Some properties of the determinant:

① If the matrix A has a row of zeros or a column of zeros then $\det A = 0$.

Example:

$$\begin{vmatrix} 3 & 4 & 5 & 1 \\ 2 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 5 & 3 \end{vmatrix} = 0 \quad ; \quad \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 4 & 0 & 5 \end{vmatrix} = 0$$

② If the matrix A has two identical rows or two identical columns then $\det A = 0$

Example:

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 3 & 1 & 2 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 1 \end{vmatrix} = 0 \quad ; \quad \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \\ 2 & 0 & 2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \\ 0 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -10 \neq 0$$

③ If the matrix A has two proportional rows or two proportional column then $\det A = 0$

Example:

$$\begin{vmatrix} 2 & 4 & 6 & -2 \\ 5 & 1 & 2 & 0 \\ 3 & 2 & 1 & 4 \\ 3 & 6 & 9 & -3 \end{vmatrix} = 0 \,; \quad \begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 6 \\ 0 & 1 & 0 \end{vmatrix} = 0$$

$$\frac{3}{2} = \frac{6}{4} = \frac{9}{6} = \frac{-3}{-2}$$

$$\frac{2}{1} = \frac{6}{3} \;;\quad 0 \leftrightarrow 0$$

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④ If A is a diagonal matrix or a triangular matrix the $\det A$ is the product of the entries in the diagonal

Example:

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 2 \times 3 \times (-1) = -6$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 0 & 5 & 2 \\ 0 & 0 & 7 \end{vmatrix} = 3 \times 5 \times 7 = 105$$

⚠ $$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{vmatrix} \neq 3 \times 2 \times 1$$

$$= -3 \times 2 \times 1$$

⚠ $$\begin{vmatrix} 3^{+} & 1 & 2 \\ 0^{-} & 4 & 1 \\ 1^{+} & 0 & 5 \end{vmatrix} = 3 \begin{vmatrix} 4 & 1 \\ 0 & 5 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 60 + (-5) = 55 \neq 3 \times 4 \times 5$$

$3 \times 4 \times 5 = 60$

$$\det I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\det O = \begin{vmatrix} O \end{vmatrix} = 0$$

(5)

$$\begin{vmatrix} 3 & 4 & 1 \\ 6 & 4 & 2 \\ 9 & 2 & 3 \end{vmatrix} = 3\times 2 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 3 & 1 & 3 \end{vmatrix} = 3\times 2\times 2 \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{vmatrix}$$

$$= 12 \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 4 & 2 \\ 4 & 2 & 4 \\ 1 & 3 & 0 \end{vmatrix} = 2\times 2 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 0 \end{vmatrix}$$

(6)

$$\begin{vmatrix} 0 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 3 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = -18$$

$$\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2$$

$$\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$$

$$\begin{vmatrix} 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 2 \\ 2 & 1 & 4 & 5 \\ 0 & 3 & 1 & 4 \end{vmatrix} = (-1)(-1)\begin{vmatrix} 2 & 1 & 4 & 5 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

$$= (-1)(-1)\, 2 \times 3 \times 3 \times 2$$

$$= 36$$

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$$\begin{vmatrix} 3 & 2 & 1 & 1 \\ 1 & 3 & 4 & 1 \\ 2 & 3 & 1 & 2 \\ 5 & 15 & 17 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 & 1 \\ 1 & 3 & 4 & 1 \\ 2 & 3 & 1 & 2 \\ 0 & 0 & -3 & -2 \end{vmatrix} \begin{matrix} \\ \\ \\ R_5 - 5R_2 \end{matrix}$$

$$= \begin{vmatrix} 0 & -7 & -11 & -2 \\ 1 & 3 & 4 & 1 \\ 0 & -3 & -7 & 0 \\ 0 & 0 & -3 & -2 \end{vmatrix} \begin{matrix} R_1 - 3R_2 \\ \\ R_3 - 2R_2 \\ \\ \end{matrix}$$