Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $4y^2 = -9x^2 + 18x + 27$, then sketch it. [4]

2. Find the standard equation of the parabola with vertex $(2, 3)$ and focus $(2, 1)$, then sketch it. [4]

3. Calculate, whenever it is possible, $A + B^T$ and AB, for matrices [4]

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & -2 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} 2x - 2y + z &= 2 \\ x - y + z &= 2 \\ 2x + 2y - z &= 2 \end{cases}$$

   (a) Solve this system by using Cramer' rule. [4]

   (b) Solve this system by using Gauss-Jordan elimination method. [4]

5. Evaluate the integrals

   (a) $\int \left(4x^3 - \frac{2}{x^3} + e^x\right)\, dx.$ [2]

   (b) $\int 20x^3 \left(x^4 + 2\right)^4\, dx.$ [2]

   (c) $\int \sec^2 x\ \ln|\sin x|\, dx.$ [3]

   (d) $\int \frac{x+1}{(x-2)(x-1)}\, dx.$ [3]