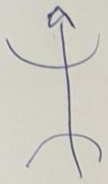
Example: Give the elements of the hyperbola of equation $\frac{y^2}{5^2} - \frac{x^2}{6^2} = 1$ and sketch it

The hyperbola bisector line

المحور القاطع للقطع الزائد

is vertical



$: x = 0.$

The vertices: $V_1(0 ; 5)$

$V_2(0, -5)$

The focci: $c = \sqrt{5^2 + 6^2} = \sqrt{61}$

$49 < 61 < 64$

$7 < \sqrt{61} < 8$

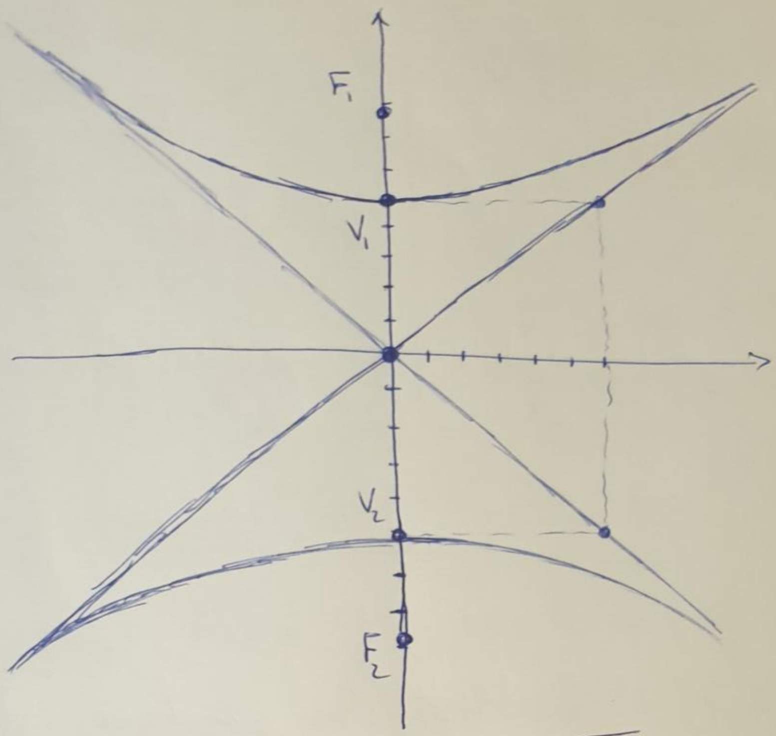
$F_1 (0, \sqrt{61})$

$F_2 (0, -\sqrt{61})$

The asymptotes:

المستقيمات المقاربة

$y = \pm \frac{5}{6} x$



The hyperbola of center $C(h,k)$ has equation :

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

The bisector line is horizontal : $y = k$

The asymptotes :

$$(y-k) = \pm \frac{b}{a}(x-h)$$

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

The bisector line is vertical : $x = h$

The asymptotes

$$y - k = \pm \frac{b}{a}(x-h)$$

The vertices:

$V_1(h+a, k)$

$V_2(h-a, k)$

The focci:

$F_1(h+c, k)$

$F_2(h-c, k)$

The vertices:

$V_1(h, k+b)$

$V_2(h, k-b)$

The focci

$F_1(h, k+c)$

$F_2(h, k-c)$

$$c^2 = a^2 + b^2$$

Examples: Find the equation of the hyperbola of vertices $V_1(5,2)$; $V_2(-1,2)$ and focus $F_1(7,2)$ and sketch it

Answer: The center is: $C(\underset{\frac{5+(-1)}{2}}{\underset{\shortparallel}{2}} ; 2)$

The bisector line: $y = 2$

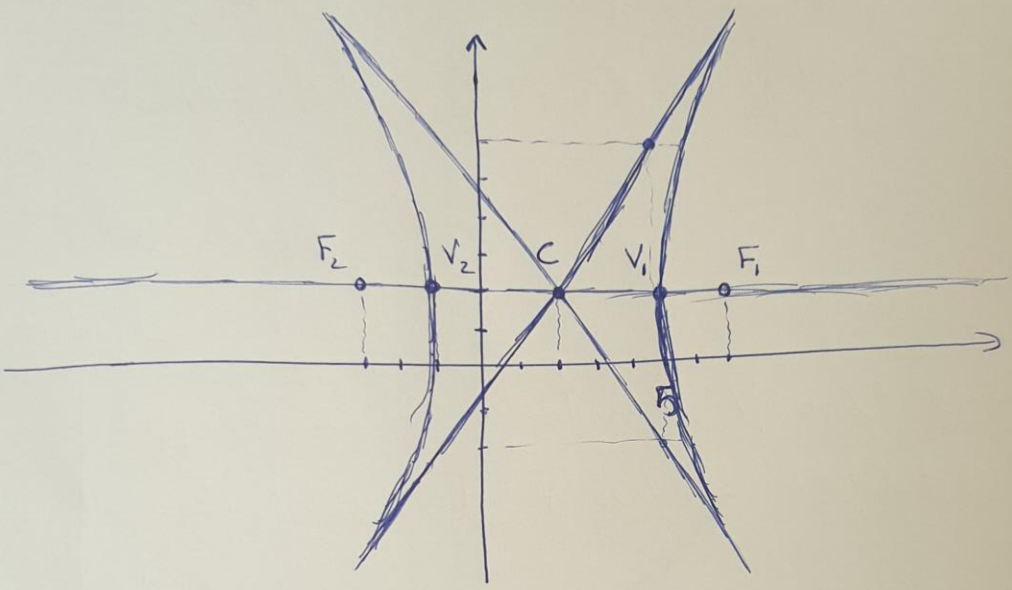
$a = 5 - 2 = 3$; $c = 7 - 2 = 5$; $b = \sqrt{5^2 - 3^2} = 4$

$$\frac{(x-2)^2}{3^2} - \frac{(y-2)^2}{4^2} = 1$$

The focci: $F_1(7,2)$; $F_2(-3;2)$

The asymptotes:

$$y-2 = \pm\frac{4}{3}(x-2)$$



① Find the equation of the hyperbola of focci $F_1(-1;6)$; $F_2(-1;-4)$; and vertex $V_1(-1,5)$ and sketch it.

② Solution : The bisector line is : $x=-1$

The center : $C(-1;\underset{\frac{6+(-4)}{2}}{1})$

$$\frac{(y-1)^2}{4^2} - \frac{(x+1)^2}{3^2} = 1$$

$b = 5 - 1 = 4$; $c = 6 - 1 = 5$; $a = \sqrt{5^2 - 4^2} = 3$

Vertices: $V_1(-1, 5)$; $V_2(-1; -3)$

$1-4$

The asymptotes:

$$y - 1 = \pm \frac{4}{3}(x+1)$$