

Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $y^2 - 4y - 8x - 12 = 0$, then sketch it. [4]
2. Find the standard equation of the ellipse with foci at $(1, 5)$, $(1, -3)$ and vertex at $(1, 6)$, then sketch it. [4]
3. Calculate, whenever it is possible, AB and $2A + B^T$, for matrices [4]

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} x - y + z = 5 \\ 2x + y + 5z = 1 \\ 2y + 3z = -6 \end{cases}$$

- (a) Solve this system by using Cramer' rule. [4]
 - (b) Solve this system by using Gauss elimination method. [4]
5. Evaluate the integrals

(a) $\int \left(2e^x + \frac{3}{x} - 4 \sin x \right) dx.$ [2]

(b) $\int 6 \cos x (\sin x)^5 dx.$ [2]

(c) $\int \frac{9x^2}{(x^3 + 1)^4} dx.$ [3]

(d) $\int (3x^2 + 2x + 1) \ln x dx.$ [3]

Mid-Term Exam

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Calculators are not permitted

1. Find the elements of the conic section of equation $4y^2 = -9x^2 + 18x + 27$, then sketch it. [4]
2. Find the standard equation of the parabola with vertex $(2, 3)$ and focus $(2, 1)$, then sketch it. [4]
3. Calculate, whenever it is possible, $A + B^T$ and AB , for matrices [4]

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & -2 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} 2x - 2y + z = 2 \\ x - y + z = 2 \\ 2x + 2y - z = 2 \end{cases}$$

- (a) Solve this system by using Cramer' rule. [4]
 - (b) Solve this system by using Gauss-Jordan elimination method. [4]
5. Evaluate the integrals

(a) $\int \left(4x^3 - \frac{2}{x^3} + e^x \right) dx.$ [2]

(b) $\int 20x^3 (x^4 + 2)^4 dx.$ [2]

(c) $\int \sec^2 x \ln |\sin x| dx.$ [3]

(d) $\int \frac{x+1}{(x-2)(x-1)} dx.$ [3]

Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $4x^2 + 9y^2 - 8x - 36y + 4 = 0$, [4]
then sketch it.
2. Find the standard equation of the hyperbola with foci $(2, 3)$, $(-6, 3)$ and the [4]
distance between its two vertices equals to 6, then sketch it.
3. Calculate, whenever it is possible, $2A - B^T$ and AB , for matrices [4]

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 3 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} 2x + y + z = 1 \\ x - y = 0 \\ y - z = 3 \end{cases}$$

- (a) Solve this system by using Cramer' rule. [4]
 - (b) Solve this system by using Gauss elimination method. [4]
5. Evaluate the integrals

(a) $\int \frac{4x^3 + 1}{\sqrt{x^4 + x + 1}} dx.$ [2]

(b) $\int (x + 1)e^{x^2+2x} dx.$ [2]

(c) $\int (2x + 1) \cos x dx.$ [3]

(d) $\int (2x + 1) \ln x dx.$ [3]

Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $y^2 - 2y + 4x = 3$, then sketch it. [4]
2. Find the standard equation of the ellipse with vertices at $(-4, 2)$, $(6, 2)$ and one of its two foci at $(5, 2)$, then sketch it. [4]
3. Calculate, whenever it is possible, the products $2AB$ and BA of matrices [4]

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} x - 2y + z = 4 \\ -x + 2y + z = -2 \\ 2x - 3y - z = 3 \end{cases}$$

- (a) Solve this system by using Cramer' rule. [4]
 - (b) Solve this system by using Gauss elimination method. [4]
5. Evaluate the integrals

(a) $\int (3x - 1)\sqrt{3x^2 - 2x + 1} dx.$ [2]

(b) $\int (5x + 4)^5 dx.$ [2]

(c) $\int x^3 \ln x dx.$ [3]

(d) $\int \frac{3 \cos(3x) + 2 \sin(2x)}{\sin(3x) - \cos(2x)} dx.$ [3]

Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $y^2 - 4x^2 + 6y - 32x - 59 = 0$, [5]
then sketch it.
2. Find the standard equation of the parabola of focus $F(-1, -2)$ and directrix of [4]
equation $y = 0$, then sketch it.
3. Calculate the product AB of matrices [3]

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -4 & 1 \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} x + y + z = 9 \\ x - y + z = 3 \\ x + y - z = 1 \end{cases}$$

- (a) Solve this system by using Cramer' rule. [4]
 - (b) Solve this system by using Gauss-Jordan elimination method. [4]
5. Evaluate the integrals

(a) $\int 10x^3 (x^4 + 1)^{\frac{3}{2}} dx.$ [2]

(b) $\int 4x \cos(2x) dx.$ [2]

(c) $\int \frac{\frac{1}{x}}{2 + \ln x} dx.$ [2]

(d) $\int x \sec^2 x dx.$ [4]

Mid-Term Exam

Allowed time: 2 hours

Calculators are not permitted

1. Find the elements of the conic section of equation $9x^2 + 4y^2 + 18x - 16y - 11 = 0$ [5]
and then sketch it.
2. Find the standard equation of the hyperbola of foci $F_1(6, 2), F_2(-4, 2)$ and one of [4]
its vertices $V(5, 2)$, then sketch it.
3. Calculate the product AB of matrices [3]

$$A = \begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & 2 \\ 2 & -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -2 \\ -2 & -1 & 2 \\ -2 & -2 & 3 \end{pmatrix}.$$

4. Consider the system of linear equations

$$\begin{cases} x + y + z = 2 \\ x - y + 2z = 3 \\ 2x + z = 3 \end{cases}$$

- (a) Solve this system by using Cramer' rule. [4]
 - (b) Solve this system by using Gauss-Jordan elimination method. [4]
5. Evaluate the integrals

(a) $\int 18x^2(x^3 + 1)^5 dx.$ [2]

(b) $\int 4xe^{2x} dx.$ [3]

(c) $\int \frac{x}{\sqrt{2x^2 + 1}} dx.$ [2]

(d) $\int \frac{3x + 5}{(x - 1)(x + 3)} dx.$ [3]

First Mid-Term Exam

Allowed time: 90min

Calculators are not permitted

- [4] 1. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$. Compute (if possible):
(i) $A + BC$, (ii) $AB + C$

- [5] 2. Compute the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$.

- [6] 3. Solve by Gauss elimination method the system of linear equations

$$\begin{cases} x - y + z = 2 \\ 2x - y - 3z = 1 \\ 3x - 2y - 4z = 1 \end{cases}$$

- [5] 4. Find the elements of the following conic section and then sketch it.

$$4x^2 - 9y^2 - 8x + 36y + 4 = 0.$$

- [5] 5. Find the standard equation of the ellipse with foci $F_1(2, 3)$ and $F_2(2, -5)$ and vertex $V_1(2, 4)$ and then sketch it.

Second Mid-Term Exam

Allowed time: 90min

Calculators are not permitted

1. Evaluate the integrals:

[2] (a) $\int (e^x + x)^3 (e^x + 1) dx;$

[2] (b) $\int x \cos(x^2) dx;$

[3] (c) $\int x \cos(x) dx;$

[3] (d) $\int x^3 \ln(x) dx;$

[2] (e) $\int \frac{3x^2 + 2}{x^3 + 2x + 2} dx;$

[3] (f) $\int \frac{3x}{(x+1)(x-2)} dx.$

[1] 2. (a) Sketch the region \mathcal{R}_1 bounded by the curves

$$y = x^2 + 2, \quad y = 3x.$$

[4] (b) Find the area of the region \mathcal{R}_1 described in part (a).

[1] 3. (a) Sketch the region \mathcal{R}_2 bounded by the curves

$$y = 4 - x^2, \quad y = 0, \quad x = 1.$$

[4] (b) Find the volume of the solid generated by rotating the region \mathcal{R}_2 in part (a) about the y -axis.