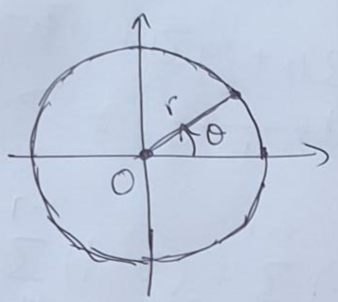
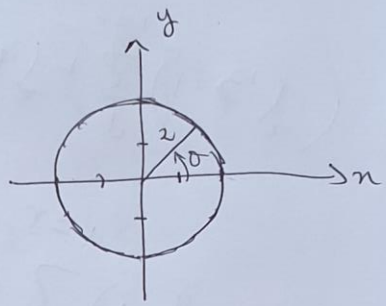
Some special equations:

① $r = \text{Constant}$

This is the equation of a circle of center O and radius the constant

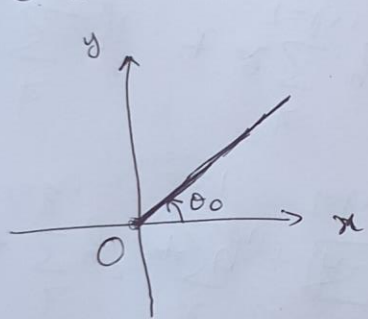


$r = 2$:

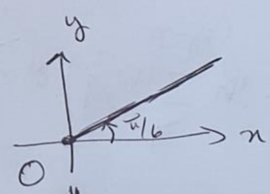


② $\theta = \text{Constant}$

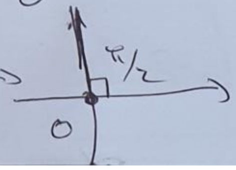
This is a half line starting from O.



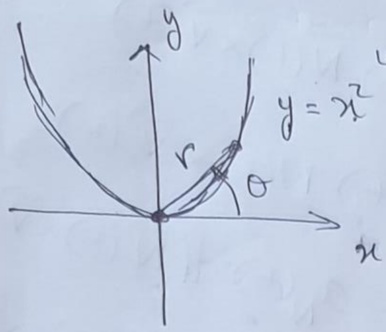
Example: $\theta = \frac{\pi}{6} \rightarrow$



$\theta = \pi/2 \rightarrow$



O. parabola of vertex



In rectangular coordinates

In polar coordinates:

$$y = r\sin\theta \ ; \ x = r\cos\theta$$

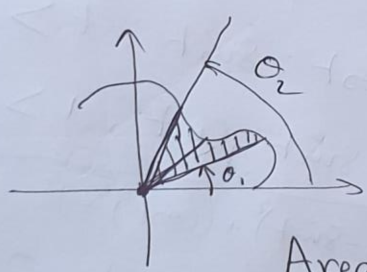
$$\underbrace{r}_{\geq 0}\underbrace{\sin\theta}_{\geq 0} = \underbrace{r^2\cos^2\theta}_{\geq 0}$$

$$0 \leq \theta \leq \pi$$

Then $\sin\theta = r\cos^2\theta$

and $\boxed{r = \dfrac{\sin\theta}{\cos^2\theta}}$

Integration:

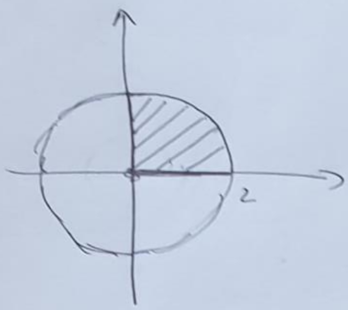


$r = f(\theta)$

$$\text{Area} = \int_{\theta_1}^{\theta_2}\int_{0}^{f(\theta)} r\,dr\,d\theta$$

$$\boxed{\text{Area} = \int_{\theta_1}^{\theta_2} \frac{f(\theta)^2}{2}\,d\theta}$$

Example: $r = 2$

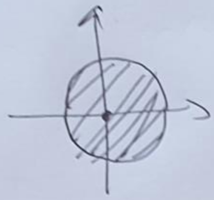


$$\text{Area} = \int_0^{\pi/2} \frac{2^2}{2}\, d\theta = 2\Big[\theta\Big]_0^{\pi/2}$$

$$= \pi$$

$r = r_0$

$$\text{Area} = \int_0^{2\pi} \frac{r_0^2}{2}\, d\theta = \frac{r_0^2}{2}\Big[\theta\Big]_0^{2\pi}$$

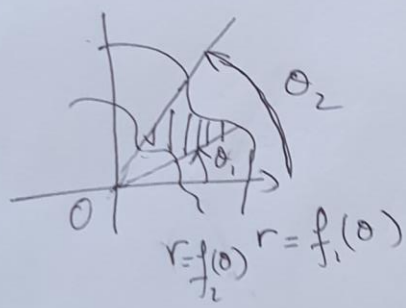


$$= \frac{r_0^2}{2}(2\pi - 0)$$

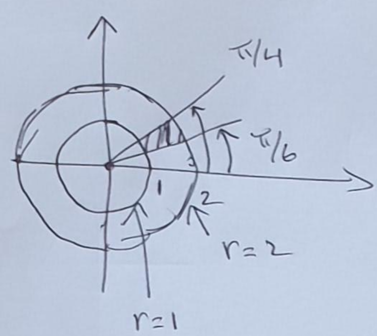
$$= \pi r_0^2$$

Example: $r = f_1(\theta)$; $r = f_2(\theta)$

$f_1(\theta) \geq f_2(\theta)$



$$\text{Area} = \int_{\theta_1}^{\theta_2} \left(\frac{f_1(\theta)^2}{2} - \frac{f_2(\theta)^2}{2} \right) d\theta$$



$$\text{Area} = \int_{\pi/6}^{\pi/4} \left(\frac{2^2}{2} - \frac{1^2}{2}\right) d\theta$$

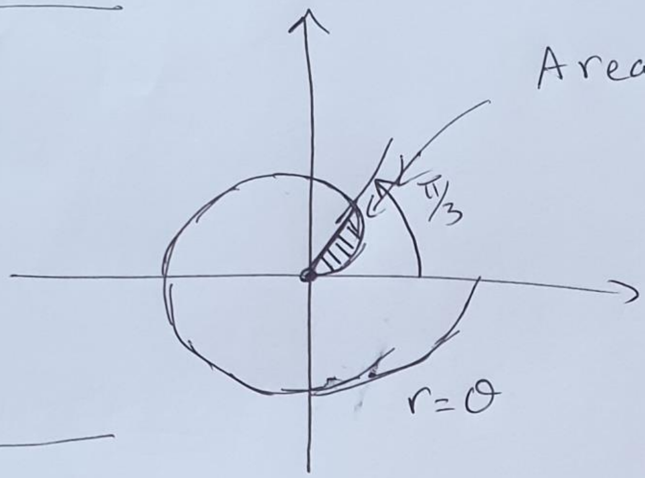
$$= \frac{3}{2}\left[\theta\right]_{\pi/6}^{\pi/4}$$

$$= \frac{3}{2}\left[\pi/4 - \pi/6\right]$$

$$= \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}\pi}{24} = \frac{\pi}{8}.$$

Example 3:

$r = \theta$



$$\text{Area} = \int_0^{\pi/3} \frac{\theta^2}{2}\, d\theta$$

$$= \left[\frac{\theta^3}{6}\right]_0^{\pi/3}$$

$$= \frac{\pi^3}{162} - 0$$

$$\text{Area} = \frac{\pi^3}{162}$$