Final Exam - Allowed time: 3 hours

Calculators are not permitted

Q1.

[3] (a) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$. Compute, if possible, AB and BA.

[3] (b) Compute the determinant $\begin{vmatrix} -1 & 6 & 2 \\ 0 & 0 & 5 \\ 0 & 3 & 4 \end{vmatrix}$.

[4] (c) Solve by using Gauss Elimination Method the linear system

$$\begin{cases} x + y + 3z &= 7 \\ -2x - y - z &= -4 \\ 3x + 2y - 2z &= -1 \end{cases}$$

Q2.

[4] (a) Find the standard equation of the ellipse with foci $(3, 6)$ and $(3, -2)$ and vertex $(3, -3)$ and then sketch it.

[4] (b) Find the elements of the conic section $9x^2 - 4y^2 - 18x - 24y + 9 = 0$ and then sketch it.

Q3.

[2,3,3] (a) Compute the integrals:
(i) $\int x\sqrt{x^2+4}\,dx$, (ii) $\int \tan^{-1} x\,dx$, (iii) $\int \frac{x+3}{(3-x)(x-2)}\,dx$.

[3] (b) Sketch and find the area of the region bounded by the curves:

$$y = 4 - x^2 \quad \text{and} \quad y = 3.$$

[4] (c) The region bounded by the curves $y = \sqrt{x}$, $y = 1$, $y = 2$ and $x = 0$ is rotated about the $y-$axis to form a solid $\mathcal{S}$. Find the volume of $\mathcal{S}$.

Q4.

[3] (a) We define $z(x, y)$ implicitly by the equation $x^2y + \sin(xyz) = 1$. Compute the partial derivative $\frac{\partial z}{\partial y}$.

[4] (b) Solve the differential equation: $xy^2 + y'e^{-x} = 0$.

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Part 1: Multiple Choice Questions:

[2] 1. The center of the conic section of equation $4x^2 + 8x - y^2 + 2y - 1 = 0$ is

a. $(-1, 1)$	*b.* $(1, -1)$	*c.* $(4, 1)$	*d.* $(4, -1)$	*Answer :*

[2] 2. The equation of the ellipse of focci $(-3, 6); (-3, 2)$ and length of major axis 14 is given by:

a. $\dfrac{(x-6)^2}{14} + \dfrac{(y-2)^2}{3} = 1$	*b.* $\dfrac{(x+3)^2}{45} + \dfrac{(y-4)^2}{49} = 1$	*Answer :*
c. $\dfrac{(x-3)^2}{9} + \dfrac{(y+6)^2}{4} = 1$	*d.* $\dfrac{(x+3)^2}{9} + \dfrac{(y-2)^2}{36} = 1$	

[2] 3. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -3 & 2 \\ 1 & 2 & -1 \end{bmatrix}$, then $A\left(B^T\right)$ equals

a. $\begin{bmatrix} 0 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$	*b.* $\begin{bmatrix} 0 & 3 \\ 0 & 2 \end{bmatrix}$	*c.* $\begin{bmatrix} 7 & -3 \\ -7 & 4 \end{bmatrix}$	*d.* *undefined*	*Answer :*

[2] 4. The determinant $\begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 2 & 2 & 3 & 4 \end{vmatrix}$ is equal to

a. -2	*b.* 0	*c.* 6	*d.* 8	*Answer :*

[2] 5. The integral $\int x^3\left(2+x^4\right)^5 dx$ is equal to

a.	b.	c.	d.	Answer :
$\frac{\left(2+x^4\right)^6}{24}+C$	$\frac{x^4\left(2+x^4\right)^6}{24}+C$	$\frac{\left(2+x^4\right)^6}{6}+C$	$\frac{x^4\left(2+x^4\right)^5}{4}+C$	

[2] 6. The volume of the solid, obtained by revolving the region bounded by the curves $y=x^2$, $y=x^3$ about the $x-$axis, is equal to:

a.	b.	c.	d.	Answer :
$\frac{\pi}{7}$	$\frac{\pi}{12}$	$\frac{2\pi}{15}$	$\frac{2\pi}{35}$	

[2] 7. The point with rectangular coordinates $\left(-1,\sqrt{3}\right)$, has polar coordinates:

a.	b.	c.	d.	Answer :
$\left(2,\frac{\pi}{3}\right)$	$\left(2,\frac{2\pi}{3}\right)$	$\left(\sqrt{3},\frac{\pi}{2}\right)$	$\left(\sqrt{3},\frac{\pi}{4}\right)$	

[2] 8. Let $f(x,y)=x^3y^2+y\sin\frac{x}{y}$. The partial derivative $\frac{\partial f}{\partial x}$ is equal to:

a.	b.	c.	d.	Answer :
$3x^2y^2+\cos x$	$6x^2y+\cos x$	$3x^2y^2+\cos\frac{x}{y}$	$6x^2y+\cos\frac{x}{y}$	

[2] 9. If $y=y(x)$ is defined implicitly by $e^{xy}=xy+1$, for $x,y>0$, then $\frac{dy}{dx}$ is equal to:

a.	b.	c.	d.	Answer :
$xe^{xy}-y$	$-\frac{e^{xy}}{xy}$	$-\frac{e^{xy}}{x}$	$-\frac{y}{x}$	

[2] 10. The general solution of the differential equation $y'-\frac{3x^2}{2y}=0$ is:

a.	b.	c.	d.	Answer :
$2y=3x^2+C$	$y-x^3\ln\lvert 2y\rvert=C$	$y^2=x^3+C$	$y-\frac{x^3}{y^2}=C$	

Part 2: Essay Questions:

[4] 11. Find the elements of the conic section $4x^2 - 9y^2 - 8x - 36y - 68 = 0$ and then sketch it.

[4] 12. Solve by using Gauss Elimination Method the system

$$\begin{cases} x + y + z & = & 2 \\ x - y + 2z & = & 0 \\ 2x + z & = & 2 \end{cases}$$

[4] 13. Compute the integral $\displaystyle\int \frac{2x^2 - 2x - 2}{x(x+1)(x-1)}\, dx$.

[4] 14. If $w = x^2 + xy + 3y^2$, $x = u^2 + v$ and $y = v^2$, use the chain rule to compute $\dfrac{\partial w}{\partial u}$.

[4] 15. Find the general solution of the linear differential equation $xy' + 2y = 5x^3$.

Make Up of The Final Exam - Allowed time: 3 hours

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Part 1: Multiple Choice Questions:

[2] 1. The center of the conic section of equation $x^2 + 2y^2 + 8x - 4y + 14 = 0$ is

a. $(-1, 2)$	*b.* $(1, -2)$	*c.* $(-4, 1)$	*d.* $(4, -1)$	*Answer* :

[2] 2. The equation of the hyperbola of vertices $(4, 2); (-2, 2)$ and focus $(5, 2)$ is given by:

a. $\frac{(x-4)^2}{5} - \frac{(y+2)^2}{2} = 1$	*b.* $\frac{(y-2)^2}{2} - \frac{(x+4)^2}{5} = 1$	*Answer* :
c. $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{7} = 1$	*d.* $\frac{(y+2)^2}{7} - \frac{(x+1)^2}{9} = 1$	

[2] 3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$, then the product AB equals

a. $\begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix}$	*b.* $\begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$	*c.* $\begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & 1 \end{bmatrix}$	*d.* *undefined*	*Answer* :

[2] 4. The determinant $\begin{vmatrix} 1 & 2 & 2 & 4 \\ 2 & 1 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 1 & 2 & 2 & 4 \end{vmatrix}$ is equal to

a. -16	*b.* 0	*c.* 4	*d.* 16	*Answer* :

[2] 5. The integral $\int (x+1)\sqrt{2x^2+4x+3}\,dx$ is equal to

a. $\left(\frac{x^2}{2}+x\right)\sqrt{\frac{2x^3}{3}+2x^2+3x}+C$	b. $\frac{1}{6}\left(2x^2+4x+3\right)^{\frac{3}{2}}+C$	Answer :
c. $\frac{1}{2\sqrt{2}}\left(4x^2+8x+6\right)^{\frac{3}{2}}+C$	d. $\sqrt{2x^2+4x+3}+C$	

[2] 6. The volume of the solid, obtained by revolving the region bounded by the curves $y=x^2$, $y=x^3$ about the $y-$axis, is equal to:

a. $\frac{\pi}{10}$	b. $\frac{\pi}{12}$	c. $\frac{2\pi}{15}$	d. $\frac{2\pi}{35}$	Answer :

[2] 7. The point with rectangular coordinates $(-1,1)$, has polar coordinates:

a. $\left(1,\frac{\pi}{2}\right)$	b. $(1,\pi)$	c. $\left(\sqrt{2},\frac{\pi}{4}\right)$	d. $\left(\sqrt{2},\frac{3\pi}{4}\right)$	Answer :

[2] 8. Let $f(x,y)=\frac{x^2}{y}e^{xy}$. The partial derivative $\frac{\partial f}{\partial x}$ is equal to:

a. $\frac{2x}{y}e^{xy}$	b. $2xe^{xy}$	c. $\frac{2x}{y}e^{xy}+x^2e^{xy}$	d. $\frac{2x}{y}e^{xy}+\frac{x^2}{y}e^{y}$	Answer :

[2] 9. If $y=y(x)$ is defined implicitly by $e^{xy}=x^2y^2+1$, for $x,y>0$, then $\frac{dy}{dx}$ is equal to:

a. $xe^{xy}-2x^2y$	b. $-\frac{e^{xy}}{x^2y^2}$	c. $-\frac{e^{xy}}{2x^2y}$	d. $-\frac{y}{x}$	Answer :

[2] 10. The general solution of the differential equation $2y'y - \frac{\sqrt{x}}{y} = 0$ is:

a.	b.	c.	d.	Answer :
$y^2 = \frac{2}{3}x^{\frac{3}{2}}ln\|y\| + C$	$y^2 - \sqrt{x}\ln\|y\| = C$	$y^3 = x^{\frac{3}{2}} + C$	$y^2 - \frac{x^{\frac{3}{2}}}{y^2} = C$	

Part 2: Essay Questions:

[4] 11. Find the elements of the conic section $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ and then sketch it.

[4] 12. Solve by using Gauss-Jordan Elimination Method the system

$$\begin{cases} 2x + y + z &= -1 \\ x + 2y + z &= 0 \\ x + y + 2z &= 1 \end{cases}$$

[4] 13. Compute the integral $\int \frac{3x+2}{x^2(x+1)}\, dx$.

[4] 14. If $w = x^2 - 3xy + 2y^2$, $x = u + v$ and $y = u - v$, use the chain rule to compute $\frac{\partial w}{\partial v}$.

[4] 15. Find the general solution of the differential equation $y' + 2xy = 3x^2 e^{-x^2}$.

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Q1.

[4] (a) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$. Compute (if possible):

(i) AB, (ii) AC.

[3] (b) Compute the determinant $\begin{vmatrix} 4 & 1 & 5 \\ 0 & 3 & 0 \\ 4 & 2 & 5 \end{vmatrix}$.

[4] (c) Solve by using Gauss Elimination Method the system $\begin{cases} x + 2z &= 1 \\ 3x + y &= -1 \\ 2y - 3z &= 1 \end{cases}$

Q2.

[4] (a) Find the standard equation of the hyperbola with foci $(1, -4)$ and $(1, 6)$ and vertex $(1, 4)$ and then sketch it.

[4] (b) Find the elements of the conic section $9x^2 + 4y^2 + 18x - 24y + 9 = 0$ and then sketch it.

Q3.

[2,2,3] (a) Compute the integrals:

(i) $\displaystyle\int 4x^3 \left(1 + 2x^4\right)^5 \, dx$, (ii) $\displaystyle\int x \cos(2x) \, dx$, (iii) $\displaystyle\int \frac{2x - 1}{x(x-1)} \, dx$.

[3] (b) Find the area of the region bounded by the curves:

$$x = y^2 \quad \text{and} \quad x = y + 2.$$

[3] (c) The region bounded by the curves $x = 4$, $y = 0$ and $y = \sqrt{x}$ is rotated about the $x-$axis to form a solid $\mathcal{S}$. Find the volume of $\mathcal{S}$.

Q4.

[4] (a) Find the partial derivatives f_x, f_y of the function

$$f(x, y) = xy^2 + \ln(x^2 + y).$$

[4] (b) Solve the linear differential equation: $xy' + y = 6x^2$ with the condition $y(1) = 1$.

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Q1.

[4] (a) Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{pmatrix}$. Compute (if possible):
(i) AB, (ii) BA.

[3] (b) Compute the determinant $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 2 & 1 \end{vmatrix}$.

[4] (c) Solve by Cramer method the system $\begin{cases} x + y + z = 0 \\ y - z = 1 \\ x + z = 0 \end{cases}$

Q2.

[4] (a) Find the standard equation of the hyperbola with foci $(1, -3)$ and $(1, 7)$ and vertex $(1, 6)$ and then sketch it.

[4] (b) Find the elements of the conic section $x^2 + 8y - 2x + 9 = 0$ and then sketch it.

Q3.

[3,2,2] (a) Compute the integrals:
(i) $\displaystyle\int \frac{x}{x^2 + 3x + 2}\, dx$, (ii) $\displaystyle\int xe^x\, dx$, (iii) $\displaystyle\int \frac{1 + \cos x}{x + \sin x}\, dx$.

[3] (b) Find the area of the surface determined by the curves:

$$y = 2 - x^2 \quad \text{and} \quad y = x.$$

[3] (c) The region R between the curves $y = 0$, $x = 1$ and $y = x^2$ is rotated about the $x-$axis to form a solid of revolution S. Find the volume of S.

Q4.

[4] (a) Find the partial derivatives f_x, f_y for the function

$$f(x, y) = xy^2 - \sin(x^2 y).$$

[4] (b) Solve the differential equation: $\dfrac{dy}{dx} + y = \dfrac{2x}{e^x}$.