CHAPTER 8

Potential Energy and Conservation of Energy

- One form of energy can be converted into another form of energy.
- Conservative and non-conservative forces

Kinetic energy: Energy associated with *motion*

Potential energy: Energy associated with *position*

Potential energy U:

- Can be thought of as stored energy that can either do work or be converted to kinetic energy.
- When work gets done on an object, its potential and/or kinetic energy increases.

⇨ There are different types of potential energy:

- Gravitational energy
- Elastic potential energy (energy in an stretched spring)
- Others (magnetic, electric, chemical, …)

Gravitational Potential Energy

Potential Energy (PE) ≡ Energy associated with position

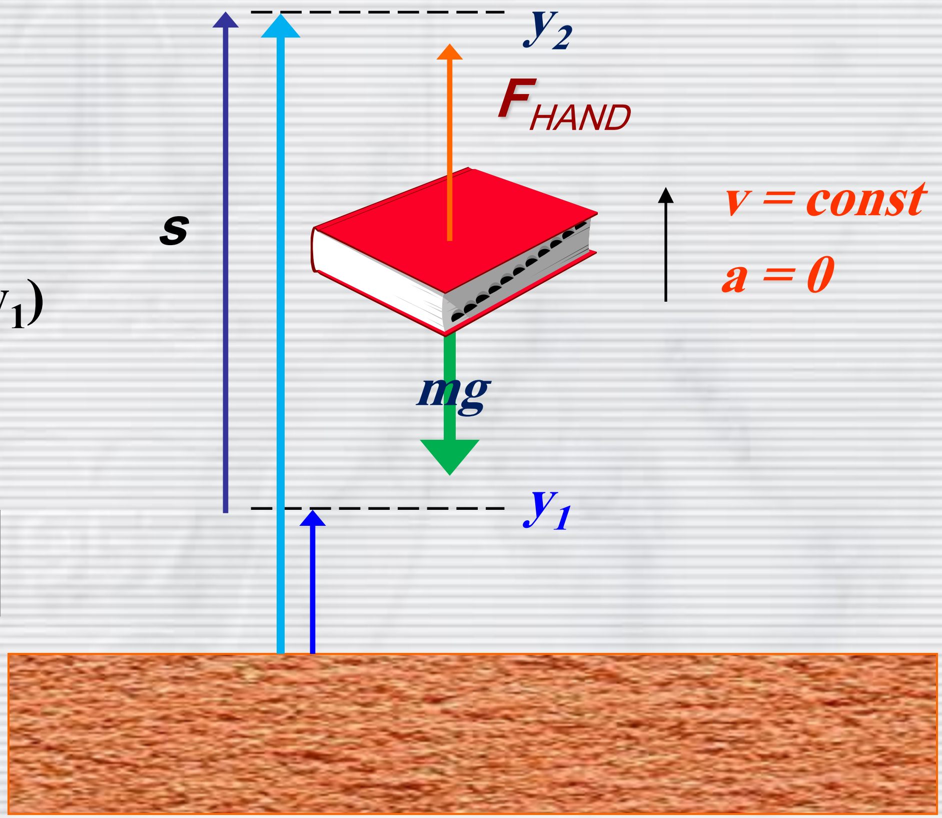
Consider a problem in which the height of a mass above the Earth changes from y_1 to y_2:

W_{grav}=?

UP ➡ $W_g = -\,mg\,s = -\,mg\,(y_2 - y_1)$

Down ➡ $W_g = +mg\,s$

$$W_g = -\,mg\,(y_2 - y_1)$$



$mgy \equiv U_g \equiv$ gravitational potential energy (PE)

➡ $U_2 - U_1 = \Delta U$

➡ $W_g = -mg(y_2 - y_1) = U_1 - U_2 = -\Delta U_g$

$$W_g = -\Delta U_g$$

Changing the configuration of an interacting system requires work

example: lifting a book

The change in potential energy is equal to the negative of the work done

$$\Delta U_g = -W$$

But Work/Kinetic Energy Theorem says: $W = \Delta K$

$$W = -\Delta U = \Delta K$$

$$\Delta K + \Delta U = 0$$

Total Mechanical Energy

The change in potential energy is equal to the negative of the work done

$$\Delta U = -W$$

But Work/Kinetic Energy Theorem says: $W = \Delta K$

$$W = -\Delta U = \Delta K \quad \Rightarrow \quad \Delta K + \Delta U = 0$$

$$\Delta K + \Delta U = 0$$

$$K_2 - K_1 + U_2 - U_1 = 0$$

$$K_2 + U_2 = K_1 + U_1 = constant = E \equiv Total\ mechanical\ energy$$

NOTE that the ONLY forces is gravitational energy which doing the work

The sum of K and U for any state of the system = the sum of K and U for any other state of the system

In an isolated system acted upon only by conservative forces

Mechanical Energy is conserved

Example;

An elevator cab of mass m = 500 kg is descending with speed 4.0 m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration a=g/5 (Fig.).

(a) During the fall through a distance d = 12 m, what is the work W_g done on the cab by the gravitational force F_g?

$$W_g = mgd \cos 0° = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})$$
$$= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ}.$$

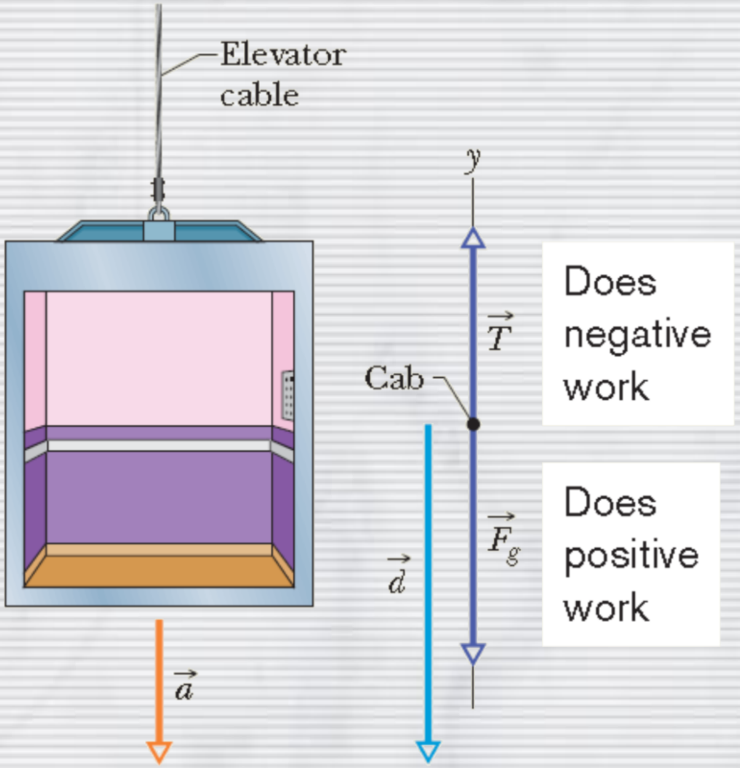
(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull of the elevator cable?

$$T - F_g = ma.$$

Solving for T, substituting mg for F_g, and then we obtain

$W_T = Td \cos \phi =$ m$(a + g)$ d cos ϕ. a=- g/5, d=12m , ϕ=180°

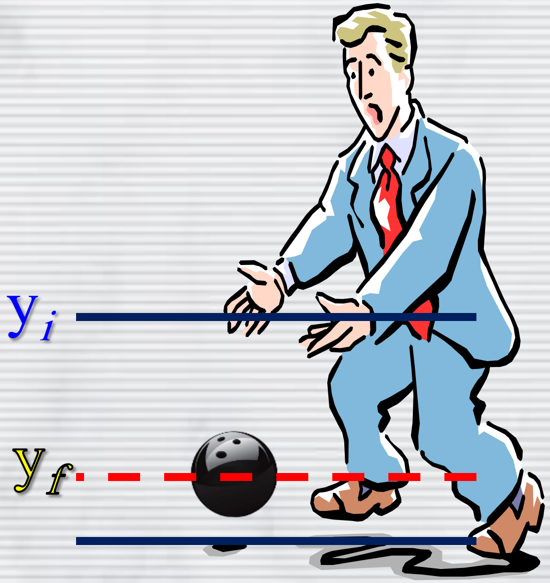
W_T= -47 kJ



Example 8.1

A bowler drops bowling ball of mass 7 kg on his toe. Choosing floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls.

Let's assume the top of the toe is 0.03 m from the floor and the hand was 0.5 m above the floor.



$$U_i = mgy_i = 7\times 9.8\times 0.5 = 34.3\,J$$

$$U_f = mgy_f = 7\times 9.8\times 0.03 = 2.06\,J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24\,J \cong 30\,J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

Assuming the bowler's height is 1.8 m

What has to change?

First we must re-compute the positions of ball at the hand and of the toe.

Assuming the bowler's height is 1.8 m, the ball's original position is –1.3 m, and the toe is at –1.77 m.

$$U_i = mgy_i = 7\times 9.8\times(-1.3) = -89.2J$$

$$U_f = mgy_f = 7\times 9.8\times(-1.77) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

$$\vec{F}_S = -k\vec{x}$$

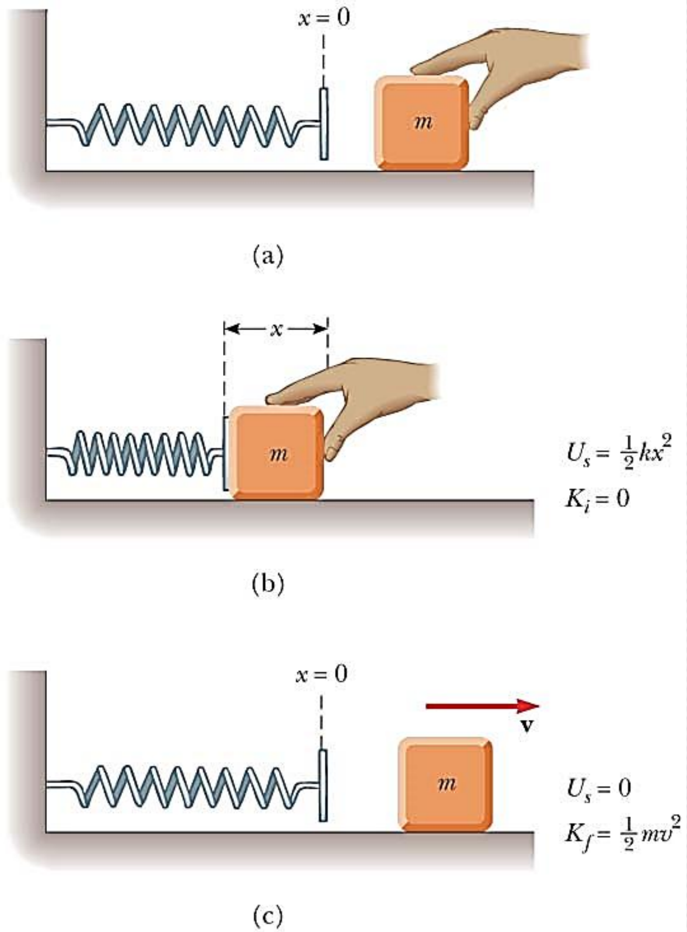
Work done by Spring

$$dW_S = \vec{F}_S \cdot d\vec{x}$$

$$W_S = \int_{x_i}^{x_f} \vec{F}_S \cdot d\vec{x} = \int_{x_i}^{x_f} (-kx) \cdot dx = -\tfrac{1}{2} k \left(x_f^2 - x_i^2 \right)$$

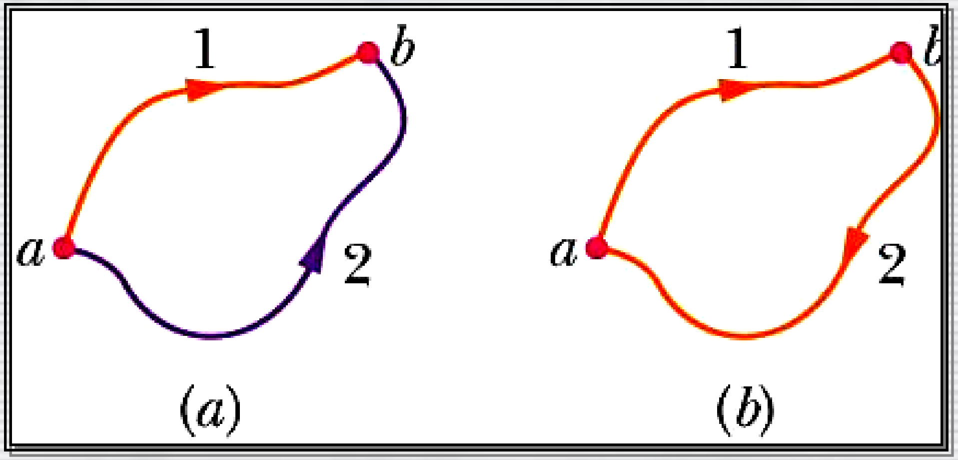
$$U_S = \tfrac{1}{2} k x^2$$

$$\Rightarrow \quad W_S = -\Delta U_S = -\left(U_{Sf} - U_{Si} \right)$$



Conservative Forces

(a) A force is conservative if work done by that force acting on a particle moving between points is independent of the path the particle takes between the two points

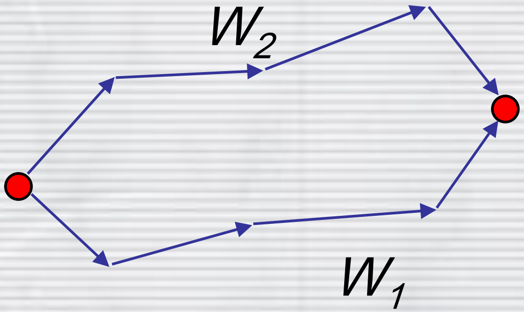


(b) The total work done by a conservative force is zero when the particle moves around any closed path and returns to its initial position

Conservative Forces

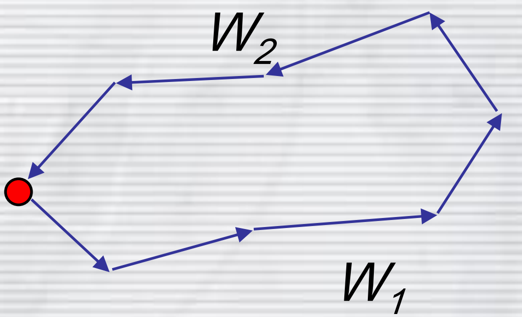
To repeat the idea on the last slide: We have seen that the work done by a conservative force does not depend on the path taken.

$$W_1 = W_2$$



Therefore the work done in a closed path is 0.

$$W_{NET} = W_1 - W_2$$
$$= W_1 - W_1 = 0$$



Work done by gravity

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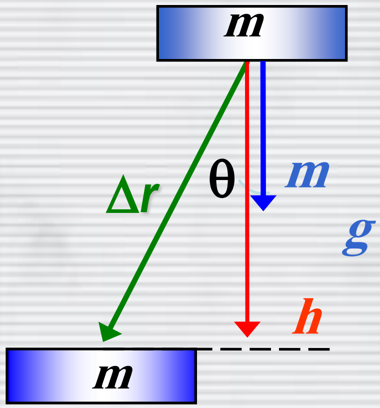
- $W_g = F.\Delta r = mg\Delta r \cos\theta = mg\,h$

$W_g = mgh$ (Depends only on h!)



$$W_{NET} = W_1 + W_2 + \ldots + W_n$$

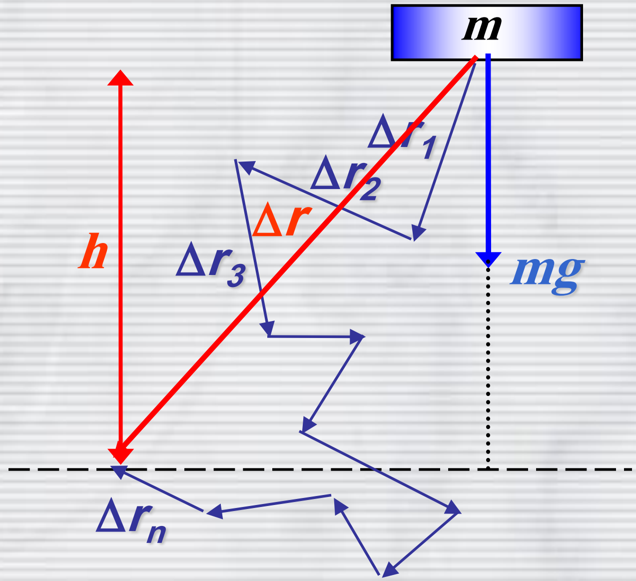
$$= F \bullet \Delta r_1 + F \bullet \Delta r_2 + \ldots + F \bullet \Delta r_n$$

$$= F \bullet (\Delta r_1 + \Delta r_2 + \ldots + \Delta r_n)$$

$$= F \bullet \Delta r$$

$$= F\,h$$

$$W_g = mg\,h$$



Depends only on h,

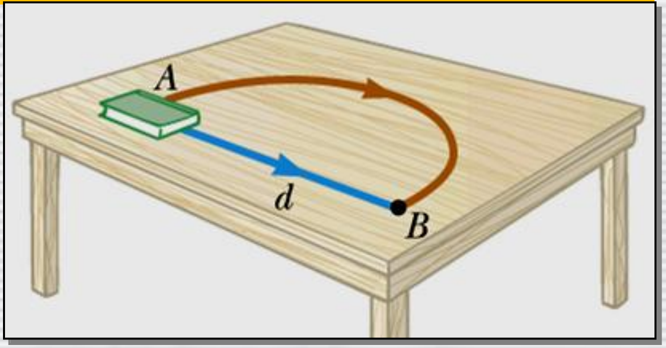
not on path taken!

Non-conservative forces:

A force is non-conservative if it causes a change in mechanical energy; mechanical energy is the sum of kinetic and potential energy.

Example: Frictional force.

- **This energy cannot be converted back into other forms of energy (irreversible).**
- **Work does depend on path.**



For straight line $W = -f\,d$

For semi-circle path $W = -f\,(\pi\, d/2)$

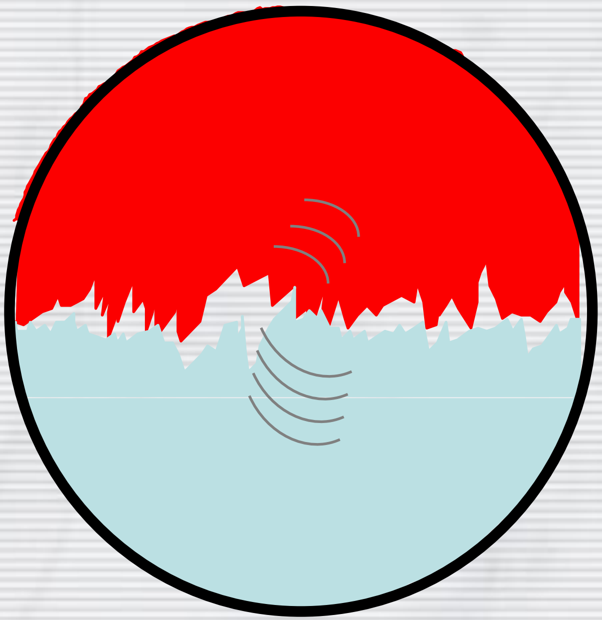
Work varies depending on the path. Energy is dissipated

The presence of a non-conservative force reduces the ability of a system to do work (*dissipative* force)

Energy dissipation: e.g. sliding friction

As the parts scrape by each other they start small-scale vibrations, which transfer energy into atomic motion

The atoms' vibrations go back and forth- they have energy, but no average momentum. The increased atomic vibrations appear to us as a rise in the temperature of the parts. The temperature of an object is related to the thermal energy it has. Friction transfers some energy into thermal energy



When there is NO work done by APPLIED FORCES , the total mechanical energy is constant or CONSEVED

If $W_a \neq 0$ ➡

$$K_2+U_2=K_1+U_1+W_a$$

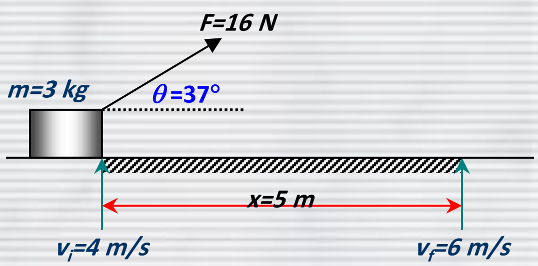
OR

$$\Delta K+\Delta U=W_{nc}$$

W_{nc}= work done by **ANY** other forces than gravitational force spring forces (e.g. any applied non-conservative force or frictional force)

سحبت كتلة مقدارها 3 kg على سطح أفقي خشن بواسطة قوة ثابتة مقدارها 16 N وتؤثر بزاوية مقدارها 37° ، فإذا زادت سرعة الكتلة من 4 m/s إلى 6 m/s خلال مسافة قدرها 5.0 m فيكون لشغل قوة الإحتكاك خلال هذه المسافة هو:

(a) –34 J (b) –64 J (c) –30 J (d) –94 J (e) +64 J

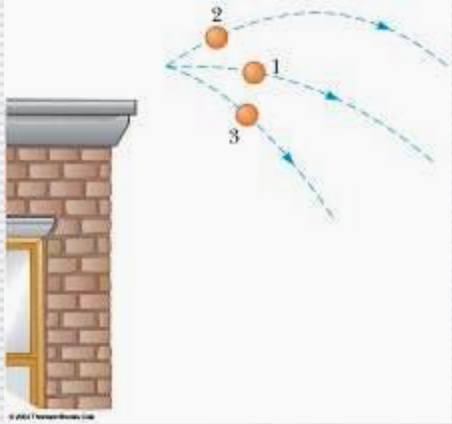


$$\Delta K + \Delta U = W_{nc}$$

$$(K_f - K_i) + 0 = W_{f_k} + (F\cos\theta)x$$

$$W_{f_k} = \frac{1}{2}m(v_f^2 - v_i^2) - (F\cos\theta)x = -33.9\ J$$

Three identical balls are thrown with the same initial speed from the top of a building.



Total Energy

$$E = K + U_g = \tfrac{1}{2}mv_0^2 + mgh$$

At y = 0

$$E = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0^2 + mgh$$

$$v = \sqrt{v_0^2 + 2gh}$$

$$v_0 = v_0\cos\theta\hat{i} + v_0\sin\theta\hat{j}$$

$$\hat{i}: v_x = v_0\cos\theta$$

$$\hat{j}: v_y = v_0\sin\theta - gt$$

$$y = h + v_0\sin\theta \cdot t - \tfrac{1}{2}gt^2 = 0$$

$$t = \frac{v_0\sin\theta + \sqrt{v_0^2\sin^2\theta + 2gh}}{g}$$

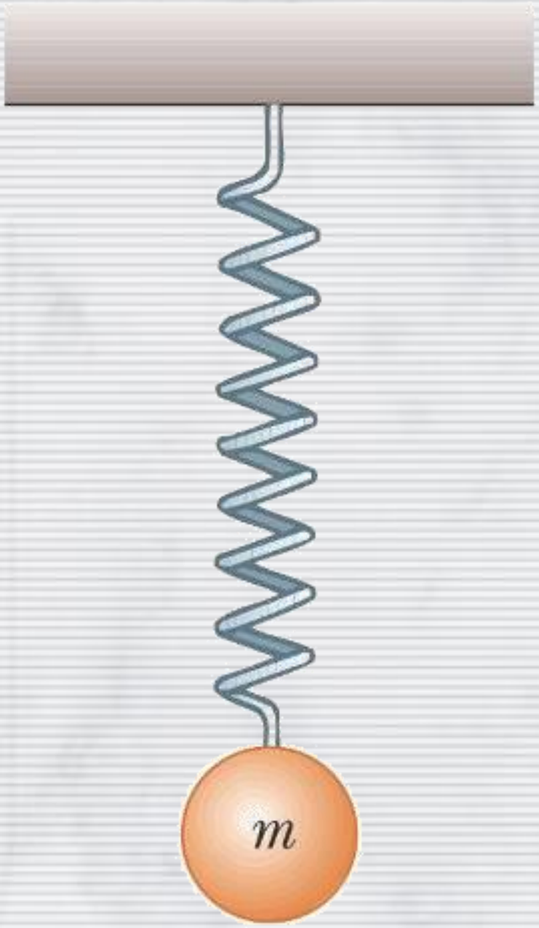
$$v_y = -\sqrt{v_0^2\sin^2\theta + 2gh}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2\sin^2\theta + 2gh + v_0^2\cos^2\theta}$$

$$= \sqrt{v_0^2 + 2gh}$$

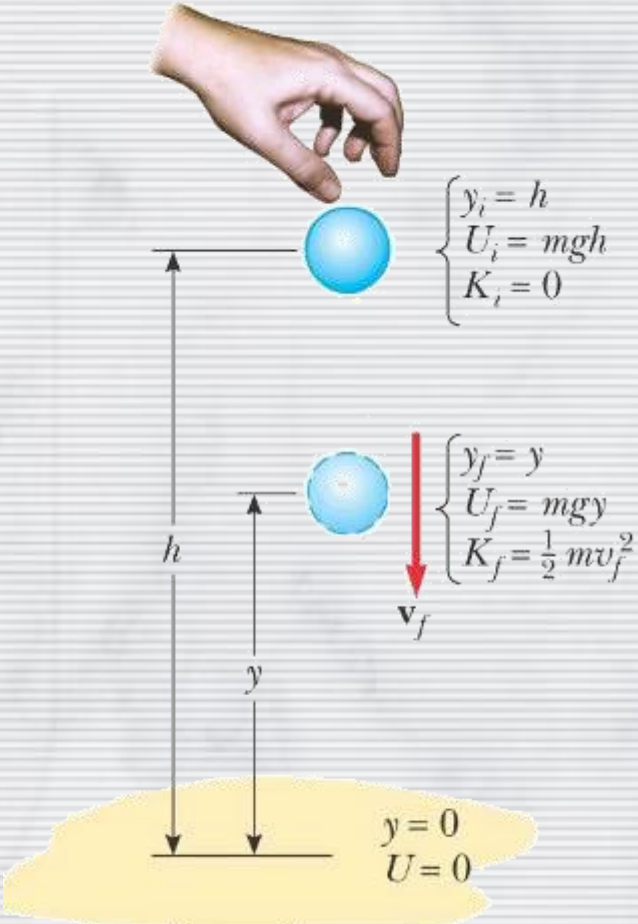
- READ Quick Quiz 8.7 & 8.8

A ball connected to a massless spring suspended vertically. What forms of potential energy are associated with the ball–spring–Earth system when the ball is displaced downward?



- READ Example 8.2

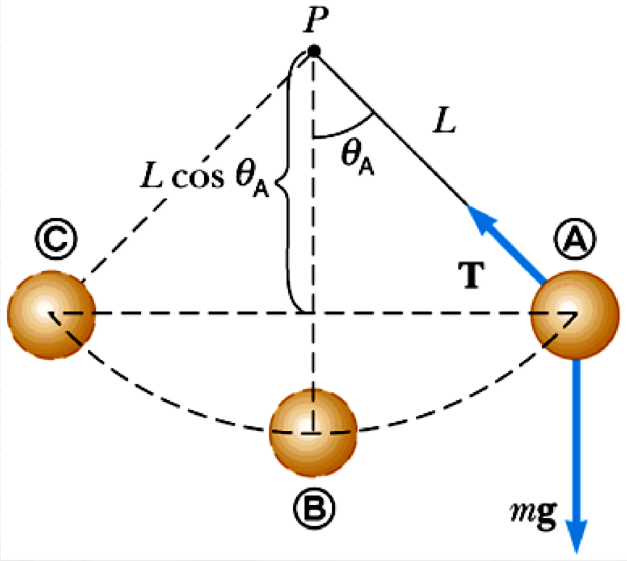
A ball is dropped from a height *h* above the ground. Initially, the total energy of the ball–Earth system is potential energy, equal to *mgh* relative to the ground. At the elevation *y*, the total energy is the sum of the kinetic and potential energies.



Example 8.3

Nose crusher?

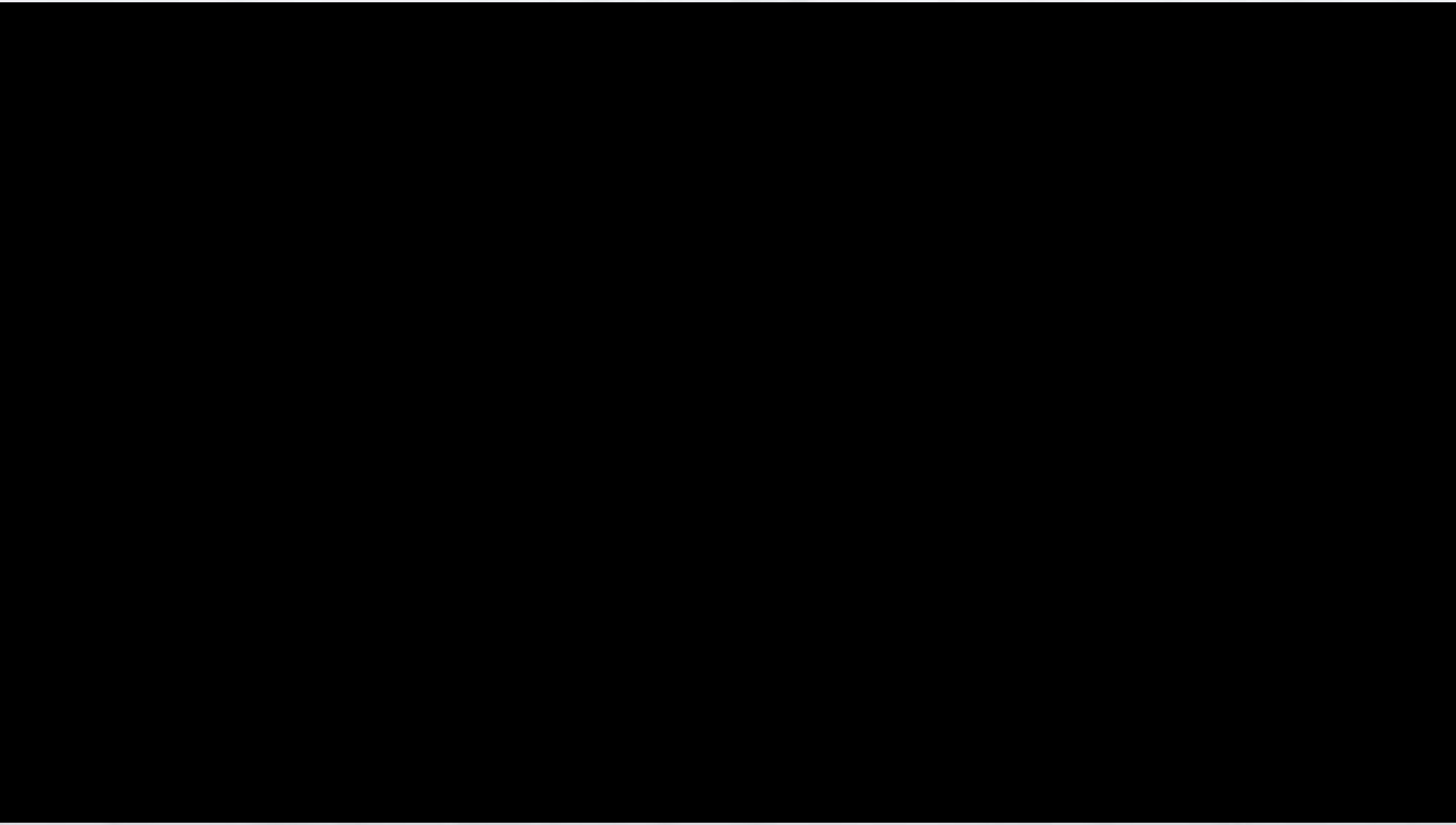
A bowling ball of mass m is suspended from the ceiling by a cord of length L. The ball is released from rest when the cord makes an angle θ_A with the vertical.



(a) Find the speed of the ball at the lowest point B.

(b) What is the tension T_B in the cord at point B?

(c) The ball swings back. Will it crush the operator's nose?




CONCEPTUAL
Physics
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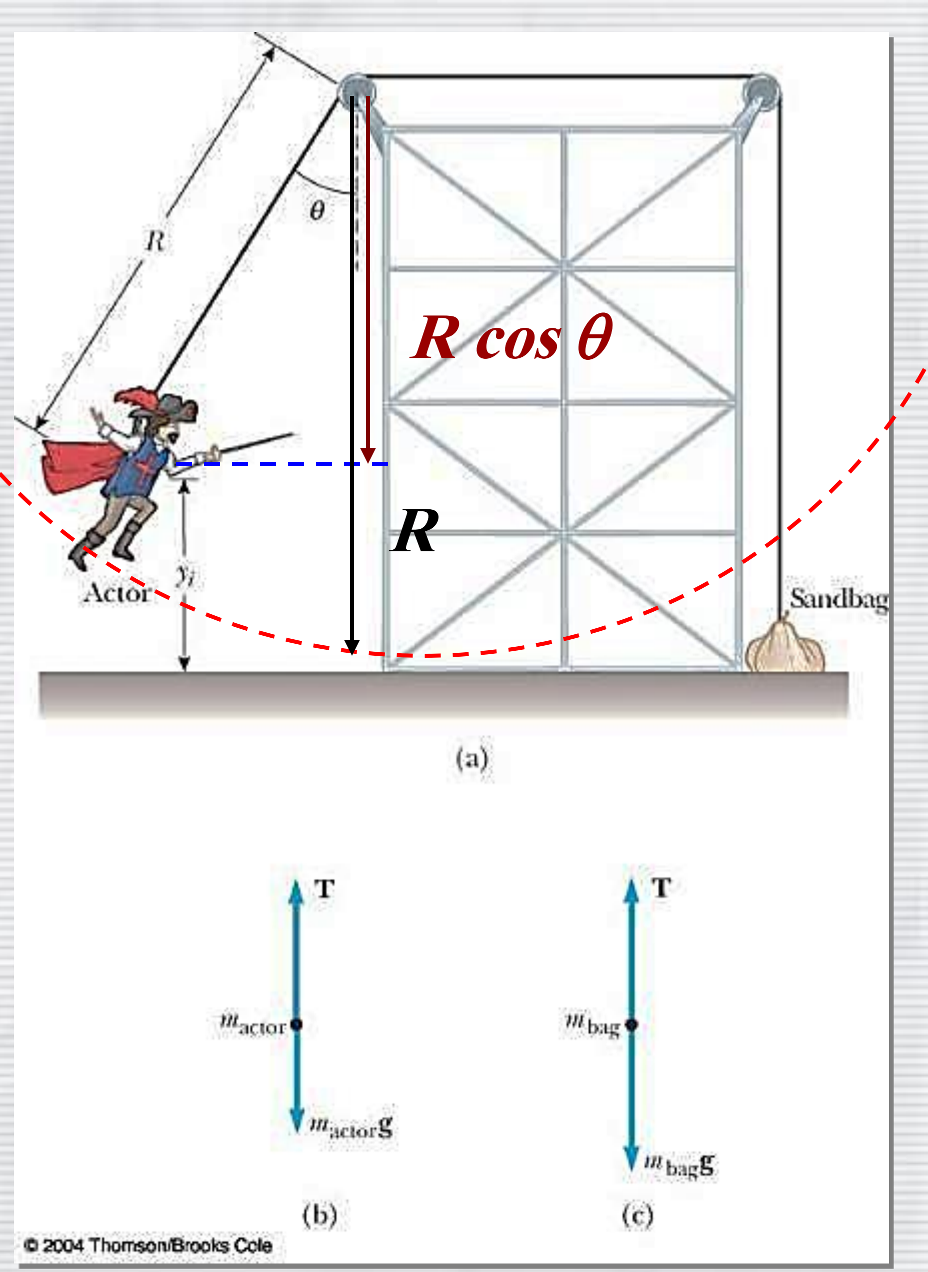
Example 8.4

(a) An actor uses some clever staging to make his entrance.

M_{actor} = 65 kg, M_{bag} = 130 kg, R = 3 m

What is the max. value of θ can have before sandbag lifts of the floor?



(b) Free-body diagram for actor at the bottom of the circular path. (c) Free-body diagram for sandbag.

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} M_{actor} v_f^2 + 0 = 0 + M_{actor} g y_i$$

$$y_i = R - R\cos\theta = R(1 - \cos\theta)$$

$$v_f^2 = 2gR(1-\cos\theta)$$

How we can obtain *v* ????

$$\sum F_y = T - M_{actor}\,g = M_{actor}\frac{v_f^2}{R}$$

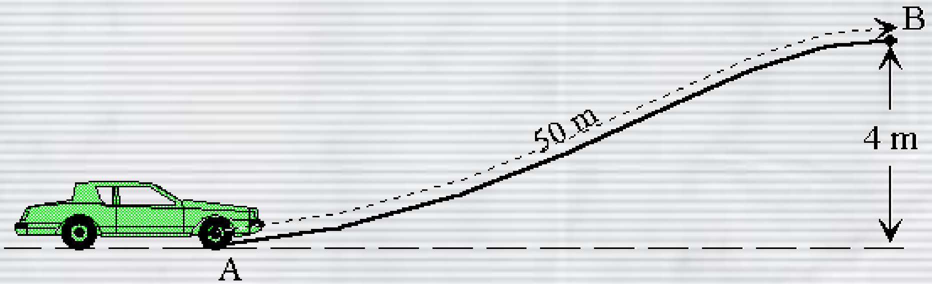
$$\Rightarrow \quad T = M_{actor}\,g + M_{actor}\frac{v_f^2}{R}$$

For the sandbag not to move ➡ $a=0$ ➡ $T=M_{bag}g$

θ =60°

EXAMPLE;

تعبر سيارة كتلتها 2500 *kg* النقطة A بسرعة مقدارها 10 *m/s* وعندما تصل الي النقطة B تصبح سرعتها 2 *m/s* ، احسب متوسط قوة الاحتكاك التي تعوق السيارة.



$$\Delta K + \Delta U = W_{nc}$$

$$\left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + \left(mgh_B - mgh_A\right) = W_{nc}$$

$$-120,000 + 98,000 = F_k.D$$

$$F_k = \frac{22,000}{50} = 440\ N$$

٨– سقطت كرة كتلتها 200 g من ارتفاع 4 m على أرض مستوية فارتدت الى ارتفاع 2.5 m ، الطاقة الحركية التي فقدت في الارتطام هي :

(a) 2.95 J (b) 4.90 J (c) 7.85 J (d) 12.70 J

٩– تبدا سيارة كتلتها 1500 kg في الحركة الي أسفل من قمة مرتفع بسرعة 30 m/s وتصل الى الأسفل ثم تصعد الى أعلى قمة مرتفع آخروتصل اليه بسرعة 20 m/s ، إذا كانت قمتي المرتفعين متساوية فإن الشغل المبذول بواسطة قوة الاحتكاك تساوي:

(a) 200,000 J (b) 400,000 J (c) 450,000 J (d) 375,000 J

١٠ – شاحنة كتلتها ثلاثة أضعاف كتلة سيارة وتتحرك ضعف سرعة السيارة ، إذا كانت K تمثل الطاقة الحركية للسيارة فإن الطاقة الحركية للشاحنة هي:

(a) K (b) 6K (c) 12K (d) 24K