Math 106
Integral Calculus
Indefinite Integrals

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About the course

Text Book:

Calculus by Swokowski, Olinick, Pence (Sixth Edition)

Course Topics:

1. Integrals
2. Transcendental functions
3. Techniques of integration
4. Applications of the definite integral.
5. Parametric equations and polar coordinates

Grading:

- First midterm 25
- Second midterm 25
- Quizzes 10
- Final 40

Antiderivatives

Antiderivatives

Definition

A function F is an antiderivative of f on an interval I if

$$F'(x) = f(x) \text{ for every } x \in I$$

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Example

$$F(x) = x^2 \qquad\qquad f(x) = 2x$$

Antiderivatives

Example

Find the antiderivatives of the following functions

1. $f(x) = x^3$
2. $f(x) = \sin x$

Antiderivatives

Theorem

If the functions F and G are antiderivatives of f on an interval I, then there exists a constant c, such that for all $x \in I$

$$G(x) = F(x) + c$$

Indefinite integrals

Indefinite integrals

Theorem

If $F'(x) = f(x)$ on the interval I and c is a constant, then the family of all antiderivatives of $f(x)$ is denoted by

$$\int f(x)dx = F(x) + c$$

$\int$ is the integral sign
$f(x)$ is the integrand
c is the constant of integration

Example

Find the antiderivatives of the following functions

1. $\displaystyle\int x^3\,dx$
2. $\displaystyle\int u^{-2}\,du$
3. $\displaystyle\int \cos t\,dt$

Integration Rules

Derivative Rule	**Integral Rule**
$\frac{d}{dx}(x) = 1$	$\int 1\,dx = x + c$
$\frac{d}{dx}(x^r) = rx^{r-1}$	$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c \quad (r \neq -1)$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + c$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + c$

Example

Evaluate the the following integrals

1. $\displaystyle\int \frac{1}{x^4}\, dx$
2. $\displaystyle\int \sqrt[2]{x^3}\, dx$
3. $\displaystyle\int \frac{1}{\cos^2 x}\, dx$
4. $\displaystyle\int \frac{\tan x}{\sec x}\, dx$

Indefinite integrals

Theorem

1. $\int \frac{d}{dx}(f(x))dx = f(x) + c$
2. $\frac{d}{dx}\left(\int f(x)dx\right) dx = f(x)$
3. $\int kf(x)\,dx = k\int f(x)\,dx$
4. $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$
5. $\int (f(x) - g(x))\,dx = \int f(x)\,dx - \int g(x)\,dx$

Indefinite integrals

Example

Evaluate the the following integrals

1. $$\int \frac{d}{dx}(\cos x)\, dx$$
2. $$\frac{d}{dx}\left(\int \sqrt{2x+1}\, dx\right) dx$$
3. $$\int (2x^2 + \sec x \tan x)\, dx$$
4. $$\int \frac{(x^2+1)^2}{x^2}\, dx$$

Common Mistakes

Example

Evaluate the the following integrals

1. $$\int \tan x \, dx$$
2. $$\int x \sin x \, dx$$
3. $$\int \sin^4 x \, dx$$

Differential equations

Example

Solve the differential equation

$$f'(x) = 6x^2 - 2x + 3$$

subject to the initial condition $f(0) = 4$

1^{st} Method: Integration By Substitution

First Method: Integration By Substitution

Rules of Integrations

$$\int dx = x + c \qquad \int x^r \, dx = \frac{x^{r+1}}{r+1} + c \quad (r \neq -1)$$

$$\int \cos x \, dx = \sin x + c \qquad \int \sin x \, dx = -\cos x + c$$

$$\int \sec^2 x \, dx = \tan x + c \qquad \int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c \qquad \int \csc x \cot x \, dx = -\csc x + c$$

Integration by Substitution

Theorem

If F is an antiderivative of f then

$$\int f(g(x))g'(x)dx = F(g(x)) + c$$

If $u = g(x)$ and $du = g'(x)dx$, then

$$\int f(u)du = F(u) + c$$

Integration by Substitution

Example

Evaluate $\int 3x^2(x^3+3)^4\,dx$

Integration by Substitution

Example

Evaluate $\int \sqrt{3x-4}\,dx$

Integration by Substitution

Example

Evaluate $\int \left(1+\frac{1}{x}\right)^4 \left(\frac{1}{x^2}\right)\, dx$

Integration by Substitution

Example

Evaluate $\int \frac{\sin^2 x + \cos^2 x}{(2x+1)^2}\, dx$

Integration by Substitution

Example

Evaluate $\int \cos 2x \, dx$

Example

Evaluate $\int x \sqrt[3]{3 - 4x^2}\, dx$

Integration by Substitution

Example

Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Integration by Substitution

Example

Evaluate $\int \frac{x^2 - 2x}{(x^3 - 3x^2 + 1)^5}\, dx$

Example

Evaluate $\int \cos^2 4x \sin 4x \, dx$

Integration by Substitution

Example

Evaluate $\int x\sqrt{x-1}\,dx$

Exam Problem

Example

Evaluate $\displaystyle\int \frac{x}{(x+2)^3}\,dx$

Exam Problem

Example

Evaluate $\int x^3 \cos x^4 (\sin x^4 + 1)^5 \, dx$

Exam Problem

Example

Evaluate $\displaystyle\int \frac{\sin(\tan x)}{\cos^2 x}\, dx$

Exam Problem

Example

Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 \, dx$

Exam Problem

Example

Use the substitution $u = x^3 + 2$ to evaluate $\int x^5 \sqrt{x^3 + 2}\, dx$