

Math 106

Integral Calculus

Indefinite Integrals

Ibraheem Alolyan

King Saud University

About the course

Text Book:

Calculus by Swokowski, Olinick, Pence (Sixth Edition)

Course Topics:

- 1 Integrals
- 2 Transcendental functions
- 3 Techniques of integration
- 4 Applications of the definite integral.
- 5 Parametric equations and polar coordinates

Grading:

- First midterm 25
- Second midterm 25
- Quizzes 10
- Final 40

Indefinite Integral

- 1 Antiderivatives
- 2 Indefinite integrals
- 3 1st Method: Integration By Substitution

Antiderivatives

Antiderivatives

Definition

A function F is an antiderivative of f on an interval I if

$$F'(x) = f(x) \text{ for every } x \in I$$

Antiderivatives

Definition

A function F is an antiderivative of f on an interval I if

$$F'(x) = f(x) \text{ for every } x \in I$$

Example

$$F(x) = x^2$$

$$f(x) = 2x$$

Antiderivatives

Example

Find the antiderivatives of the following functions

① $f(x) = x^3$

② $f(x) = \sin x$

Antiderivatives

Theorem

If the functions F and G are antiderivatives of f on an interval I , then there exists a constant c , such that for all $x \in I$

$$G(x) = F(x) + c$$

Indefinite integrals

Indefinite integrals

Theorem

If $F'(x) = f(x)$ on the interval I and c is a constant, then the family of all antiderivatives of $f(x)$ is denoted by

$$\int f(x)dx = F(x) + c$$

$\int$ is the integral sign

$f(x)$ is the integrand

c is the constant of integration

Indefinite integrals

Example

Find the indefinite integral

① $\int x^3 dx$

② $\int u^{-2} du$

③ $\int \cos t dt$

Integration Rules (Integration Table)

Derivative Rule	Integral Rule
$\frac{d}{dx}(x) = 1$	$\int 1 dx = x + c$
$\frac{d}{dx}(x^r) = rx^{r-1}$	$\int x^r dx = \frac{x^{r+1}}{r+1} + c \quad (r \neq -1)$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x dx = \sin x + c$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x dx = -\cos x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + c$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + c$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + c$

Indefinite integrals

Example

Evaluate the the following integrals

① $\int \frac{1}{x^4} dx$

② $\int \sqrt[2]{x^3} dx$

③ $\int \frac{1}{\cos^2 x} dx$

④ $\int \frac{\tan x}{\sec x} dx$

Indefinite integrals

Theorem

$$\textcircled{1} \quad \int \frac{d}{dx}(f(x))dx = f(x) + c$$

$$\textcircled{2} \quad \frac{d}{dx} \left(\int f(x)dx \right) dx = f(x)$$

$$\textcircled{3} \quad \int k f(x) dx = k \int f(x) dx$$

$$\textcircled{4} \quad \int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

$$\textcircled{5} \quad \int (f(x) - g(x)) dx = \int f(x) dx - \int g(x) dx$$

Indefinite integrals

Example

Evaluate the the following integrals

$$\textcircled{1} \int \frac{d}{dx}(\cos x) dx$$

$$\textcircled{2} \frac{d}{dx} \left(\int \sqrt{2x+1} dx \right) dx$$

$$\textcircled{3} \int (2x^2 + \sec x \tan x) dx$$

$$\textcircled{4} \int \frac{(x^2 + 1)^2}{x^2} dx$$

Common Mistakes

Example

Evaluate the the following integrals

① $\int \tan x \, dx$

② $\int x \sin x \, dx$

③ $\int \sin^4 x \, dx$

Differential equations

Example

Solve the differential equation

$$f'(x) = 6x^2 - 2x + 3$$

subject to the initial condition $f(0) = 4$

1^{st} Method: Integration By Substitution

First Method: Integration By Substitution

Rules of Integrations

$$\int dx = x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + c \quad (r \neq -1)$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \csc^2 x \, dx = -\cot x + c$$

$$\int \csc x \cot x \, dx = -\csc x + c$$

Integration by Substitution

Theorem

If F is an antiderivative of f then

$$\int f(g(x))g'(x)dx = F(g(x)) + c$$

If $u = g(x)$ and $du = g'(x)dx$, then

$$\int f(u)du = F(u) + c$$

Integration by Substitution

Example

Evaluate $\int 3x^2(x^3 + 3)^4 dx$

Integration by Substitution

Example

Evaluate $\int \sqrt{3x - 4} \, dx$

Integration by Substitution

Example

Evaluate $\int \left(1 + \frac{1}{x}\right)^4 \left(\frac{1}{x^2}\right) dx$

Integration by Substitution

Example

Evaluate $\int \frac{\sin^2 x + \cos^2 x}{(2x + 1)^2} dx$

Integration by Substitution

Example

Evaluate $\int \cos 2x \, dx$

Integration by Substitution

Example

Evaluate $\int x \sqrt[3]{3 - 4x^2} dx$

Integration by Substitution

Example

Evaluate $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

Integration by Substitution

Example

Evaluate $\int \frac{x^2 - 2x}{(x^3 - 3x^2 + 1)^5} dx$

Integration by Substitution

Example

Evaluate $\int \cos^2 4x \sin 4x \, dx$

Integration by Substitution

Example

Evaluate $\int x\sqrt{x-1} dx$

Exam Problem

Exam problem

Evaluate $\int \frac{x}{(x+2)^3} dx$

Exam Problem

Exam problem

Evaluate $\int x^3 \cos x^4 (\sin x^4 + 1)^5 dx$

Exam Problem

Exam problem

Evaluate $\int \frac{\sin(\tan x)}{\cos^2 x} dx$

Exam Problem

Exam problem

Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$

Exam Problem

Exam problem

Use the substitution $u = x^3 + 2$ to evaluate $\int x^5 \sqrt{x^3 + 2} dx$