# Matrices المصفوفات

Matrix مصفوفة
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① Definition: A matrix is a rectangular table with entries numbers.

Examples: $\begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & 5 \end{pmatrix}$ ; $\begin{pmatrix} 4 & 1 \\ 2 & 1 \\ -1 & 1 \end{pmatrix}$ ; $\begin{pmatrix} 1 & 3 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ ; $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$ ; $\begin{pmatrix} \sqrt{3} & \pi & 4 \\ -1 & 1/3 & 5 \\ 4 & 2 & 2 \end{pmatrix}$

② The size of a matrix: مقياس المصفوفة

Size = Number of rows × Number of columns
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Examples: $\text{Size} \begin{pmatrix} 1 & 2 & -1 \\ 5 & 4 & 3 \end{pmatrix} = 2 \times 3$

$\text{Size} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 3 \times 1$

$\text{Size} \begin{pmatrix} 1 & 2 & 4 \end{pmatrix} = 1 \times 3$

Remark: $3 \times 1 \neq 1 \times 3$

$2 \times 3 \neq 6$

Some types of matrices:

① A horizontal vector matrix
مصفوفة : متجه أفقي.

Any matrix with only one row

A row matrix:

Examples:

$\begin{pmatrix} 1 & 2 \end{pmatrix}$ ; $\begin{pmatrix} 1 & 2 & 1 & 3 \end{pmatrix}$ ; $\begin{pmatrix} 3 & -1 & 0 \end{pmatrix}$

Size $= 1 \times m$

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② A vertical vector matrix
مصفوفة متجه عمودي

Any matrix with only one column.

Examples:

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ ; $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ ; $\begin{pmatrix} \sqrt{2} \\ -1 \\ 5 \\ 4 \end{pmatrix}$ ; ---

Size $= m \times 1$

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③ A square matrix مصفوفة مربعة

Number of rows = Number of columns

Examples of square matrices:

2x2 matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$$

3x3 matrix

$$\begin{pmatrix} 1 & 3 & -1 \\ 0 & 5 & 4 \\ -1 & 2 & 1 \end{pmatrix}$$

4x4 matrix

$$\begin{pmatrix} 2 & -1 & 0 & 1 \\ 4 & 3 & 1 & 2 \\ 2 & 5 & 2 & 1 \\ 3 & 1 & 0 & -1 \end{pmatrix}$$

In a square matrix:

$$\begin{pmatrix} * & & & \\ & * & & \\ & & * & \\ & & & * \end{pmatrix}$$ The main diagonal

<u>A diagonal matrix</u> is a square matrix with all entries outside the main diagonal are zeros.

<u>Examples:</u>

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} ; \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} ; \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

A triangular matrix مصفوفة مثلثية

is a square matrix with all entries below the diagonal are 0 / or above the diagonal are 0.

$$\begin{pmatrix} 2 & 5 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad \begin{pmatrix} 5 & 0 & 0 \\ 2 & -1 & 0 \\ 5 & 0 & 3 \end{pmatrix}$$

Upper triangular
مثلثية فوقية

lower triangular
مثلثية تحتية

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The identity matrix مصفوفة الوحدة

We denote it $I$ ; $I_n$ ; $Id$ ; $Id_n$

It is a diagonal matrix with diagonal of 1s only

$$I \quad \underset{I_2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} , \underset{I_3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} ; \underset{I_4}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

A zero matrix is a matrix where all the entries are 0.

We denote it $O$ ; $O_n$ ; $O_{m\times n}$

$O$: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ ; $\begin{pmatrix} 0 & 0 \end{pmatrix}$ ; $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ; $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$O_2$ $\quad O_{1\times 2}$ $\quad O_{2\times 1}$ $\quad O_{2\times 3}$

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## Operations on matrices:

### ① Sum / Difference

We can add or substract two matrices if they have the same size.

**Example:**

$$\underset{2\times 3}{\begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & -1 \end{pmatrix}} + \underset{2\times 3}{\begin{pmatrix} 5 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix}} = \underset{2\times 3}{\begin{pmatrix} 6 & 5 & 3 \\ 5 & 5 & 0 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & -1 \end{pmatrix} - \begin{pmatrix} 5 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -4 & -1 & -1 \\ 1 & 3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 4 \end{pmatrix} \pm \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 3 & -1 \end{pmatrix} : \text{undefined}$$

$$2\times 3 \neq 3\times 2$$

## ② Multiplication by a scalar
الضرب بعدد

Always defined

$$3\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 9 & -3 \end{pmatrix}$$

$$2\begin{pmatrix} 1 & 2 & 1 \\ -1 & \frac{1}{2} & 5 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 2 \\ -2 & 1 & 10 \end{pmatrix}$$

$$3\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 3 \end{pmatrix}$$

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## ③ Product of two matrices
ضرب مصفوفتين

To be defined, the product $AB$ must satisfy the condition $n = p$

size $A = m \times n$

size $B = p \times q$

Examples:

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \end{pmatrix} = \text{undefined}$$

$1 \times 2 \qquad 1 \times 2$

$\neq$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \text{defined}$$

$1 \times 2 \qquad 2 \times 1$

$=$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 5 \\ 3 & 0 & -1 \end{pmatrix} \quad \text{defined}$$

$2 \times 2 \qquad 2 \times 3$

$=$

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If size $A = m \times n$ and size $B = n \times p$

then size$(AB) = m \times p$.

Examples:

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \underset{1 \times 1}{\begin{pmatrix} \quad \end{pmatrix}}$$

$1 \times 2 \qquad 2 \times 1$

$$\underset{2 \times 2}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}} \underset{2 \times 3}{\begin{pmatrix} 2 & 1 & 5 \\ 3 & 0 & -1 \end{pmatrix}} = \underset{2 \times 3}{\begin{pmatrix} \qquad \qquad \end{pmatrix}}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 & 4 \end{pmatrix} = \begin{pmatrix} - & - \\ - & - \end{pmatrix}$$

$2 \times 1 \quad 1 \times 2 \qquad 2 \times 2$

The product of a horizontal vector by a vertical vector matrices.

<u>Examples:</u>

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 1 \times 3 + 2 \times 4 = 11$$

$1 \times 2 \qquad 2 \times 1$

$$\begin{pmatrix} 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = 1 \times (-1) + 3 \times 0 + 1 \times 2 = 1$$

The product of two matrices:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 5 \\ 3 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 8 & 1 & 3 \\ 18 & 3 & 11 \end{pmatrix}$$

$2 \times 2 \qquad 2 \times 3 \qquad 2 \times 3$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$$

$2 \times \underline{1}$ $\quad \underline{1} \times 2$ $\quad 2 \times 2$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & -1 & 5 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 0 & 5 \end{pmatrix}$$

$2 \times 3$ $\quad 3 \times 2$ $\quad 2 \times 2$