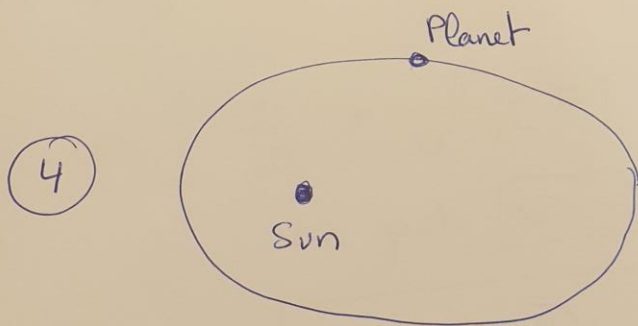
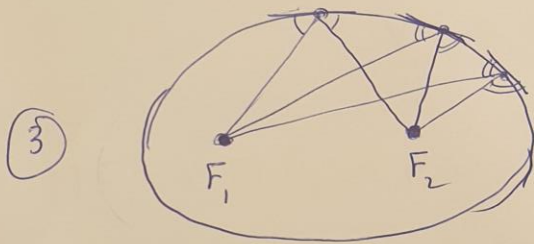
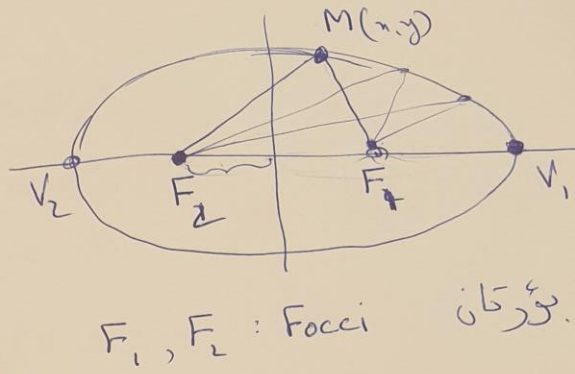
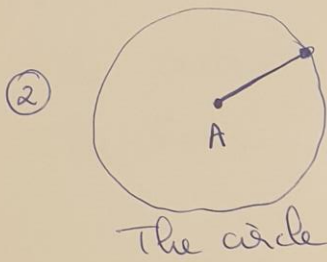
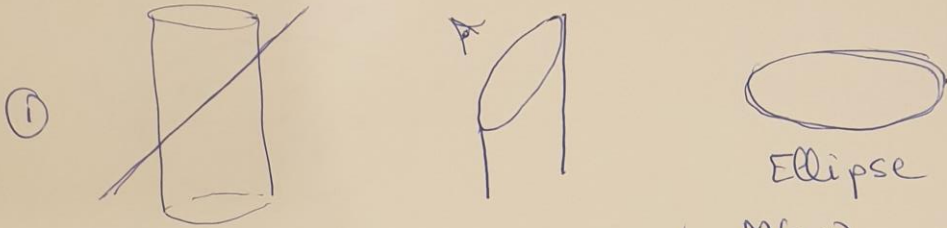
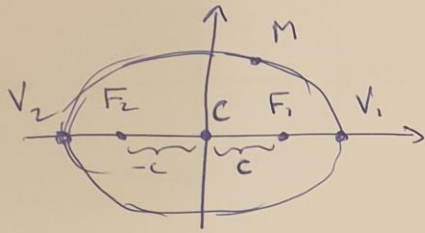


2 - The Ellipse القطع الناقص



# The equation of an Ellipse



The center  $C(0,0)$

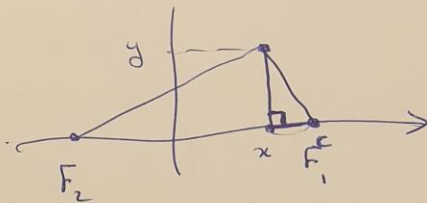
The foci  $F_1(c,0)$   
 $F_2(-c,0)$

$$2a > |b| > 0$$

The vertices  $V_1(a,0)$   
 $V_2(-a,0)$

$$MF_1 + MF_2 = 2a$$

$$\sqrt{(c-x)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$$



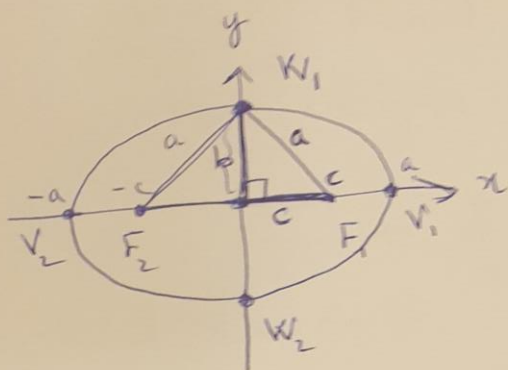
$$\Leftrightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

We put  $b^2 = a^2 - c^2$

$$\Leftrightarrow a^2 = b^2 + c^2$$

$a > b, c$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$



$$W_1(0, b)$$

$$W_2(0, -b)$$

$x$ -axis : The major axis  
 $y$ -axis : The minor axis.

---

The elements of the ellipse of equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

- ① The center :  $C(0, 0)$
- ② The foci :  $F_1(c, 0)$   
 $F_2(-c, 0)$   $c = \sqrt{a^2 - b^2}$
- ③ The vertices :  $V_1(a, 0)$   
 $V_2(-a, 0)$   
 $W_1(0, b)$   
 $W_2(0, -b)$
- ④ The axis : The major  $x$ -axis :  $y=0$   
 The minor  $y$ -axis :  $x=0$

Example: Find the elements of the ellipse of equation

$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

and sketch it.

Answer:  $a=5$  ;  $b=4$  ;  $c=\sqrt{5^2-4^2}=3$ .

① The center  $C(0,0)$

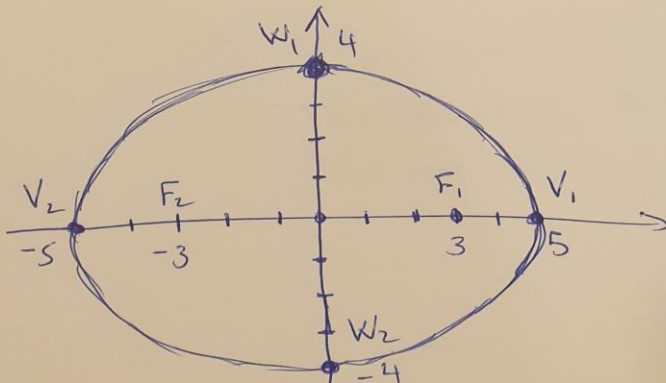
② The foci  $F_1(3,0)$   
 $F_2(-3,0)$

③ The vertices  $V_1(5,0)$   
 $V_2(-5,0)$

$W_1(0,4)$

$W_2(0,-4)$

④ The axis: Major :  $y=0$  x-axis  
Minor :  $x=0$  y-axis



Example: Give the elements of the ellipse of equation

$$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$$

and sketch it.

Solution:  $a=5$  ;  $b=3$  ;  $c=\sqrt{5^2-3^2}=4$

The elements:

① The center  $C(0,0)$

② The foci  $F_1(0,4)$

$F_2(0,-4)$

③ The vertices  $V_1(0,5)$

$V_2(0,-5)$

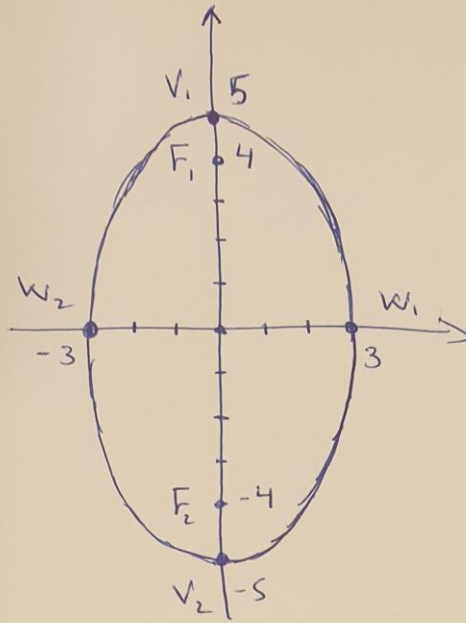
$W_1(3,0)$

$W_2(-3,0)$

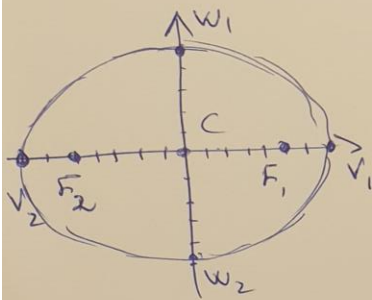
④ The axis:

The major :  $x=0$  (y-axis)

The minor :  $y=0$  (x-axis)



Example: Give the equation of the ellipse of foci  $F_1(5, 0); F_2(-5, 0)$  and vertex  $V_1(7, 0)$ .



$$a = 7 ; c = 5$$

$$b = \sqrt{7^2 - 5^2} = \sqrt{24} = 2\sqrt{6}$$

$$\frac{x^2}{7^2} + \frac{y^2}{(2\sqrt{6})^2} = 1$$