# Final Exam - Allowed time: 3 hours

**Calculators are not permitted**

## Q1.

[3] (a) Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$. Compute, if possible, $AB$ and $BA$.

[2] (b) Compute the determinant $\begin{vmatrix} 1 & 1 & -1 \\ 0 & -2 & -5 \\ 1 & 2 & 1 \end{vmatrix}$.

[4] (c) Solve by using Gauss-Jordan Elimination Method the linear system

$$\begin{cases} x + y - z = -3 \\ -2y + 5z = 1 \\ x + 2y + z = 1 \end{cases}$$

## Q2.

[4] (a) Find the standard equation of the ellipse with endpoints of minor axis are $(1, 4)$ and $(1, -2)$ and the distance between foci is 8, and then sketch its graph.

[3] (b) Find the elements of the conic section $y = 4x - x^2$ and then sketch it.

## Q3.

[2,3,3] (a) Compute the integrals:

(i) $\int 8x\left(x^2 + 24\right)^3 \, dx$, (ii) $\int (\ln x)^2 \, dx$, (iii) $\int \frac{3x}{x^2 - 2x - 8} \, dx$.

[3] (b) Sketch the region bounded by the curves $y = x^2$, $y = x + 3$, $x = 1$ and $x = 2$ and compute its area.

[4] (c) The region bounded by the curves $y = 4x - x^2$ and $y = x$ is rotated about the $y-$axis to form a solid $\mathcal{S}$. Use the method of cylindrical shells to find the volume of $\mathcal{S}$.

[2] (d) Give the Cartesian coordinates of the points in polar coordinates

$$M\left(\sqrt{2}, \frac{\pi}{4}\right) \quad \text{and} \quad N(2, \pi).$$

## Q4.

[3] (a) Let $z = xy^2 + \sin(xy)$, where $x = s^2t$ and $y = \frac{t}{s}$. Use the chaine rule to compute the partial derivatives $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

[4] (b) Solve the differential equation: $xy' + y = 3x^2 + 1$.