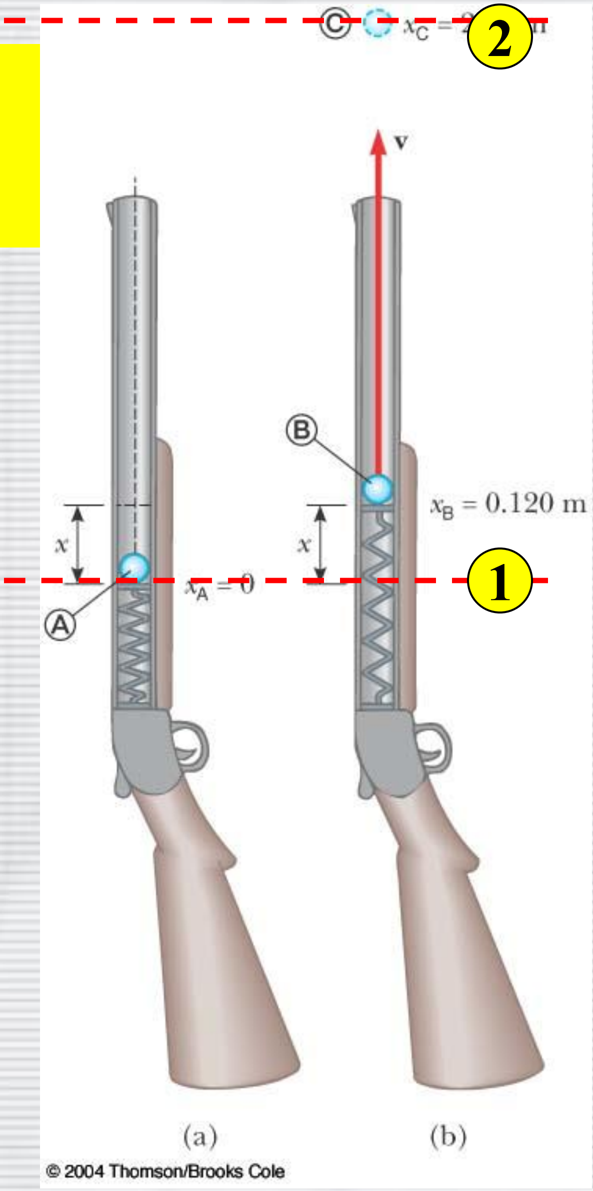
Example 8.5 :

A spring-loaded popgun $x = 0.12$ m, $m = 35$ g, $h = x_c = 20$ m,

Neglect all resistive forces ， k = ?

$$\Delta K + \Delta U = 0$$
$$K_2 - K_1 + U_2 - U_1 = 0$$
$$K_2 + U_2 = K_1 + U_1 = constant = E \equiv Total\ mechanical\ energy$$



$$K_2 + U_2 = K_1 + U_1$$
$$0 + U_2 = 0 + U_1$$
$$U_2 = U_{2s} + U_{2g} = 0 + mgh$$
$$U_1 = U_{1s} + U_{1g} = \frac{1}{2}kx^2 + 0$$
$$mgh = \frac{1}{2}kx^2$$
$$k = \frac{2mgh}{x^2} = 953\ N/m$$

What is the speed of projectile as it moves through equilibrium position of the spring

$$K_2 + U_2 = K_1 + U_1$$

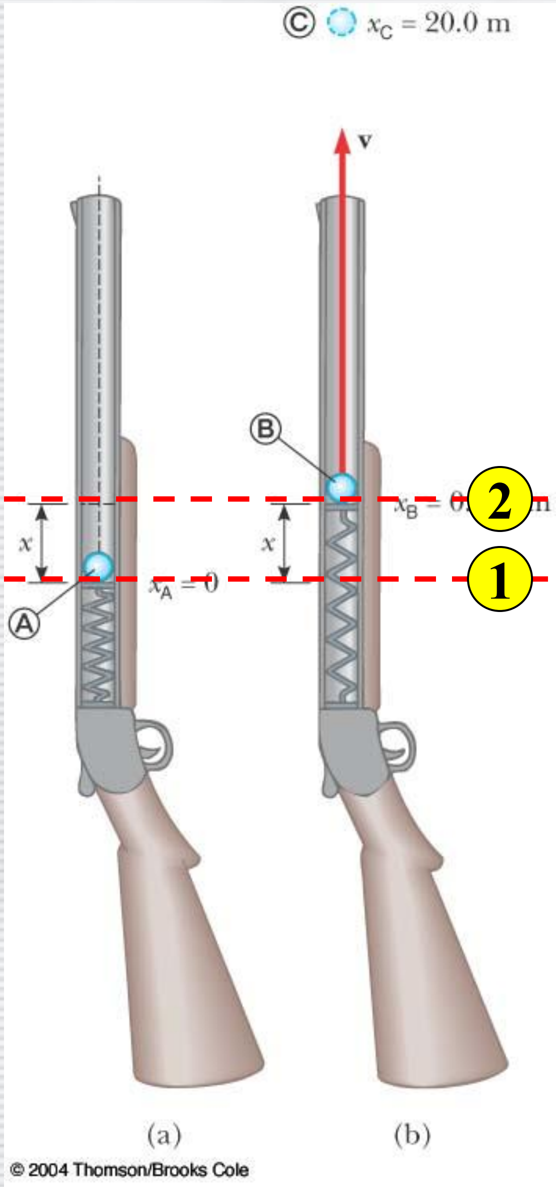
$$K_2 + U_2 = 0 + U_1$$

$$U_2 = U_{2s} + U_{2g} = 0 + mgx$$

$$U_1 = U_{1s} + U_{1g} = \frac{1}{2}kx^2 + 0$$

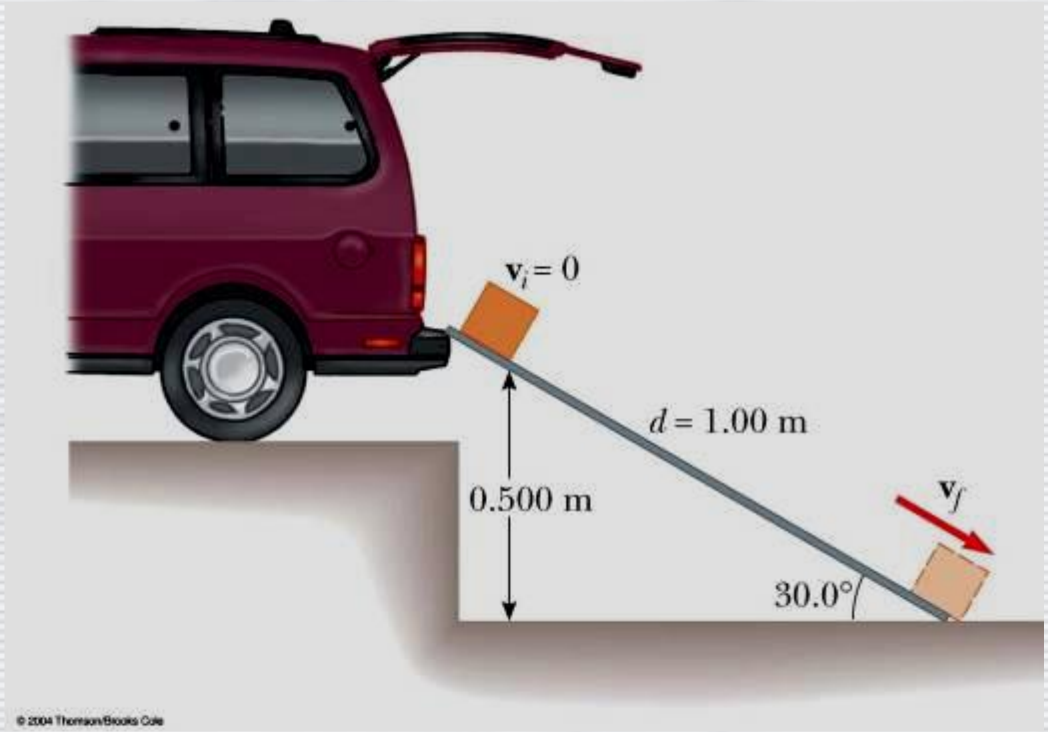
$$\frac{1}{2}mv_2^2 + mgh = \frac{1}{2}kx^2$$

$$v_2 = \sqrt{\frac{kx^2}{m} - 2gx} = 19.7\ m/s$$



Example 8.6

m =3 kg , d = 1 m , θ = 30° ,
v_i = 0 , f_k = 5 N , h = 0.5 m ,
v_f = ?



$$\Delta K + \Delta U = W_{nc}$$

$$K_f - K_i + U_f - U_i = W_{f_k}$$

$$\frac{1}{2} m v_f^2 - 0 + 0 - mgh = -f_k d$$

$$v_f = \sqrt{\frac{2}{m}\left(mgh - f_k d\right)} = 2.54\ m/s$$

What happen when you don't know h?

Example 8.6

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.12. The child starts from rest at the top.

(A) Determine his speed at the bottom, assuming no friction is present.

$v \approx 6.26$ m/s

(B) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.

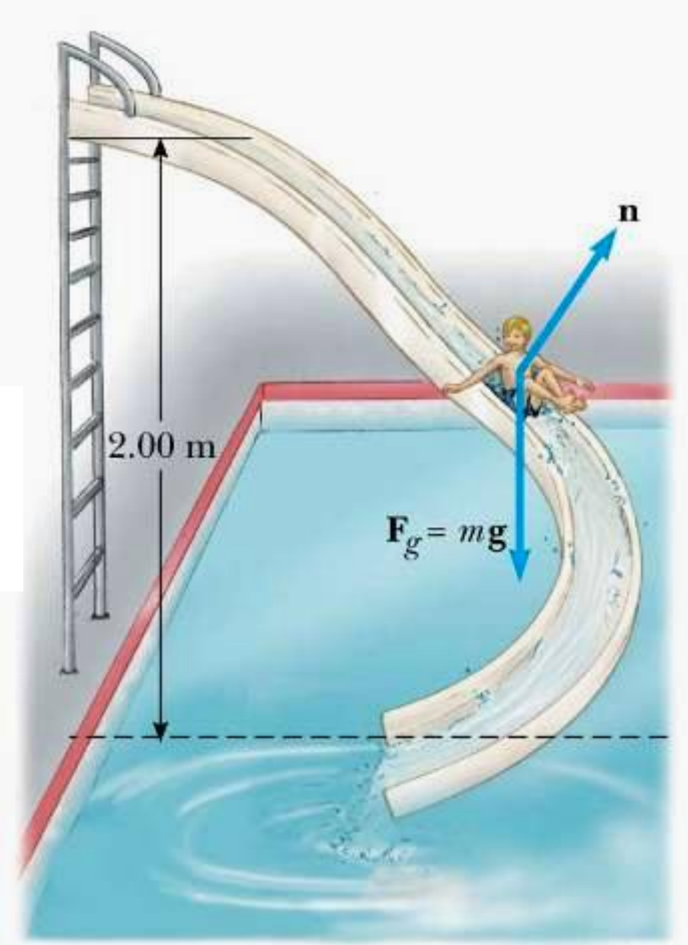
$$\Delta E_{\text{mech}} = (K_f + U_f) - (K_i + U_i)$$

$$= (\tfrac{1}{2} m v_f^2 + 0) - (0 + mgh) = \tfrac{1}{2} m v_f^2 - mgh$$

$$= \tfrac{1}{2}(20.0\text{ kg})(3.00\text{ m/s})^2$$

$$- (20.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})$$

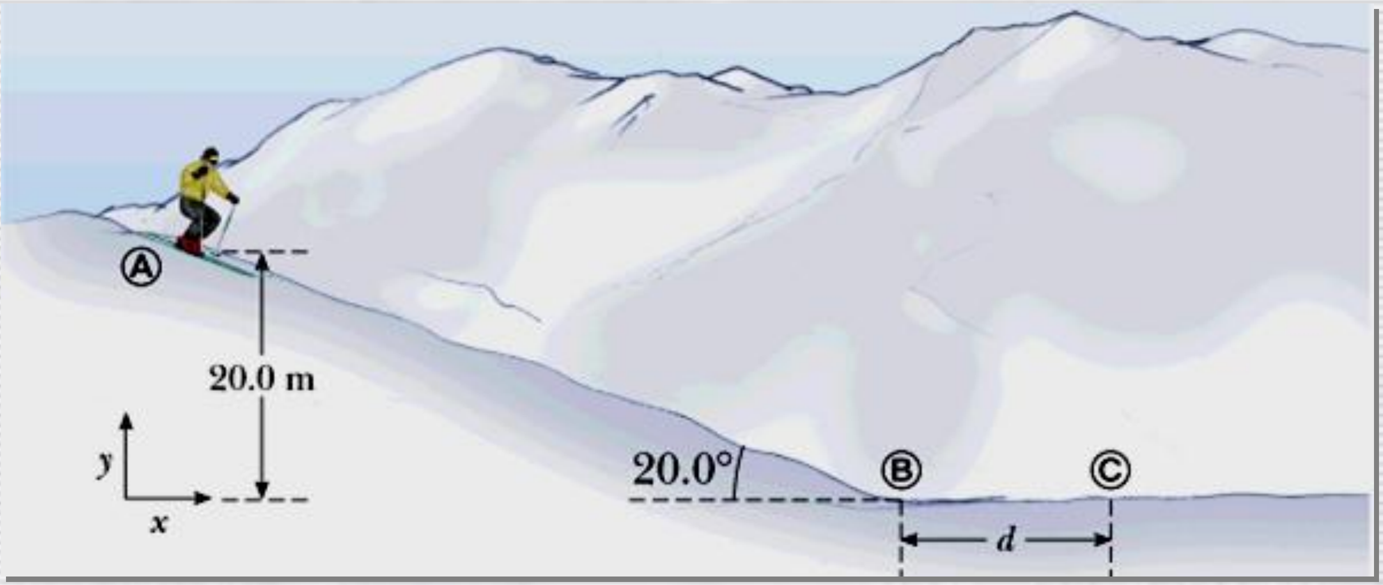
$$= -302\text{ J}$$



example 8.8

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210.

(a)How far does she travel on the horizontal stretch.



$$\Delta E = 0 = \Delta K + \Delta U$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 20.0} = 19.8\ m/s$$

$$\Delta K = K_f - K_i = -f_k d$$

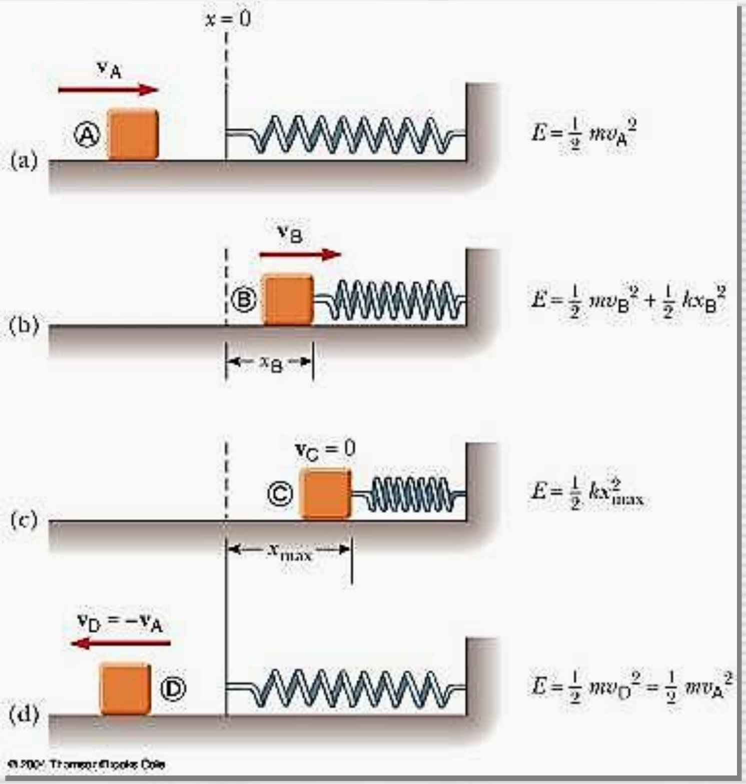
Since $K_f = 0$ $\quad -K_i = -f_k d;\ f_k d = K_i$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2 \times 0.210 \times 9.80} = 95.2\ m$$

Example 8.9

A block sliding on a smooth, horizontal surface collides with a light spring. (a) Initially the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy of the system remains constant throughout the motion.



(i) $\mu_k = 0$

$$E_t = \tfrac{1}{2} m v_i^2$$

$$E_t = \tfrac{1}{2} k x_m^2 \quad \Rightarrow x_m = \sqrt{\frac{m}{k}}$$

$$E_t = \tfrac{1}{2} m v_f^2 \quad \Rightarrow \vec{v}_f = -\vec{v}_i$$

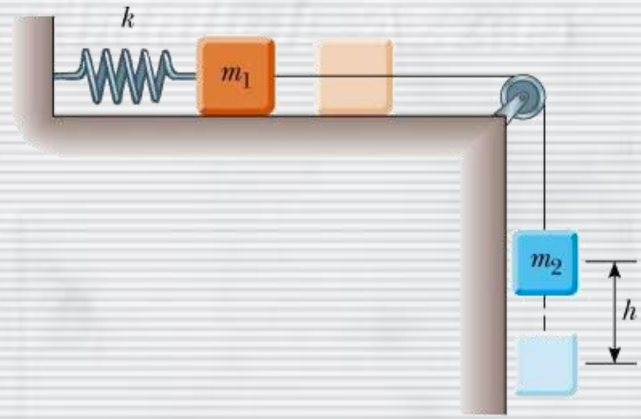
(i) $\mu_k \neq 0$

$$E_t - \mu_k m g (x_m + x_0) = \tfrac{1}{2} k x_m^2$$

$$E_t - \mu_k m g 2 (x_m + x_0) = \tfrac{1}{2} m v_f^2$$

Example

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.15. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



$$\Delta E_{\text{mech}} = \Delta U_g + \Delta U_s$$

$$\Delta E_{\text{mech}} = -f_k h = -\mu_k m_1 g h$$

$$\Delta U_s = U_{sf} - U_{si} = \frac{1}{2}kh^2 - 0$$

$$-\mu_k m_1 g h = -m_2 g h + \frac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Work done by constant force: $W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta$

e.g. Work done by gravity: $W_g = -mg\Delta y$

Change in gravitational PE: $\Delta U_g = -W_g = mg\Delta y$

Work done by variable force: $W = \int_{x_1}^{x_2} F_x \, dx$

e.g. Work ***by*** spring: $W_{x_1 \to x_2} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$

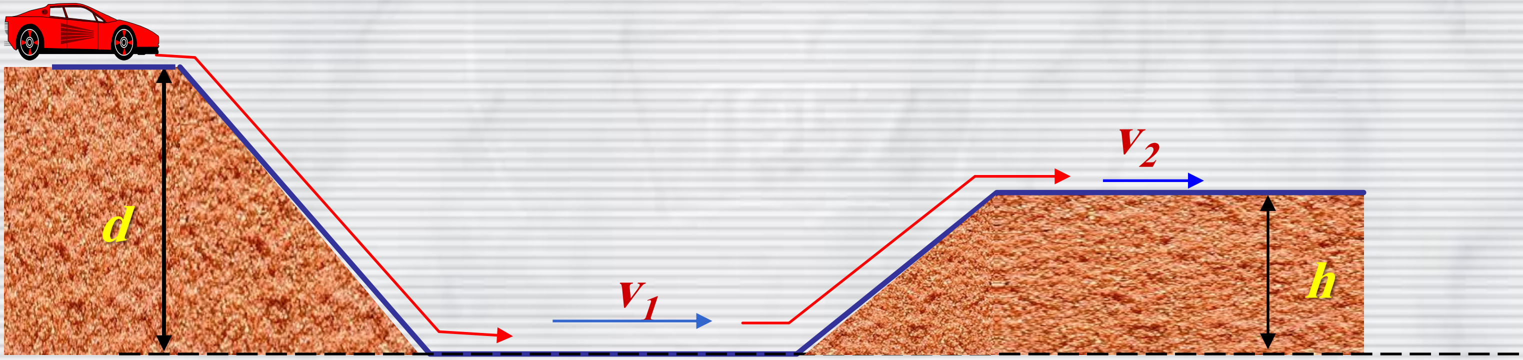
Change in spring PE: $\Delta U = -W_s = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$

Work – Energy Thm: $\sum W_{non\text{-}con} = \Delta E_{mech} = \Delta K + \Delta U$

Example : Hotwheel

A toy car slides on the frictionless track shown below. It starts at rest, drops a distance *d*, moves horizontally at speed v_1, rises a distance *h*, and ends up moving horizontally with speed v_2.

Find v_1 and v_2.



- K+U energy is conserved, so $\Delta E = 0$

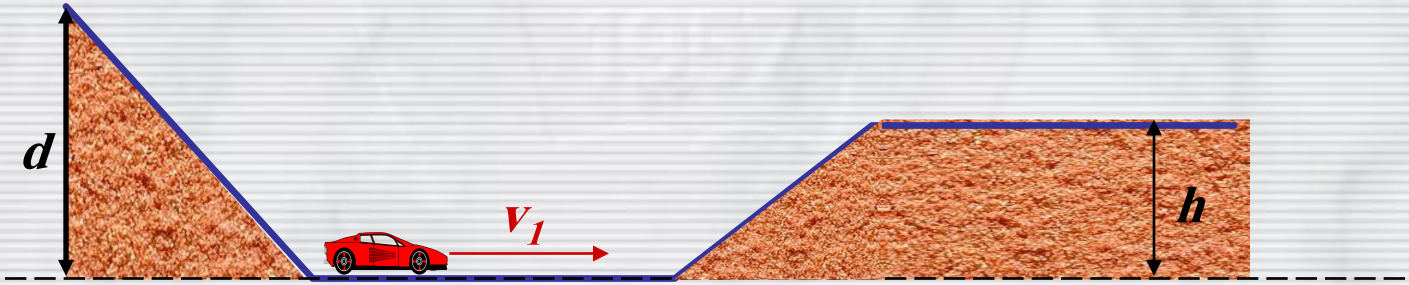
$$\Delta K = -\Delta U$$

- Moving down a distance *d*,

$$\Delta U = -mgd, \quad \Delta K = \frac{1}{2} m v_1^2$$

- Solving for the speed:

$$v_1 = \sqrt{2gd}$$

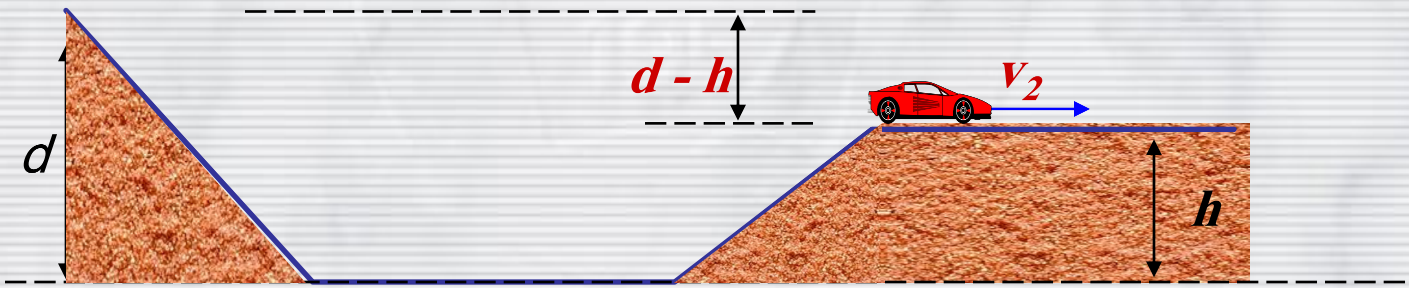


- **At the end, we are a distance *d - h* below our starting point.**

$$\Delta U = -mg(d - h), \quad \Delta K = \frac{1}{2} m v_2^2$$

Solving for the speed:

$$v_2 = \sqrt{2g(d-h)}$$



Example:

With what speed does the weight have just before contact with the nail?

$$\Delta K + \Delta U = 0$$

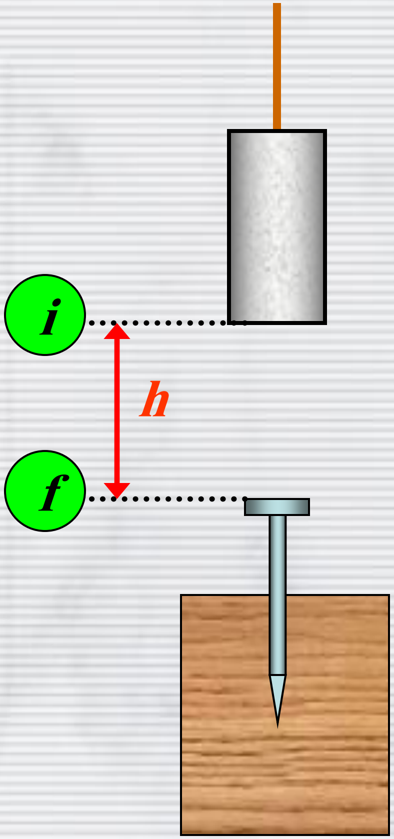
$$U_i = mgh$$

$$U_f = 0$$

$$K_i = 0$$

$$K_f = \frac{1}{2} m v^2$$

$$v = \sqrt{2gh}$$



What is the force of resistance between the nail and the block?

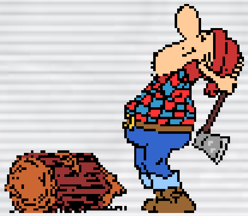
$$\Delta K + \Delta U = W_{nc} = -fd$$

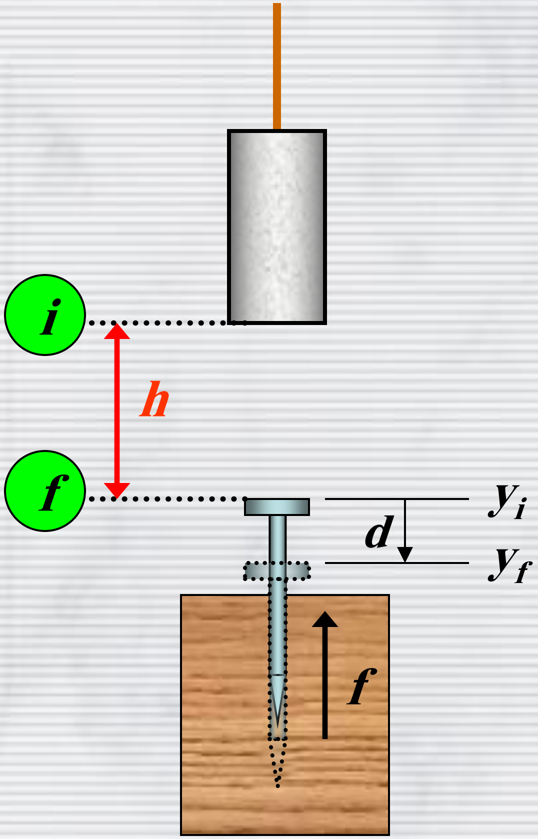
$U_i = 0$

$U_f = -mgd$

$K_i = mgh$

$K_f = 0$



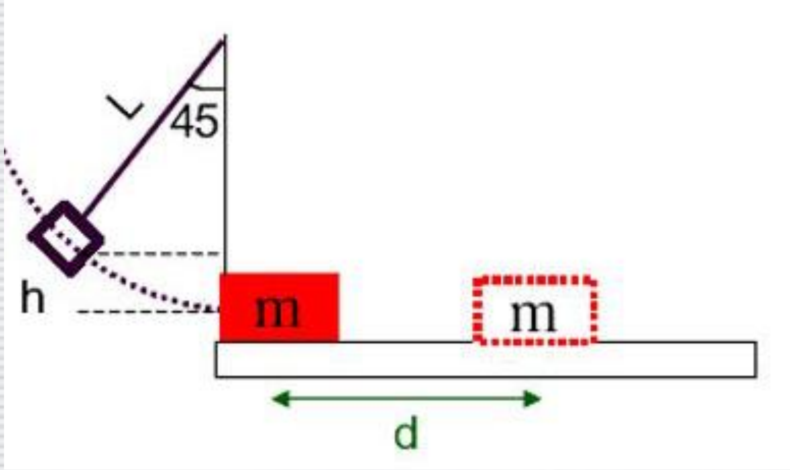


Solve you get

$$f = mg\left(\frac{h+d}{d}\right)$$

Using Energy to Find Resistive Forces

Pendulum & Sliding Block



What is the work done by friction?

$$\Delta K + \Delta U = W_{nc}$$

$K_i = K_f = 0$

$U_i = mgh = mgL(1-\cos\theta)$

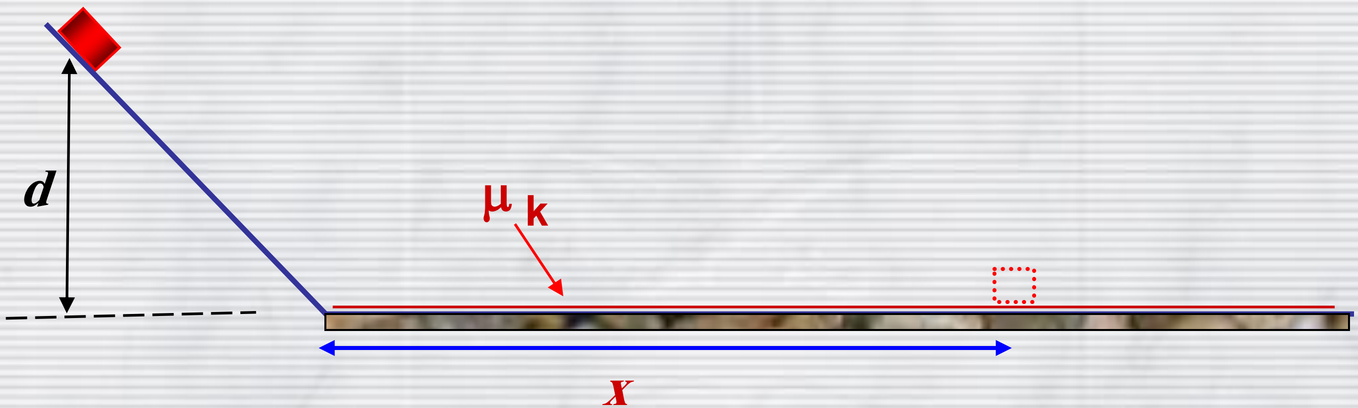
$U_f = 0$

$$W_{nc} = \Delta U = U_f - U_i = mgL(1-\cos\theta) = -f\,d$$

Example;

A block slides down a frictionless ramp. Suppose the horizontal (bottom) portion of the track is rough, such that the coefficient of kinetic friction between the block and the track is μ_k.

How far, *x*, does the block go along the bottom portion of the track before stopping?



Using $W_{nc} = \Delta K + \Delta U$

As before, $\Delta U = -mgd$

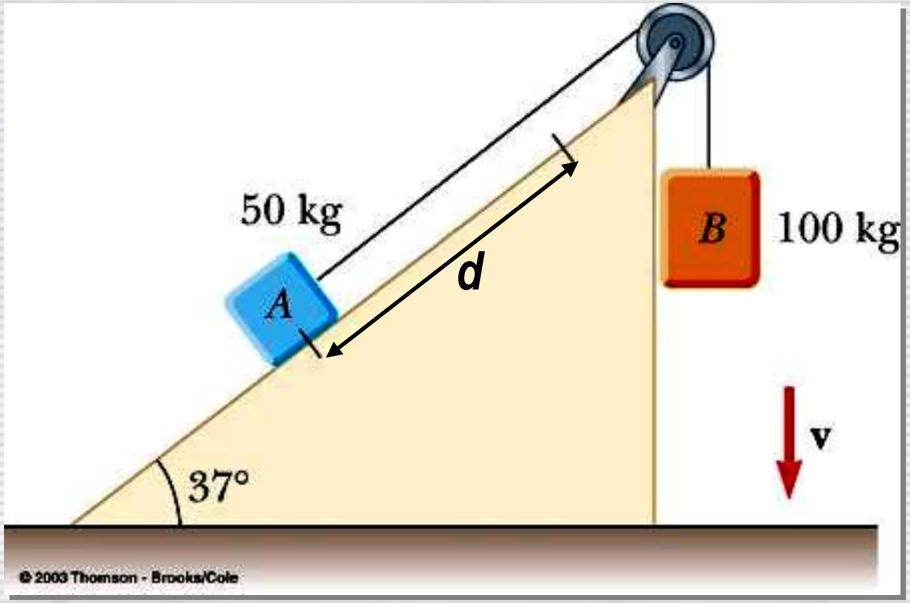
W_{nc} = work done by friction $= -\mu_k mgx$.

$\Delta K = 0$ since the block starts out and ends up at rest.

$W_{nc} = \Delta U$ ⇨ $-\mu_k mgx = -mgd$ ⇨ $x = d/\mu_k$

Example

Two blocks, *A* and *B* (m_A=*50 kg* and m_B=100 kg), are connected by a string as shown. If the blocks begin at rest, what will their speeds be after *A* has slid a distance *d* = 0.25 m? Assume the pulley and incline are frictionless.

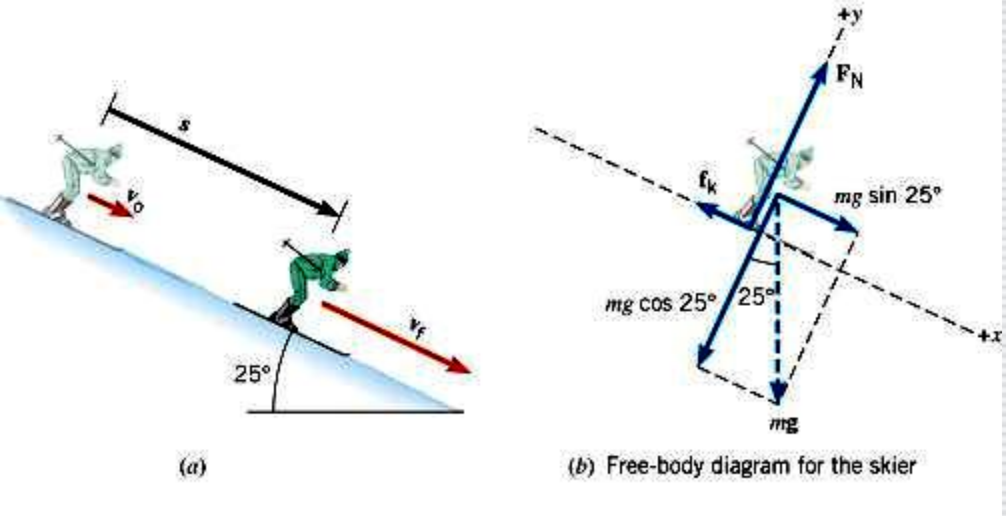


ANS: 1.51 m/s

Example:

A skier (m=58 kg) is traveling down a 25 degree slope. His skies against the snow exert a frictional force of 70 N. He starts out with a velocity of 3.6 m/s. What velocity does he end up with after traveling 57 m downhill?

What is the net force along the direction of the displacement?



$$\sum F_s = mg\sin\theta + (-70N)$$

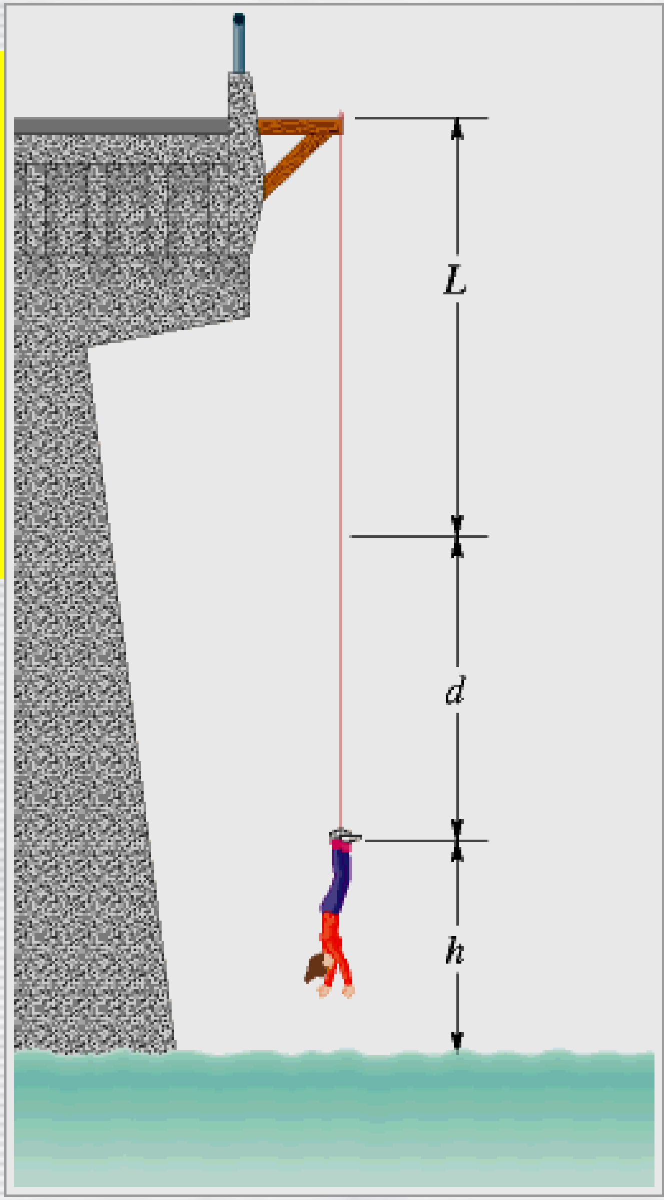
$$W = \sum F_s \times s = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$[mg\sin\theta + (-70N)] \times s = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

From this, we can solve for v!

Sample Problem

A 61.0 kg bungee-cord jumper is on a bridge 45.0 m above a river. The elastic bungee cord has a relaxed length of *L* = 25.0 m. Assume that the cord obeys Hooke's law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height *h* of her feet above the water at her lowest point?



$$\Delta K + \Delta U_e + \Delta U_g = 0$$

$$\Delta U_g = mg\Delta y = -mg(L + d)$$

$$\Delta U_e = \frac{1}{2}kd^2$$

$$0+\frac{1}{2}kd^2-mg\,(L+d)=0$$

$$\frac{1}{2}kd^2-mgL-mgd=0$$

$$\frac{1}{2}(160\,N/m)\,d^2-(61.0\,kg)(9.8\,m/s^2)(25.0\,m)$$

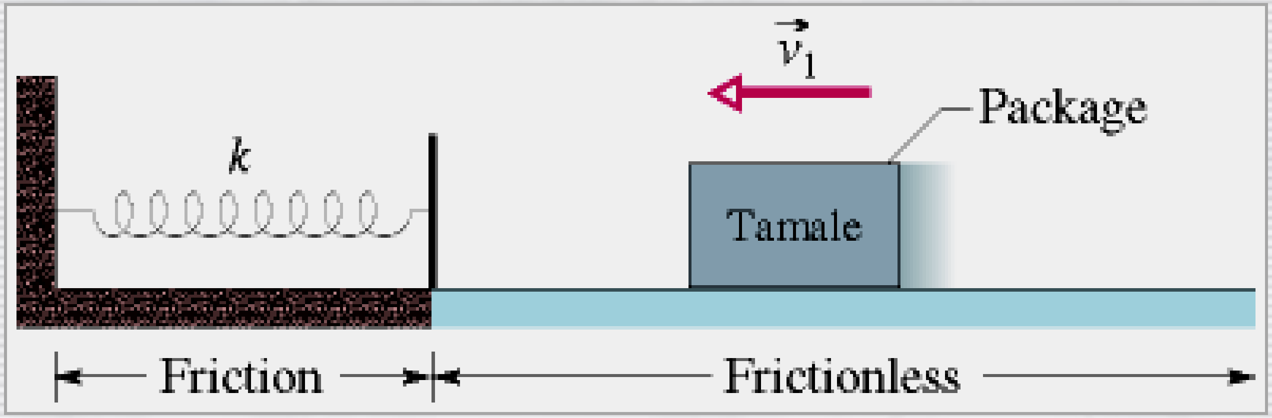
$$-(61.0\,kg)(9.8\,m/s^2)\,d=0$$

$$d=17.9\,m$$

$$h=45.0\,m-42.9\,m=2.1\,m$$

Sample Problem

In Fig., a 2.0 kg package of tamale slides along a floor with speed v_1 = 4.0 m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on it. The spring constant is 10,000 N/m. By what distance d is the spring compressed when the package stops?



SOLUTION:

$$E_{mec,2} = E_{mec,1} - \Delta E_{th}$$

$$E_{mec,1} = K_1 + U_1 = \frac{1}{2} m v_1^2 + 0$$

$$E_{mec,2} = K_2 + U_2 = 0 + \frac{1}{2} k d^2$$

$$\frac{1}{2} k d^2 = \frac{1}{2} m v_1^2 - f_k d$$

$$5000\, d^2 + 15\, d - 16 = 0$$

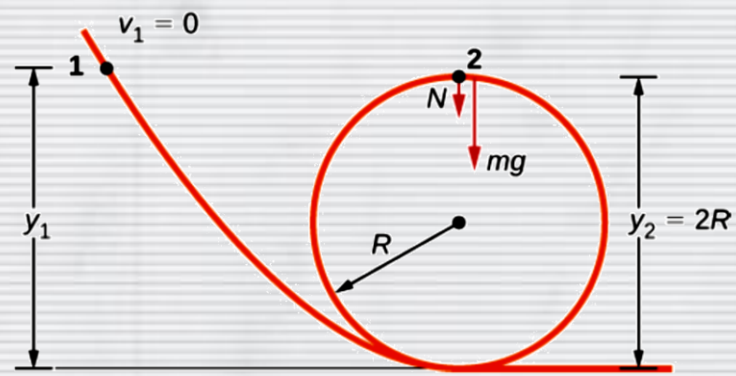
$$d = 0.055\ \text{m} = 5.5\ \text{cm}$$

Example:

The frictionless track for a toy car includes a loop-the-loop of radius R. How high, measured from the bottom of the loop, must the car be placed to start from rest on the approaching, section of track and go all the way around the loop?

$$\Delta k + \Delta U = w_{nc} = 0$$

$$-mg(y_2 - y_1) = \frac{1}{2} m v_2^2$$



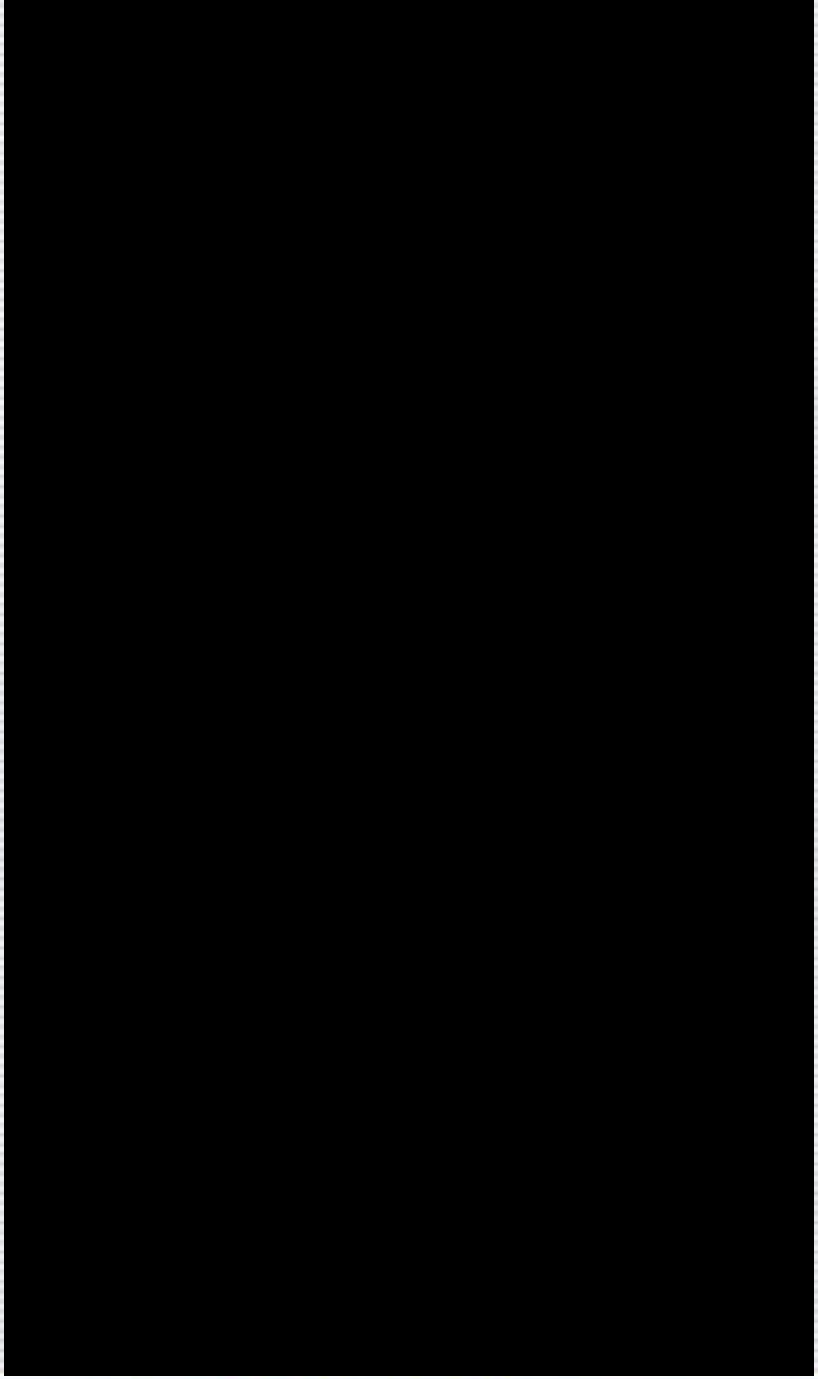
At the top of the loop, the normal force and gravity are both down and the acceleration is centripetal, so

$$a_{top} = \frac{F}{m} = \frac{N + mg}{m} = \frac{v_2^2}{R}$$

The condition for maintaining contact with the track is that there must be some normal force, however slight; that is, N > 0.

Substituting for v_2 and N, we can find the condition for y_1

$$N = \frac{-mg + m v_2^2}{R} = \frac{-mg + 2mg\,(y_1 - 2R)}{R} > 0 \qquad or \quad y_1 > \frac{5R}{2}$$



8.5 Relationship Between Conservative Forces and Potential Energy

A conservative force does not depend on the path

The work depends only on the initial and final coordinates.

Potential energy function U such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by a conservative force **F**

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

where F_x is the component of **F** in the direction of the displacement.

the work done by a conservative force acting equals the negative of the change in the potential energy associated with that force

Where $\Delta U = U_f - U_i$

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

Therefore, ΔU is negative when F_x and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

The term *potential energy* implies that the system has the potential, or capability, of either gaining kinetic energy or doing work when it is released under the influence of a conservative force exerted on an object by some other member of the system.

We can then define the potential energy function as

$$U_f = -\int_{x_i}^{x_f} F_x dx + U_i$$

- The value of U_i is often taken to be zero for the reference configuration. It really does not matter what value we assign to U_i because any nonzero value merely shifts $U_f(x)$ by a constant amount and only the *change* in potential energy is physically meaningful.

$$dU = -F_x dx$$

Therefore, the conservative force is related to the potential energy function through the relationship[3]

$$F_x = -\frac{dU}{dx}$$

That is, **the x component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to x.**

We can easily check this relationship for the two examples already discussed. In the case of the deformed spring, $U_s = \frac{1}{2}kx^2$, and therefore

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

which corresponds to the restoring force in the spring (Hooke's law). Because the gravitational potential energy function is $U_g = mgy$, it follows from Equation 8.18 that $F_g = -mg$ when we differentiate U_g with respect to y instead of x.

We now see that U is an important function because a conservative force can be derived from it. Furthermore, Equation 8.18 should clarify the fact that adding a constant to the potential energy is unimportant because the derivative of a constant is zero.

Quick Quiz 8.11 What does the slope of a graph of $U(x)$ versus x represent? (a) the magnitude of the force on the object (b) the negative of the magnitude of the force on the object (c) the x component of the force on the object (d) the negative of the x component of the force on the object.

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles $m_{car} = 1450kg$ $Weight = mg = 14200N$

Coefficient of Rolling Friction; μ=0.016 $\mu n = \mu mg = 227N$

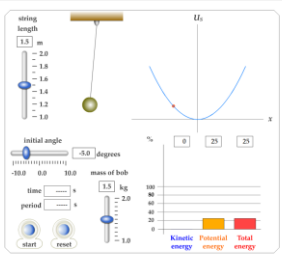
Air Drag $f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$

Total Resistance $f_t = f_r + f_a$

Total power to keep speed v=26.8m/s=60mi/h $P = f_t v = (691N) \cdot 26.8 = 18.5kW$

Power to overcome each component of resistance $P_r = f_r v = (227) \cdot 26.8 = 6.08kW$

$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$



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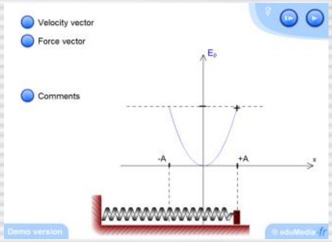

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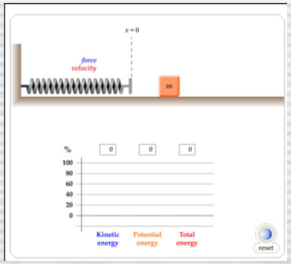

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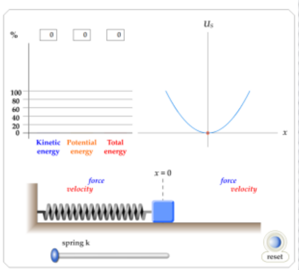

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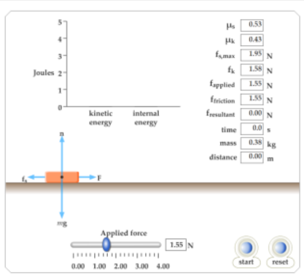

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SUMMARY

$$\sum W_{nc} = \Delta K + \Delta U$$

If zero, then mechanical energy is conserved!

$\Delta K = \frac{1}{2} m(v_f^2 - v_i^2)$

e.g. work by friction $f_K d$

e.g. $\Delta U_g = mg\Delta y$

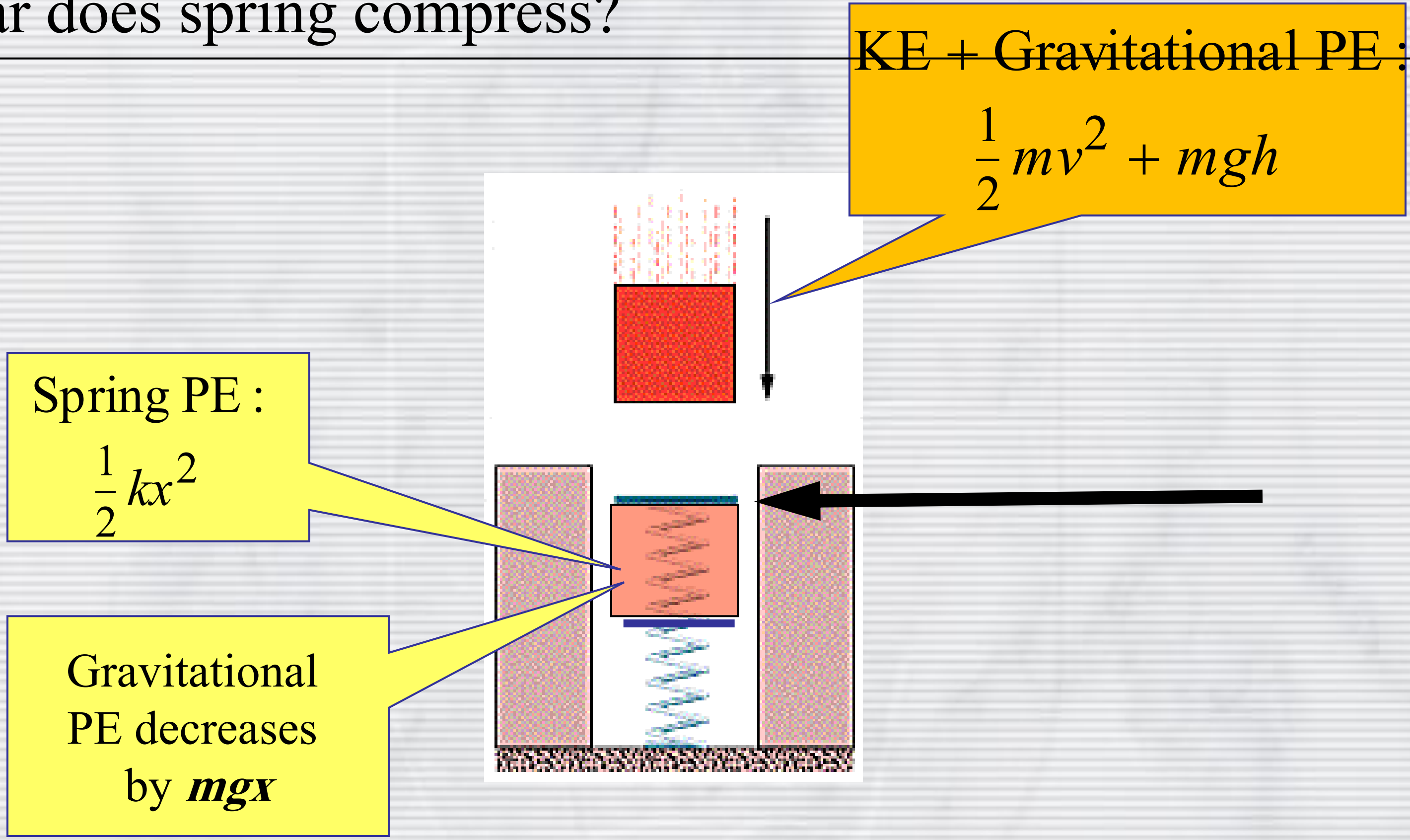
$\Delta U_{el} = \frac{1}{2} k \Delta x^2$

NOT very good for:

1) **times**
2) **directions**
3) **accelerations**

PROBLEM (?)

Mass is dropped on spring. …
How far does spring compress?



$$\frac{1}{2}mv^2 + mgh = -mgx + \frac{1}{2}kx^2$$