOLUTION

IDENTIFY and SET UP: Table indicates that water is nearly incompressible, so we can treat it as having uniform density. The level of the top of the tank corresponds to point 2 in Fig. 12.5, and the level of the bottom of the tank corresponds to point 1. Our target variable is p in Eq. (12.6). We have $h = 12.0$ m and $p_0 = 1$ atm $= 1.01 \times 10^5$ Pa.

EXECUTE: From the pressures are absolute:

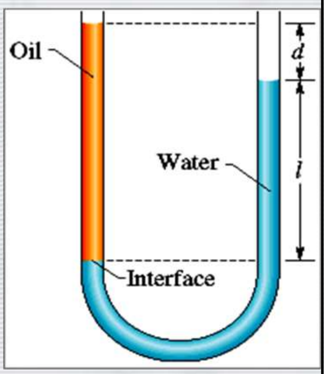
$$p = p_0 + \rho g h$$
$$= (1.01 \times 10^5 \text{ Pa}) + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(12.0 \text{ m})$$
$$= 2.19 \times 10^5 \text{ Pa} = 2.16 \text{ atm} = 31.8 \text{ lb/in.}^2$$

gauge:
$$p - p_0 = (2.19 - 1.01) \times 10^5 \text{ Pa}$$
$$= 1.18 \times 10^5 \text{ Pa} = 1.16 \text{ atm} = 17.1 \text{ lb/in.}^2$$

EVALUATE: A pressure gauge at the bottom of such a tank would probably be calibrated to read gauge pressure rather than absolute pressure.

Example ;

The U-tube in Figure contains two liquids in static equilibrium: Water of density ρ_w (= 998 kg/m^3) is in the right arm, and oil of unknown density ρ_x is in the left. Measurement gives l = 135 mm and d = 12.3 mm. What is the density of the oil?



SOLUTION:

We equate the pressure in the two arms

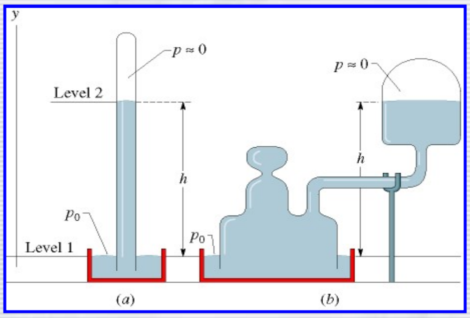
at the level of the interface :

$$P_{int} = p_0 + \rho_w\, g\, l \quad (right\ arm)$$

$$P_{int} = p_0 + \rho_x\, g\,(l + d) \quad (left\ arm)$$

$$\rho_x = \rho_w \frac{1}{l+d} = (998\, kg/m^3)\frac{135\, mm}{135\, mm + 12.3\, mm} = 915\, kg/m^3$$

The Mercury Barometer



$$p_0 = \rho g h$$

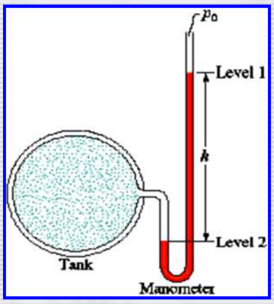
For normal atmospheric pressure, h is 76 cm Hg.

The Open Tube Manometer

- **The gauge pressure is the difference between the absolute pressure and the atmospheric pressure.**

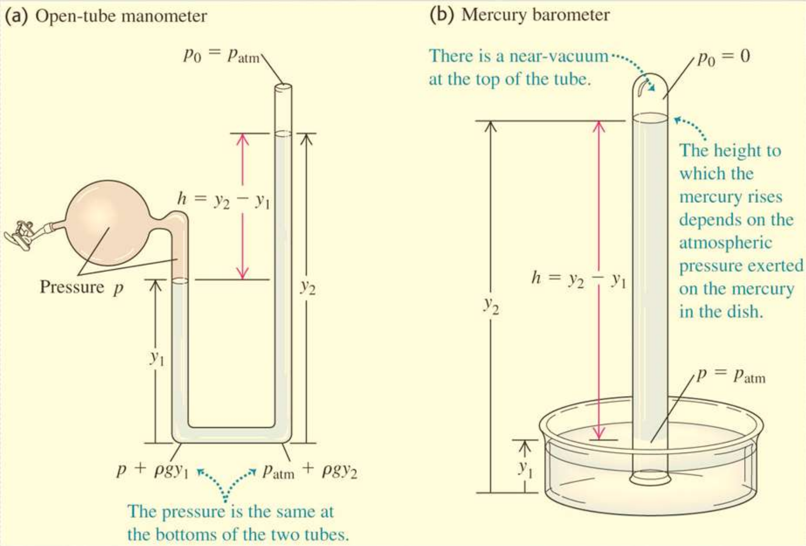
$$p_g = p_A - p_0 = \rho g h$$

- **The gauge pressure is directly proportional to h. It can be positive or negative depending on whether the absolute pressure is greater or less than the atmospheric pressure.**
- **We can suck fluids up a straw because at that time the absolute pressure in the lungs is less than the atmospheric pressure.**



Two types of pressure gauge

- Figure below shows two types of gauges for measuring pressure.



105 PHYS Dr. Abdallah M.Azzeer

Example ;

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

$$P - P_0 = \rho_W gh = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 \, Pa$$

Estimating the surface area of the eardrum at $1.0 cm^2 = 1.0 x 10^{-4} m^2$, we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9 \, N$$

105 PHYS Dr. Abdallah M.Azzeer

مثال (2-6)

غواصة تستطيع أن تغوص إلى عمق أقصاه 1000 m تحت سطح البحر. احسب أقصى ضغط يتحمله غلافها الخارجي إذا كان الضغط الجوي يعادل 76 سم زئبق وكثافة ماء المحيط $1.3\ gm/cm^3$ وكثافة الزئبق $13.6\ gm/cm^3$.

الحل:

أقصى ضغط تستطيع أن تتحمله الغواصة عبارة عن الضغط الجوي مضاف إليه ضغط عمود الماء الذي طوله 1000 m.

أولاً: حساب الضغط الجوي:

$$P_0 = \rho_{Hg}\, g\, h_0 = 13.6 \times \underline{1000} \times 9.8 \times 0.76 = 1.013 \times 10^5\ N/m^2$$

ثانياً: حساب ضغط عمود الماء:

$$P_1 = \rho_w\, g\, h = 1.3 \times 1000 \times 9.8 \times 1000 = 1.274 \times 10^7\ N/m^2$$

ويكون الضغط الكلي هو مجموع كل من P_0 ، P_1:

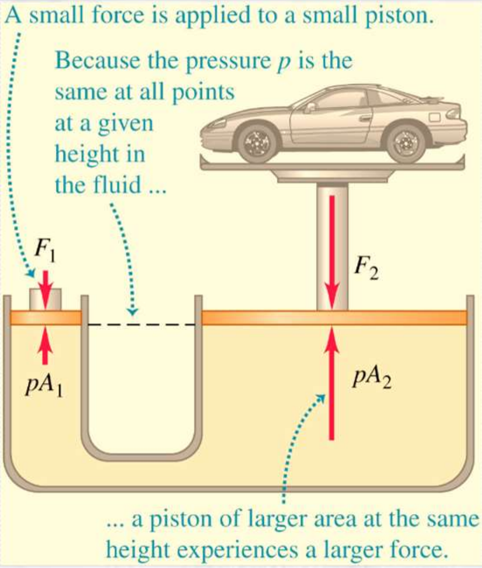
$$P = P_o + P1 = 1.284 \times 10^7\ N/m^2$$

Pascal's law

- *Pascal's law*: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.
- The hydraulic press is an important application of Pascal's Principle

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

- Also used in hydraulic brakes, forklifts, car lifts, etc



ترفع شاحنة كتلتها 22500 kg في مغسلة على ذراع رافعة هيدروليكية قطرها 30 cm. ما القوة اللازم تطبيقها على الذراع الأخرى إذا كان قطرها 10 cm وما ضغط الزيت هناك؟

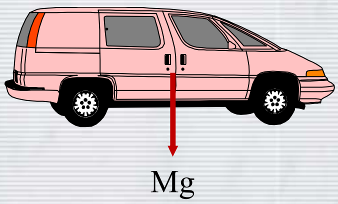
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \left(\frac{A_2}{A_1}\right)F_1 = \frac{\pi(0.05\,\text{m})^2}{\pi(0.15\,\text{m})^2}(22500\ \text{N}) = 2500\ \text{N}$$

وهذه القوة تعادل وزن كتلة 225 kg فقط. ونجد ضغط الزيت بكتابة:

$$p_2 = \frac{F_2}{A_2} = \frac{2500\ \text{N}}{\pi(0.05\ \text{m})^2} = 3.18 \times 10^5\ \text{Pa}$$

اذا كان وزن سيارة يستند بالتساوي علي أربعة إطارات وكل إطار له مساحة تلامس مع الأرض تساوي 150 cm^2

والضغط القياسي (Gauge Pressure) في الإطارات يساوي 1.8 bar فماهي كتلة السيارة؟ (1 bar = 10^5 Pa)



$A = 150\ \text{cm}^2$; $P_G = 1.8\ \text{bar} = 1.8\times10^5\ \text{Pa}$

حيث أن للسيارة أربعة إطارات فإن كل إطار سوف يسند ربع وزن السيارة وبالتالي يكون الضغط الناتج علي كل إطار يساوي

$P = F/A = Mg/A$

والضغط الناتج علي كل الإطارات الأربعة يساوي

$P = Mg/4A$

$\therefore\ M = 4PA/g = 4(1.8\times10^5)(150\times10^{-4})/9.8 = 1102\ \text{kg}$

Calculating Force on Wheel Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in Figure 14.18. Suppose a force of 100 N is applied to the brake pedal, which acts on the pedal cylinder (acting as a “master” cylinder) through a lever. A force of 500 N is exerted on the pedal cylinder. Pressure created in the pedal cylinder is transmitted to the four wheel cylinders. The pedal cylinder has a diameter of 0.500 cm and each wheel cylinder has a diameter of 2.50 cm. Calculate the magnitude of the force F_2 created at each of the wheel cylinders.

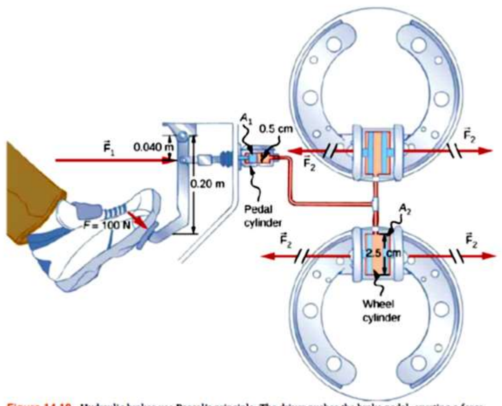


Figure 14.18 Hydraulic brakes use Pascal's principle. The driver pushes the brake pedal, exerting a force that is increased by the simple lever and again by the hydraulic system. Each of the identical wheel cylinders receives the same pressure and, therefore, creates the same force output F_2. The circular cross-sectional areas of the pedal and wheel cylinders are represented by A_1 and A_2, respectively.

We are given the force F_1 applied to the pedal cylinder. The cross-sectional areas A_1 and A_2 can be calculated from their given diameters. Then we can use the following relationship to find the force F_2:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

Manipulate this algebraically to get F_2 on one side and substitute known values.

Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

$$F_2 = \frac{A_2}{A_1}F_1 = \frac{\pi r_2^2}{\pi r_1^2}F_1$$

$$= \frac{(1.25\text{ cm})^2}{(0.250\text{ cm})^2} \times 500\text{ N} = 1.25 \times 10^4\text{ N}.$$

Fluid flow

- *Main difference between a moving fluid and a static fluid is that a moving fluid exerts a force parallel to a surface.*
- **Steady flow – velocity of fluid "particles" constant as time passes (laminar).**
- **Unsteady flow – the velocity at a point changes as time passes.**
  - ***Turbulent flow*** **- extreme kind of unsteady flow. The velocity changes erratically from moment to moment both in time and direction.**

Fluid flow can be compressible or incompressible.
- Most liquids virtually incompressible.
- Most gases are compressible.

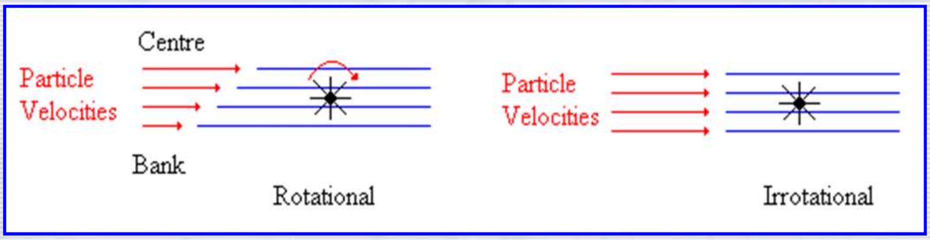
(certain situations can be treated as incompressible)

Fluid Flow can be viscous or nonviscous.
- Viscous (has large viscosity).
- Water is less viscous (low viscosity) and flows more readily.


Vortex shedding behind cylinder

Ideal Fluid is incompressible and nonviscous

Fluid flow can be rotational or irrotational



- **Rotational flow occurs when fluid has a rotational as well as a translational motion.**

Streamlines

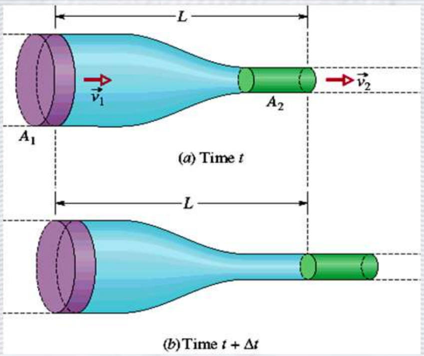
- **Streamlines – used in steady fluid flow to represent the trajectories of the fluid particles**

Fluids in Motion

The Equation of Continuity

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 \quad \text{(equation of continuity)}$$



$$Q = Av = \text{constant} \quad \text{(volume flow rate, equation of continuity)}$$

- The quantity Q is the volume flow rate (volume per unit time). The SI unit is m^3/s.
- If the density of the fluid is uniform, the mass flow rate Q_m (mass per unit time) is constant.

$$Q_m = \rho Q = \rho Av = \text{constant} \quad \text{(mass flow rate)}$$

- The SI unit of Q is kg/s

105 PHYS — Dr. Abdallah M.Azzeer

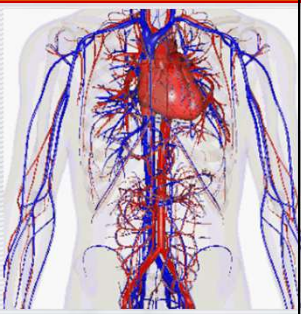
Example

The cross-sectional area A_0 of the aorta (the major blood vessel emerging from the heart) of a normal resting person is 3 cm^2, and the speed v_0 of the blood through it is 30 cm/s. A typical capillary (diameter ≈ 6 μm) has a cross-sectional area A of 3 x 10^{-7} cm^2 and a flow speed v of 0.05 cm/s. How many capillaries does such a person have?

$$A_0 v_0 = nAv$$

$$n = \frac{A_0 v_0}{Av} = \frac{(3\ cm^2)(30\ cm/s)}{(3\ x\ 10^{-7}\ cm^2)(0.05\ cm/s)}$$

$$= 6\ x\ 10^9 \quad or \quad 6\ billion$$



The combined cross-sectional area of the capillaries is about 600 times the cross-sectional area of the aorta.

105 PHYS — Dr. Abdallah M.Azzeer

يسري الماء من صنبور مساحة مقطعه 4 cm^2 بمعدل ستة لترات في الدقيقة ، إذا وُصّل الصنبور بأنبوب نصف قطره 1.6 cm ، فأحسب:
(i) سرعة الماء في الانبوب. (ii) سرعة الماء الخارجة من الصنبور.

$Q = v_1 A_1 = v_2 A_2$

$6/(10^3 \times 60)\ m^3/s = v_1 \times (4/10^4) = v_2\,(3.142)\,(1.6/10^2)^2$

(i) $v_1 = 0.250$ m/s

(ii) $v_2 = 0.124$ m/s

حوض سباحة عمقه 6 m ، إذا كان الضغط عند عمق 4.8 m اسفل سطح الحوض يساوي 1.5×10^5 Pa ، فاحسب: (i) الضغط الجوي.(ii) الضغط عند قاع الحوض .
علما بأن كثافة الماء (ρ_{water} =1000 kg/m^3)

$P_p = P_{atm} + \rho g d$

$1.5 \times 10^5\ Pa = P_{atm} + (1000)(9.8)(4.8)\ Pa$

$P_b = P_{atm} + (1000)(9.8)(6)\ Pa$

(i) $P_{atm} = 1.0296\ 10^5$ Pa (ii) $P_b = 1.617\ 10^5$ Pa.

105 PHYS Dr. Abdallah M.Azzeer 35

يسري الماء من لي مزرعة نصف قطره الداخلي 6 mm بسرعة مقدارها 2.5 m/s

(1) كم يجب أن يكون نصف قطر فتحة اللي لكي يخرج الماء بسرعة 10 m/s

(2) احسب الزمن اللازم لملء وعاء سعته 34 L [1 L = 10^{-3} m^3]

(1)

$r_1 = 6$ mm ; $v_1 = 2.5$ m/s
$r_2 = ?$; $v_2 = 10$ m/s
$A_1 v_1 = A_2 v_2 \Rightarrow (\pi r_1^2) v_1 = (\pi r_2^2) v_2 \quad \Rightarrow r_2 = r_1 \sqrt{\frac{v_1}{v_2}} = (6\ mm)\sqrt{\frac{2.5\ m/s}{10\ m/s}} = 3\ mm$

(2) حيث أن معدل السريان هو R=Av فإن:

$R = A_1 v_1 = (\pi r_1^2) v_1 = (2.5)\left[\pi (6\times10^{-3})^2\right] = 5.65\times10^{-4}\ m^3/s$

بالطبع يمكن الحصول على نفس النتيجة من $R=A_2v_2$

وحيث أن 1 L = 10^{-3} m^3

$\therefore R = 5.65\times10^{-4} \times 10^3$ L/s = 0.565 L/s

لملء وعاء سعته 34 L فإنه يحتاج إلي زمن

$t = \frac{34\ L}{0.565\ L/s} = 60\ s = 1\ min$

105 PHYS Dr. Abdallah M.Azzeer 36

Example;

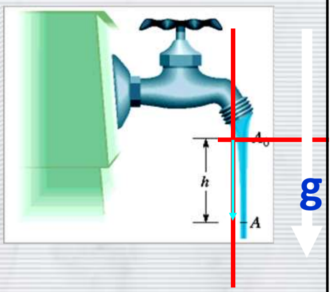
Figure shows how the stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are A_0 = 1.2 cm^2 and A = 0.35 cm^2. The two levels are separated by a vertical distance h = 45 mm. What is the volume flow rate from the tap?

$$A_0 v_0 = A v \qquad \because\; v^2 = v_0^2 + 2gh$$

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$= \sqrt{\frac{(2)(9.8\,m/s^2)(0.045\,m)(0.35\,cm^2)^2}{(1.2\,cm^2)^2 - (0.35\,cm^2)^2}}$$

$$= 0.286\,m/s = 28.6\,cm/s$$



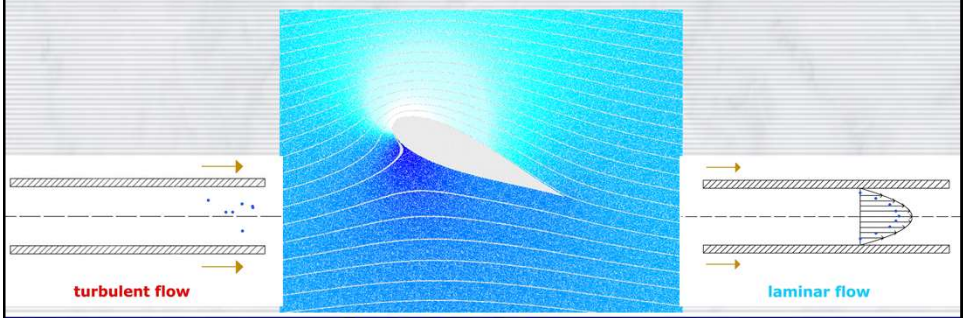
The volume flow rate is :

$$Q_v = A_0 v_0 = (1.2\,cm2)(28.6\,cm/s) = 34\,cm^3/s$$

105 PHYS — Dr. Abdallah M.Azzeer

Ideal Fluids

- Laminar Flow
  - No turbulence
- Non-viscous
  - No friction between fluid layers
- Incompressible
  - Density is same everywhere

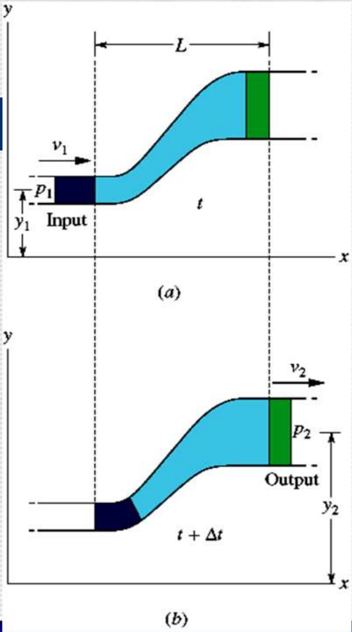


105 PHYS — Dr. Abdallah M.Azzeer

Bernoulli's Equation

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant} \quad \text{(Bernoulli's equation)}$$

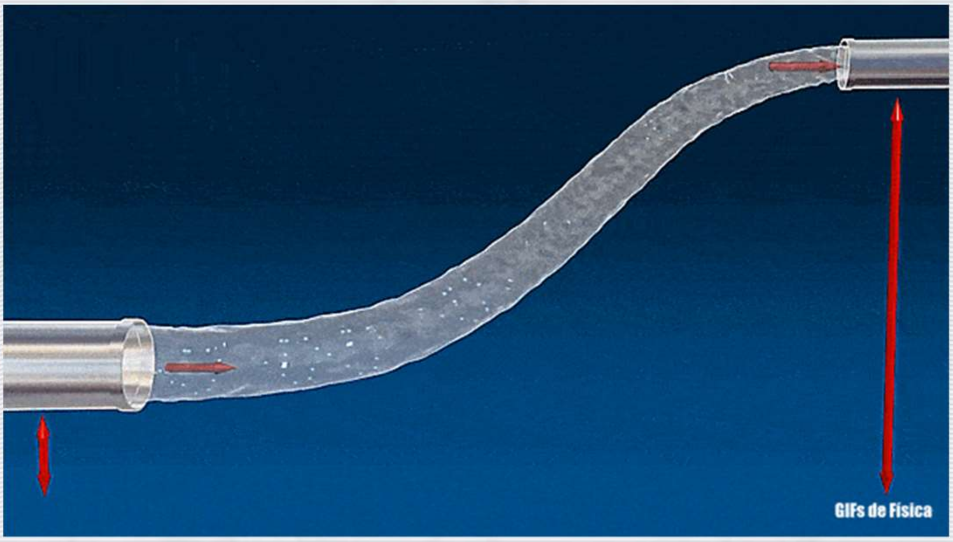


- If $y_1 = y_2$, then

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

- If the speed of a fluid element increases as it travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.
- Large speed means small pressure.

105 PHYS Dr. Abdallah M.Azzeer



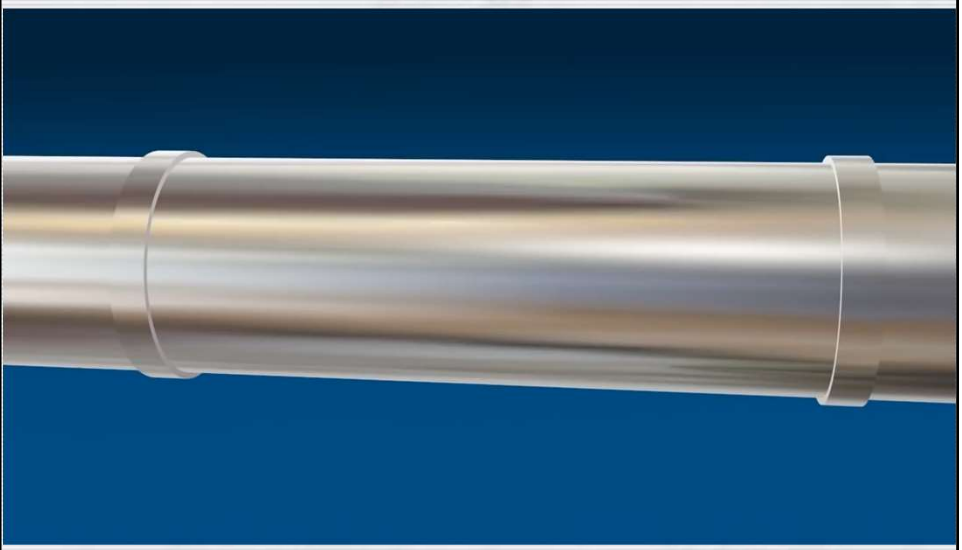
105 PHYS Dr. Abdallah M.Azzeer 40

Proof of Bernoulli's Equation

- **Work done on system = (K.E. + P. E.) gained**
- **Work done on system at** $p_1 = F_1 \Delta x_1 = p_1 A_1 \Delta x_1 = p_1 \Delta V$
- **Work done by system at** $p_2 = p_2 A_2 \Delta x_2 = p_2 \Delta V$
- $\Delta V = A_1 \Delta x_1 = A_2 \Delta x_2$
- **Net work done on system =** $(p_1 - p_2)\Delta V$
- **Net K.E. gained =** $\frac{1}{2}\rho (v_2^2 - v_1^2)\Delta V$
- **Net P.E. gained =** $\rho g (y_2 - y_1)$
- **Therefore,**

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

105 PHYS Dr. Abdallah M.Azzeer



105 PHYS Dr. Abdallah M.Azzeer 42

Example

Ethanol of density ρ = 791 kg/m^3 flows smoothly through a horizontal pipe that tapers in cross-sectional area from A_1 = 1.20 x 10^{-3} m^2 to $A_2 = A_1/2$. The pressure difference between the wide and narrow sections of pipe is 4120 Pa. What is the volume flow rate Q_V of the ethanol?

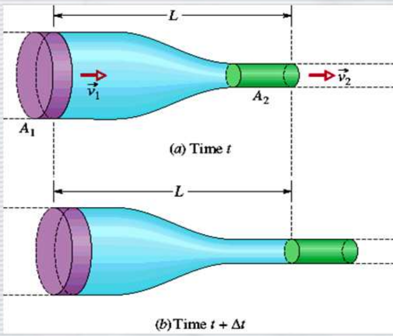
$$Q_v = v_1 A_1 = v_2 A_2$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y$$

$$v_1 = \frac{Q_v}{A_1} \quad and \quad v_2 = \frac{Q_v}{A_2} = \frac{2Q_v}{A_1}$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

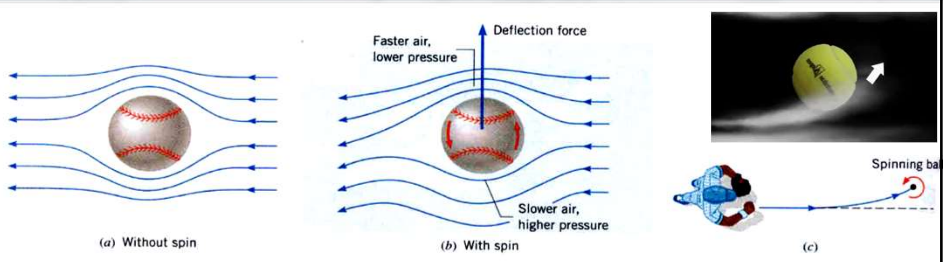
$$\frac{2(p_1 - p_2)}{\rho} = v_2^2 - v_1^2 = \frac{3Q_v^2}{A_1^2} \qquad Q_v = A_1\sqrt{\frac{2(p_1 - p_2)}{3\rho}}$$



The lower speed v_1 means that p_1 is greater, we have :

$$Q_v = 1.20 \ x \ 10^{-3} \ m^2 \sqrt{\frac{(2)(4120 \ Pa)}{(3)(791 kg/m^3)}}$$

105 PHYS Dr. Abdallah M.Azzeer

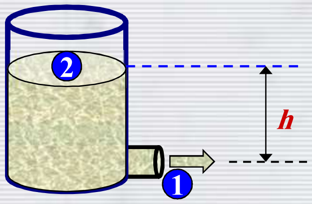


These views of a baseball are from above, looking down toward the ground, with the ball moving to the right. (a) Without spin, the ball does not curve to either side. (b) A spinning ball curves in the direction of the deflection force. (c) The spin in part b causes the ball to curves as shown here.

105 PHYS Dr. Abdallah M.Azzeer

Torricelli's equation

A tank is open to atmosphere at the top. What is the velocity of the liquid leaving the pipe at the bottom (assume ideal fluid).



Apply Bernoullis equation

$P_1 + \rho g h_1 + ½\rho v_1^2 = P_2 + \rho g h_2 + ½\rho v_2^2$

Since at (1) and (2) $P_1 = P_2 = P_{atm}$

$\rho g h_1 + ½\rho v_1^2 = \rho g h_2 + ½\rho v_2^2$ dividing by density and since $h_2 - h_1 = h$

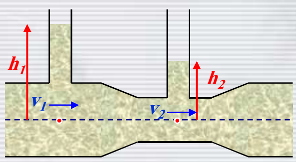
$v_1^2 = v_2^2 + 2gh$

If rate of fall of surface very slow can set $v_2 = 0$.

$$v_1 = \sqrt{2gh}$$ Torricelli's equation

Venturi Tube - application of Bernoullis equation to measure fluid flow

- Fluid flows through different cross sectional areas in different portions of the tube.
- As tube narrows (2), velocity of fluid increases thus dropping pressure
- Velocity of the fluid can be found by measuring the pressure



Apply Bernoulli's equation to points at the same height in the flow stream just below the columns.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

From the continuity equation: $A_1 v_1 = A_2 v_2$ So: $v_2 = \frac{A_1}{A_2} v_1$

Hence, $P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$

Thus measuring P_1-P_2 and a knowledge of the areas determines v_1

v_2 can then be found from the continuity equation

Example

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second 5.0 m above? Assume the pipes do not divide into branches.

Using the equation of continuity, flow speed on the second floor is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = 0.5 \times \left(\frac{0.020}{0.013}\right)^2 = 1.2\, m/s$$

Using Bernoulli's equation, the pressure in the pipe on the second floor is

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(y_1 - y_2\right)$$

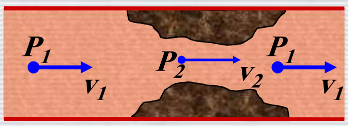
$$= 3.0\times10^5 + \frac{1}{2}1\times10^3\left(0.5^2 - 1.2^2\right) + 1\times10^3\times9.8\times(-5)$$

$$= 2.5\times10^5\, N/m^2$$

Example Vascular/Heart Flutter

If artery area A_1 is reduced by factor of 5 (A_2 = 1/5 A_1) what is the effect on the arterial blood pressure? (Blood density 1060 kgm^{-3} , person lying down, velocity in a healthy artery is $0.3ms^{-1}$).

From the continuity equation:



$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = 5v_1 = 5(0.3ms^{-1}) = 1.5ms^{-1}$$

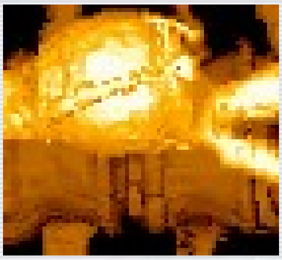
Applying Bernoulli's equation ($y_1 = y_2$ since horizontal)

$$P_1 + ½\rho v_1^2 = P_2 + ½\rho v_2^2$$

Pressure difference $= P_2 - P_1 = ½\rho(v_1^2 - v_2^2)$

$$= ½(1060kgm^{-3})(0.3^2 - 1.5^2) = -1145\ Pa$$

Applications of Bernoullis equation

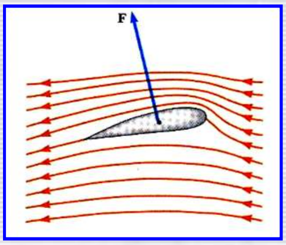


Aortic Aneurysm:

Opposite effect to heart flutter:

The blood will travel more slowly through the aneurysm thus increasing pressure.

$P_1 + ½\rho v_1^2 = P_2 + ½\rho v_2^2$



Air wing designed so that air velocity above wing faster than below.

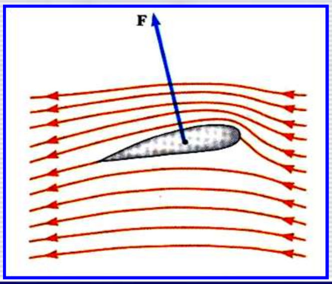
Hence above wing has lower pressure thus get lift.

Angle of attack

105 PHYS Dr. Abdallah M.Azzeer 49

The Flight of Animals and Aeroplanes

- A full discussion requires a combination of elaborate mathematics and experimental data
- Bernoulli's equation can derive some qualitative results about flying.
- Note air around plane disturbed only briefly. Not steady state. To person on plane airflow around wing approx. steady state. Hence take coordinate frame at rest relative to plane.



105 PHYS Dr. Abdallah M.Azzeer 50

Applying Bernoulli above & below the wing

Since $v_a > v_b$ then the pressure $P_a < P_b$.

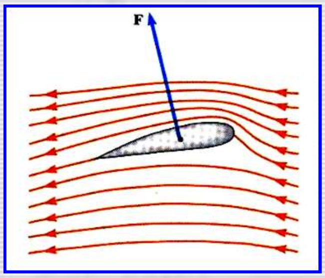
Note the ρgh terms can be ignored because the wing is thin.

From Bernoullis equation can write:

$$P_b - P_a = \frac{1}{2}\rho\left(v_a^2 - v_b^2\right)$$

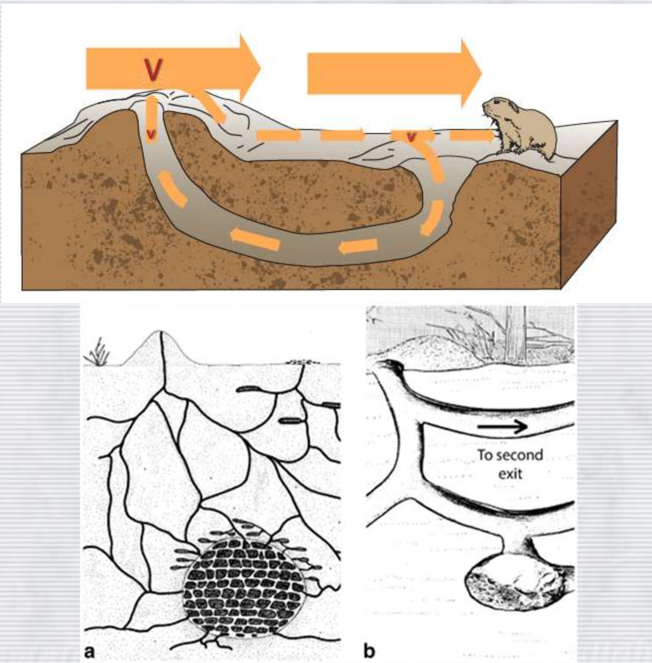
Multiplying by Area can get the lift force F_L

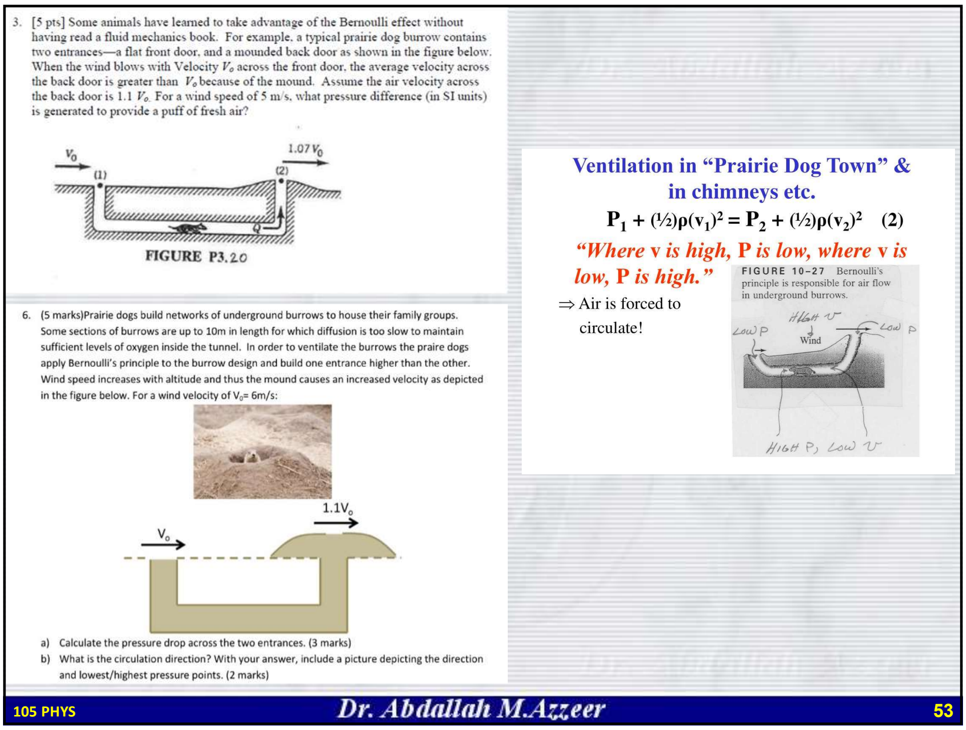
$$F_L = A(P_b - P_a) = A\frac{1}{2}\rho\left(v_a^2 - v_b^2\right)$$



Note v_a and v_b are proportional to the air velocity v and this equation can be rewritten in terms of a proportionality factor C_L called the lift coefficient (which is dimensionless)

$$F_L = AC_L\frac{1}{2}\rho v^2$$




3. [5 pts] Some animals have learned to take advantage of the Bernoulli effect without having read a fluid mechanics book. For example, a typical prairie dog burrow contains two entrances—a flat front door, and a mounded back door as shown in the figure below. When the wind blows with Velocity V_o across the front door, the average velocity across the back door is greater than V_o because of the mound. Assume the air velocity across the back door is 1.1 V_o. For a wind speed of 5 m/s, what pressure difference (in SI units) is generated to provide a puff of fresh air?
V_0
(1)
$1.07 V_0$
(2)
Q
FIGURE P3.20
6. (5 marks)Prairie dogs build networks of underground burrows to house their family groups. Some sections of burrows are up to 10m in length for which diffusion is too slow to maintain sufficient levels of oxygen inside the tunnel. In order to ventilate the burrows the praire dogs apply Bernoulli's principle to the burrow design and build one entrance higher than the other. Wind speed increases with altitude and thus the mound causes an increased velocity as depicted in the figure below. For a wind velocity of V_0= 6m/s:
$1.1V_o$
V_o
a) Calculate the pressure drop across the two entrances. (3 marks)
b) What is the circulation direction? With your answer, include a picture depicting the direction and lowest/highest pressure points. (2 marks)
Ventilation in "Prairie Dog Town" & in chimneys etc.
$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2$ (2)
"Where v is high, P is low, where v is low, P is high."
⇒ Air is forced to circulate!
FIGURE 10–27 Bernoulli's principle is responsible for air flow in underground burrows.
HIGH v
LOW P
Wind
LOW P
HIGH P, LOW v
105 PHYS
Dr. Abdallah M.Azzeer
53