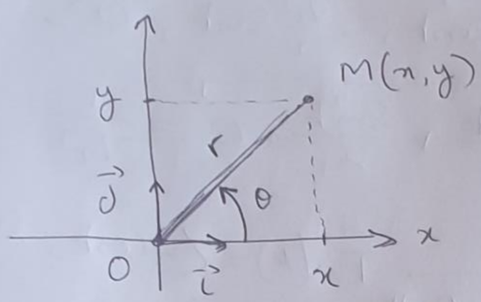
The polar coordinates

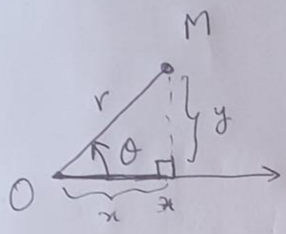


(x,y) : Rectangular coordinates

(r,θ) : Polar coordinates

r : The distance between O and M

θ : The angle between the ray Ox and OM



We know that:

$\cos\theta = \dfrac{x}{r} \iff x = r\cos\theta$

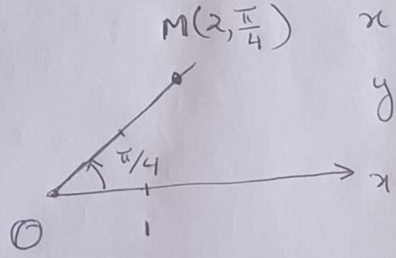
$\sin\theta = \dfrac{y}{r} \iff y = r\sin\theta$

Example:

$\pi \equiv 180^\circ$ $\quad \frac{\pi}{4} \equiv 45^\circ$

In polar coordinates:

$M\left(\underset{\uparrow \atop r}{2} ; \underset{\uparrow \atop \theta}{\frac{\pi}{4}}\right)$

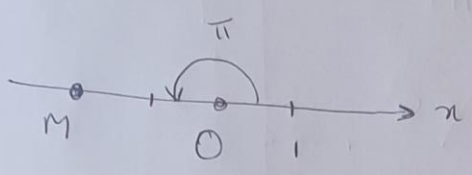


Rectangular coordinates

$x = 2\cos\frac{\pi}{4} = 2\frac{\sqrt{2}}{2} = \sqrt{2}$

$y = 2\sin\frac{\pi}{4} = 2\frac{\sqrt{2}}{2} = \sqrt{2}$

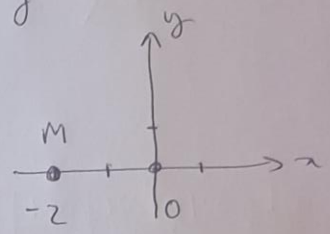
In polar coordinates: $M(2, \pi)$



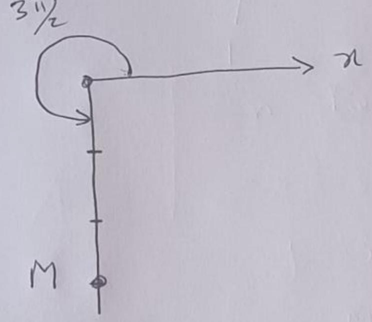
In Rectangular:

$x = 2\cos\pi = -2$

$y = 2\sin\pi = 0$



In polar coordinates: $M\left(3, \frac{3\pi}{2}\right)$

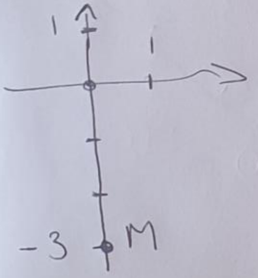


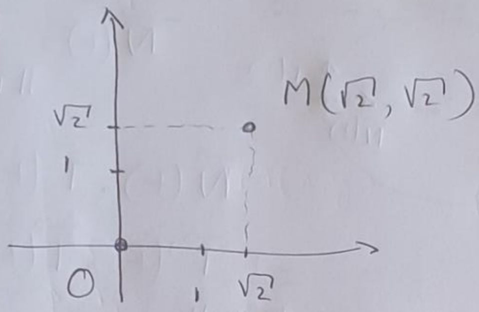
In rectangular :

$x = 3\cos\left(\frac{3\pi}{2}\right) = 0$

$y = 3\sin\left(\frac{3\pi}{2}\right) = -3$

$M(0, -3)$





θ: Degree	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
Sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$0 \quad \frac{1}{2} \quad 1$

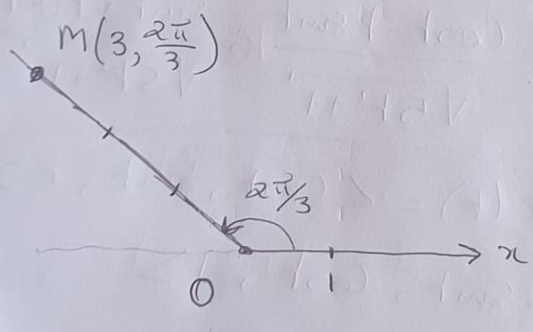
In polar

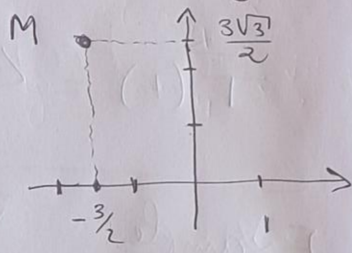
$$M\left(3 ; \frac{2\pi}{3}\right)$$

In Rectangular

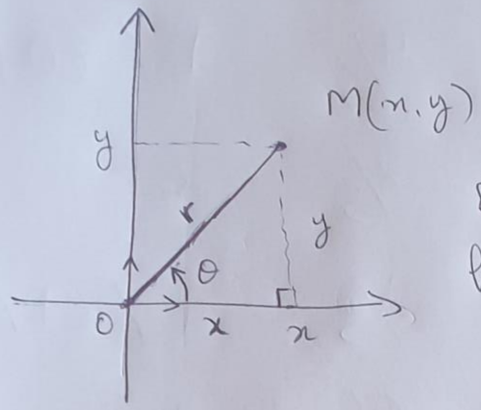
$$x = 3\cos\frac{2\pi}{3} = -\frac{3}{2}$$

$$y = 3\sin\frac{2\pi}{3} = \frac{3\sqrt{3}}{2}$$





How to deduce polar coordinates from rectangular coordinates



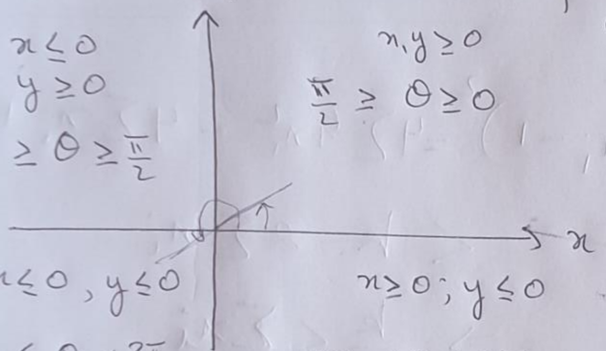
How to find r and θ

From Pythagoras we have $r^2 = x^2 + y^2$

$$r = \sqrt{x^2 + y^2} \geq 0$$

$$\cos\theta = \frac{x}{r} \quad ; \quad \sin\theta = \frac{y}{r}$$

If $r = 0$ we take any value for θ.



$x \leq 0$, $y \geq 0$

$\pi \geq \theta \geq \frac{\pi}{2}$

$x, y \geq 0$

$\frac{\pi}{2} \geq \theta \geq 0$

$x \leq 0$, $y \leq 0$

$\pi \leq \theta \leq \frac{3\pi}{2}$

or

$-\pi \leq \theta \leq -\frac{\pi}{2}$

$x \geq 0$; $y \leq 0$

$\frac{3\pi}{2} \leq \theta \leq 2\pi$

or

$-\frac{\pi}{2} \leq \theta \leq 0$

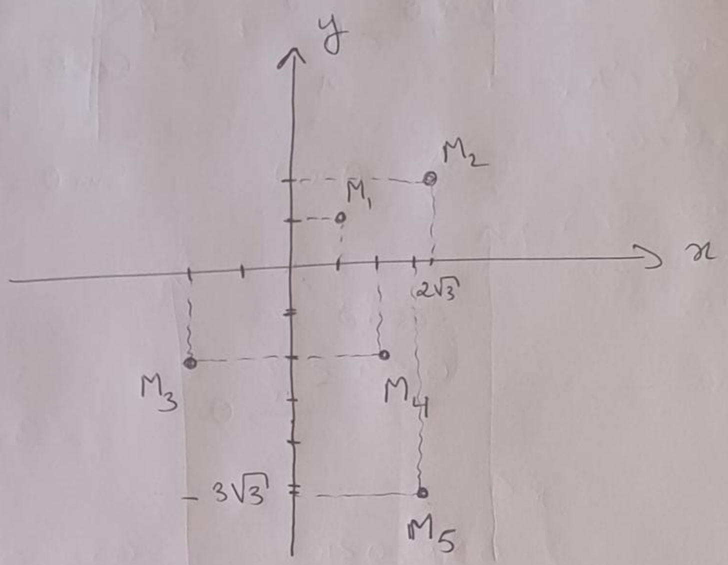
$$\tan\theta = \frac{y}{x} \quad \text{If } x \neq 0$$

Example:

In rectangular coordinates:

$M_1(1, 1)$; $M_2(2\sqrt{3}; 2)$; $M_3(-2; -2)$

$M_4(2, -2)$; $M_5(3, -3\sqrt{3})$



Polar coordinates:

M_1 : $r = \sqrt{1^2 + 1^2} = \sqrt{2}$; $\left.\begin{array}{l}\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\theta = \frac{\sqrt{2}}{2}\end{array}\right\} \theta = \frac{\pi}{4}$

$M_1(\sqrt{2}, \pi/4)$

M_2 : $r = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{12 + 4} = 4$

$\cos\theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$; $\sin\theta = \frac{2}{4} = \frac{1}{2}$; $\theta = \frac{\pi}{6}$

$M_2\left(4 ; \pi/6\right)$

$M_3 : r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$

$\cos\theta = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$; $\sin\theta = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

$M_3\left(2\sqrt{2} ; \frac{5\pi}{4}\right)$

$M_4 : r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$

$\cos\theta = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$; $\sin\theta = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\theta = -\frac{\pi}{4}$

$2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ gi

$M_4\left(2\sqrt{2}, -\frac{\pi}{4}\right)$

$M_5 : \sqrt{3^2 + (-3\sqrt{3})^2} = 6$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$; $\sin\theta = \frac{-3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$

$\theta = -\frac{\pi}{3}$ or $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

$M_5\left(6 ; -\pi/3\right)$